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MATHEMATICAL INSTITUTIONS.

In Three PARTS.

I. *CLAVIS* KEY,
II. *JANUA* The GATE,
III. *ANCILLA* HAND-MAID,

TO THE

Mathematical Sciences.

WHEREIN,
The Doctrine of *Plain* and *Spherical* TRIANGLES, is Succinctly Handled, *Geometrically Demonstrated*, Arithmetically, Geometrically, Instrumentally Performed;

And Practically Apply'd to
GEOMETRY, SCIOGRAPHIA,
COSMOGRAPHY, NAVIGATION,
GEOGRAPHY, And THEORIES
ASTRONOMY, of the PLANETS.

By ***WILL. LEYBOURN***, Philom.

Ingredere ut Proficias.

LONDON,
Printed for ***Rob. Billing***, at the *White Swan* in St. *Paul's* Church-Yard.
and ***Geo. Briant***, at the *King's Head* near the *Royal-Exchange*
in *Cornhil*. MDCCIV.

TO THE
Young STUDENT
IN THE
MATHEMATICKS.

IN the *Study* (as well as in the *Teaching*) of any *Art* or *Science*, a Gradual Proceeding therein, ought Firſt, and Principally, to be conſidered: For, It is one thing to *Teach*; and another *How* to *Teach*: And, according to the Adage, *Qui bene diſtinguit, bene Docet.*

Now, the Deſign of this *Book*, being for the *Directing* and *Inſtructing* of ſuch as would apply themſelves to the Study of any *Mathematical Science*, ſo as to become ſuch a *Proficient* therein, as to give a *Demonſtrable Account* of what he does; muſt not attempt, at the firſt Onſet, to fall directly upon that *Part* he principally aims at, but firſt acquaint himſelf well with ſuch *Elements* as concern, and are ſubſervient to, them all.

And to that end, The ſeveral *Parts* and *Sections* of this *Book*, are diſpoſed in ſuch *Order*; ~~That, Beginning with the~~ *Firſt* Part (which is as a KEY to give you entrance into the GATE; which is the *Second Part*; Through which, having perfectly paſſed, you may confidently proceed to fall upon the *Study* (or *Practice*) of any *Mathematical Science*; And not only on thoſe that I have here (by my HAND-MAID, in the *Third Part*) directed you to; but to any other you ſhall attempt: TRIGONOMETRIA being the *Baſis*, *Foundation*; nay, the very *Primum Mobile*, of all the reſt.

For Illuſtration whereof, I have applyed the *Doctrine* of *Triangles* to *Practice* (in the ſeveral *Sections* of the *Third Part*) in the Solution of ſuch Problems, as are of frequent uſe in ſeveral Parts of the Mathematicks. So in the Firſt, Which teacheth how to take all manner of *Heights* and *Diſtances*, acceſſible or inacceſſible: And in the Sixth, which treateth of *Navigation* both by the *Plain Sea-Chart*, and that called *Mercators*; in both which, I have firſt ſhewed how *Geometrically*, by *Scale* and *Compaſſes*, to lay down (upon Paper) a *Figure* anſwerable to the *Queſtion* Propounded: In which *Figure*, ſo laid down, you will have conſtituted a *Right-Lined Triangle* or *Triangles*, of ſome kind or other, *i. e.*

either *Right* or *Oblique-Angled*; And in it diſcovered what Parts thereof are Given, by the demand in the *Queſtion*; and then the *Part* or *Parts Unknown* will be ſuch (as being found) will anſwer the *Queſtion* demanded: And herein the Excellency of the *Doctrine* of *Plain Triangles* is in part made manifeſt.

Likewiſe, In the 1*ſt.* 2*d.* 3*d.* 4*th.* and 5*th.* Sections, which treateth of *Coſmography*; and therein of *Geography*, *Aſtronomy* and *Dialling*: I do firſt declare, and ſhew how the Operation of the *Queſtion* or *Proportion*, is to be performed upon the *Terreſtrial* or *Celeſtial Globe* (according as the thing propoſed does require.) Which *Queſtion* being reſolved upon the *Globe*; and the *Globe* reſting in the ſame *Poſition* it was when the *Queſtion* was reſolved thereupon: You will upon the *Body* of the *Globe*, (by the great *Circles* thereon deſcribed, and thoſe appendant to it) apparently diſcover a *Spherical Triangle* or *Triangles* Conſtituted; in which (according to the Tenor of the *Queſtion*) you will plainly perceive what *Parts* thereof are *Given*; and then, the other *Parts* being found, muſt neceſſarily Anſwer the *Queſtion* Propounded: And herein, in ſome meaſure, is the *Doctrine* of *Spherical Triangles* made applicable to all the above-named Practices. And Moreover, by Projecting of the Sphere, as is done for the *Geographical Problems* in *Part* III. *Section* the *Second*. And for *Aſtronomical Problems*, as is ſhewed in PART II. Section II. Of Projection of the Sphere in Plano, *&c.*

Alſo in the VIIth. *Section*, Which treats of the *Theories*, and finding the *Places* of the *Planets*, *&c.* there the *Doctrine* of *Triangles* both *Plain* and *Spherical*, are joyntly concerned: So that, let your *Mathematical Practice* or *Study* be, *Opticks*, *Perſpective*, *Fortification*, *Gunnery*, or any other *Mathematical Art*: The moſt *Problems* relating to any of them, muſt be beholding to *Trigonometry* for their Solutions.

And now I have but one thing more to advertiſe the Reader of: That, Whereas in the performances of all the *Operations* in the ſeveral *Parts* of this Book, the Firſt (which treats of *Practical Geometry* only) excepted, there is continual Uſe to be made of a CANON, TRIGONOMETRICAL; or TABLES of Artificial SINES, TANGENTS and LOGARITHMS; So that the Reader cannot expect but that ſuch TABLES ſhould have been *Added* unto theſe *Precepts*. And, indeed, it was ſo intended, but that almoſt in every Book that hath any thing of the *Menſuration* of *Triangles* in it, there are ſuch *Tables Printed*; and that one or other of them may be in every Man's Hands; and becauſe all the *Problems* in this Book may be ſolved by (almoſt) any of thoſe *Canons* or *Tables*; was the occaſion of the omiſſion of ſuch Tables here at preſent. But chiefly, becauſe there is intended (in ſome ſhort time) to be Printed,) A CANON of Artificial SINES and TANGENTS, both **Sexagenary** and **Centecimal**; differing from any yet Extant; it ſupplying both ways of *Numeration* at one *view*, and yet in the ſame *Room*: And to this CANON ſhall be Added Ten CHILIADS of LOGARITHMS commodiouſly Contracted; and the Uſe thereof in Logarithmical Arithmetick.

Thus having given the Reader a *General Account* in this *Preface*, I refer him to a more *Particular* in the following *Contents*, and ſo leave him to the Practice of them: In which I wiſh him good ſuceeſs; and till he hears farther from me, bid him Farewell.

WILL. LEYBOURN.

ERRATA.

Page	Line	For	Read
9	20	the given Point	the given Point P.
11	16	fame	ſame
13	28	from N to O,	from M to O,
20	19	*M ans* 8 is	*Means* 8 in 8 is
21	5	to D E.	to A E.
52	14	E C, intercepted	E F, intercepted
52	16	E and D ;	E and F ;
52	17	*Arch* E D is ;	Arch E F is ;
59	30	The *Angle* B P Q	The *Angle* P B G
63	1	*Diſtinct*	*Disjunct.*
69	17	contrived :	continued :
95	29	½ Sun < s::	½ Sum < s:
110	37	Meaſure is ⊙ O,	Meaſure is Q æ,
111	15	T æ,	Q æ
111	16	and T,	and Q
142	23	of R, S, P, T.)	of R, A, P, T.)
144	29	draw a Line L,	draw a Line L *b*,
ibid	*ib.*	alſo b, an	alſo an
148	21	*Angle* at F,	*Angle* at D,
151	14	From A G to	From G to
164	1	of the	The
196	*ult*	Plain Triangles	Spherical Triangles
279	23	*Point* B :	*Point* in B :
281	22	Hour	Hours
282	2	26 de.	36 de.
282	16	23.25	23.35
287	2	Declination	Reclination
ibid	21	Reclination 120	Reclination 20
297	8	is to be moſt	is moſt
308	37	*Problems* and	Problems in
327	*ult*	6.50 deg.	5.50 deg.
328	32	M P O of A B,	M P of A B,
330	2	6.50 de.	5.50 de.
347	8	B T E.	B F E.

Figures in the Margins Omitted, or Miſplaced.

Page 7. Againſt *Probl.* I. put *Fig.* VIII. and againſt *Prob.* II. put *Fig.* IX. in the Margin.

Page 9. Againſt *Probl.* V. put *Fig.* XIII. in the Margin.

Page 20. For *Fig.* XXXIV. put *Fig.* XXXVII. in the Margin.

Page 50. For *Fig.* VI. put *Fig.* XII. in the Margin.

Page 89, 90. 91. *Fig.* XXXI. in the Margin, muſt be *Fig.* XXXII.

Page 274. *Fig.* LIII. in the Margin, is omitted

Page 285, and 286. for *Fig.* LIX. read *Fig.* LVIII. in the Margin.

Page 53. The three laſt Lines are thus to be read :

$$\text{The Triangle}\left\{\begin{array}{l}\text{B A C}\\ \text{B D C}\\ \text{C D E}\end{array}\right\}\text{hath}\left\{\begin{array}{l}\text{One Right Angle, and Two Acute.}\\ \text{Two Obtuse, and One Right.}\\ \text{One Obtuse, and Two Acute.}\end{array}\right.$$

CON-

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CLAVIS MATHEMATICÆ.
THE KEY TO THE Mathematical Sciences.

PART I.

OF Practical Geometry.

THIS Firſt Part conſiſts only of ſuch DEFINITIONS, PROBLEMS and THEOREMS, GEOMETRICAL, which of Neceſſity ought to be underſtood and practiced, before farther Progreſs be made in any other Mathematical Science: And ſo I will begin It with theſe following

Geometrical Definitions.

I. *A* Point *is that which hath no Part.*

That is, it hath no Parts into which it may be divided; It being the leaſt thing that by Mind and Underſtanding can be imagined or conceived; than which there can be nothing leſs: As the *Point* in the Margin, noted with the Letter A over it: It being neither *Quantity*, nor any Part of *Quantity*; but only the *Term* or *End* of *Quantity*.

II. *A* Line *is a Length without Breadth; as the Line* A B.

A————B

Unto *Quantity* there appertain Three *Dimenſions*; viz. *Length*, *Breadth* and *Depth* (or *Thickneſs*;) of which, a *Line* is the firſt, and hath *Length* only, without *Breadth* or *Thickneſs*; as the *Line* C D,

C|————E|———F|———|D

which may be divided into *Parts*; either *Equally*, in the Point E; or *Unequally*, in the Point F.

III. *The* Ends, *or* Limits, *of a* Line, *are* Points G————H.

For a *Line* hath its beginning from a *Point*, and likewiſe endeth in a *Point*: So the *Points* G and H are the *Ends* of the *Line* G H, and are no *Parts* of it.

IV. *A* Right Line *is that which lyeth* Equally *between its* Points.

Or, it is the *Shorteſt Diſtance* that can be drawn between *Point* and *Point*; ſo the *Right Line* G H, is the *Shorteſt Diſtance* between the *Points* G and H.

V. *Parallel (or Equidiſtant)* Right Lines *are ſuch, which being drawn upon the ſame Plain, and infinitely produced, would never meet.*

And ſuch are theſe Two *Lines*, A B and C D.

A————B
C————D

VI. *A* Plain Angle *is the Inclination (or Bowing) of Two* Right Lines, *one to the other, and the one touching the other; and not being directly joined together.*

Fig. I. So the Two *Lines* A B and B C incline one to the other, and touch each other in the Point B; in which Point (by reaſon of the *Inclination* of the ſaid Two *Lines*) is made the *Angle* A B C; But if the Two *Lines* which incline one to the other, do (when they meet) make one *Streight Line*, then do they make no *Angle* at all: As the *Lines* D E and E F incline one to the other, and meet each other in the Point E, and yet they make no *Angle*.

And here *Note*, That an *Angle* (generally) is noted with *Three Letters*, of which, the middlemoſt Letter repreſents the *Angular Point*; ſo, in this *Angle* A B C, the Letter B denotes the *Angular Point*. —And of *Angles* there are Three Kinds; *viz. Right*, *Acute* and *Obtuſe*.

VII. *When a* Right Line *ſtanding upon a* Right Line *maketh the* Angles *on either Side thereof* Equal, *then either of thoſe* Angles *is a* Right Angle; *and the* Right Line *which ſtandeth erected is called a* Perpendicular Line *to that* Line *upon which it ſtandeth.*

So upon the *Right Line* C D, ſuppoſe there do ſtand another *Right Line* A B, in ſuch ſort, that it maketh the *Angles* A B C and A B D (on either ſide of the *Line* A B) equal; then are either of thoſe *Angles*, A B C and A B D, *Right Angles*; and the *Line* A B, which ſtandeth erected upon the *Line* C D, (without inclining on either ſide) is a *Perpendicular* to the *Line* C D. *Fig.* II.

VIII. *An* Obtuſe Angle *is that which is* Greater *than a* Right Angle:

So the *Angle* C B E is an *Obtuſe Angle*, it being greater than the *Right Angle* A B C, by the Quantity of the Angle A B E.

IX. *An* Acute Angle *is that which is* Leſs *than a* Right Angle.

So the Angle E B D is an *Acute Angle*, it being *Leſs* than the *Right Angle* A B D, by the Quantity of the Angle A B E.

X. *A* Limit *or* Term *is the End of any thing.*

Foraſmuch as there is no *Quantity* (or *Magnitude*) of which *Geometry* treateth, but it hath *Bounds* or *Limits*: And as *Points* are the *Bounds* or *Limits* of *Lines*, ſo *Lines* are the *Bounds* or *Limits* of *Plains* or *Superficies*; and *Plains* (or *Superficies*) of *Solids* (or *Bodies*.)

XI. *A* Figure *is that which is contained under* One Term, *or* Limit; *or* Many.

So A is a Figure contained under one *Line* or *Limit*: B is a Figure under Three *Lines* or *Limits*: C under *Four*: D under *Five*, &c. which are their reſpective *Bounds* or *Limits*. *Fig.* III.

XII. *A* Circle *is a* Plain Figure *contained under* One Line, *which is called a* Circumference *or* Periferie; *unto which all* Right Lines *drawn from one certain* Point *within the* Figure *unto the* Circumference, *are equal one to the other.*

Fig. IV. So the Figure B C D contained under One crooked Line, is a *Circle*, whoſe *Circumference* or *Periferie* is B C D. In the middle whereof there is a *Point* A, from which all the *Right Lines*, A B, A C, A D, being drawn to the *Circumference* B C D, are *Equal.* And that *Point* A is called the *Centre* of the *Circle* B C D.

XIII. *The* Diameter *of a* Circle *is any* Right Line *drawn through the* Centre, *and ending at the* Circumference *on either Side, dividing the* Circle *into Two* Equal Parts.

Fig. V. So the Line E K F is a *Diameter*, becauſe it paſſeth from the Point E of the *Circumference* on the one Side, to the Point F on the other Side; and paſſeth alſo by the Point K, which is the *Centre* of the *Circle:* And moreover, it divideth the *Circle* into Two equal *Parts*, *viz.* into the Part E G F above, and E H F below, the *Diameter*; which Two *Parts* are termed *Semicircles.*

XIV. *A* Section, Segment *or* Portion, *of a* Circle, *is a Figure contained under one* Right Line, *and a* Part *of the* Circumference; Greater *or* Leſſer *than a* Semicircle.

So the *Right Line* L M divideth the *Circle* E G F M H L into Two unequal *Sections*; namely, into the *Section* L G M above, *Greater*, and the *Section* L H M below, *Leſſer*, than a *Semicircle.*

XV. *The* Semidiameter *of a* Circle, *is half of the* Diameter *of that* Circle.

So K E or K F are *Semidiameters* of the *Circle* E G F H: And ſo is any *Right Line* drawn from K the *Center* to the *Circumference*; which Lines are frequently called the *Radius* of the *Circle.*

XVI. Right Lined Figures, *are ſuch* Figures *as are contained under* Right Lines; *of which none can conſiſt of leſs than* Three; *and thoſe are called* Trilaterals, *or* Triangles.

And *Triangles* are denominated partly from the Differences of their *Sides*, and partly from the Differences of their *Angles*: As for the *Differences* of their *Sides*, they may be, (1.) All *Equal*, and ſuch a *Triangle* is called an *Equilateral Triangle*, as the Figure N. Or, (2.) *Two Sides* may be *Equal*, and the third unequal; and ſuch a *Triangle* is called an *Iſoſceles Triangle*, as the the Figure O. Or, (3.) All the *Three Sides* may be unequal, and ſuch a *Triangle* is called a *Schalenum Triangle*, as the Figure P. And theſe are the *Diſtinctions* in relation to their *Sides*. *Fig.* VI.

Now for the Diſtinction of *Triangles* in relation to their *Angles*, they are Three alſo: For, (1.) If a *Triangle* have *One Right Angle*, it will have Two *Acute* ones, as the Figure Q, where the *Angle* at A is a *Right Angle*, and the *Angles* at B and C are *Acute*, and ſuch a *Triangle* is called *Orthogonium*. Or, (2.) If the *Triangle* have all the *Angles Acute*, as the Figure R; all whoſe *Angles* at D E and F are *Acute*; ſuch a *Triangle* is called an *Oxogonium Triangle*. Or, (3.) If the *Triangle* have *One Obtuſe Angle*, as the Figure S, whoſe *Angle* at I is *Obtuſe*, and the other Two at G and H are *Acute*; ſuch a *Triangle* is called *Ambligonium*. And theſe are the *Denominations* in relation to their *Angles*.

XVII. *Of* Four-ſided Figures *(or* Quadrilaterals*) a* Quadrat, *or* Square, *is that whoſe* Sides *are* Equal, *and* Angles Right.

And as *Triangles* have their various *Denominations* from the Species of their *Sides* and *Angles*, ſo have *Quadrilateral Figures* alſo. For, (1.) If all the *Sides* be *Equal*, and all the *Angles Right Angles*, as the Figure O, ſuch a *Figure* is called a *Quadrat* or *Square*. But, (2.) If of the Four Sides, Two be longer than the other, each to its Correſpondent (or Oppoſite) and the *Angles* all *Right Angles*, as the Figure P; ſuch a *Figure* is called a *Parallelogram*, or *Long Square*, (and ſometimes) a *Rectangle*. But, (3.) If all the *Sides* be *Equal*, but the *Angles Unequal*, that is, *Two Acute*, and *Two Obtuſe*, each to his Correſpondent (as the Figure R) ſuch *Figure* is called a *Rhombus* or *Diamond Form*. And farther, (4.) If ſuch a *Figure* have *Two Sides Longer*, and *Two Sides Shorter*; and alſo *Two Angles Acute*, and *Two Obtuſe*, *Fig.* VII.

each to it oppoſite correſponding, as the Figure Q; ſuch a *Figure* is called a *Rhomboyades*, or Diamond-like Figure. But, (5.) If *Quadrilateral Figures* have all their *Sides*, and all their *Angles*, *Unequal*, as the Figures S and T; ſuch *Figures* are called *Trapezia*, or Table Forms.

PRA-

PRACTICAL PROBLEMS, GEOMETRICAL.

PROBLEM I.

To divide a Right Line A B, *into Two* Equal Parts A E *and* B E, *and at* Right Angles.

Practice. FIrſt, Open your Compaſſes to any Diſtance greater than half the Length of the given Line A B.

2. With that Diſtance ſet one Foot in A, and with the other deſcribe the obſcure Arch *b c*,—and (with the ſame Diſtance) One Foot ſet in B, with the other deſcribe the obſcure Arch *d e*, croſſing the former Arch in the Points C and D.

3. Through the Points C and D, draw a Right Line C D, which will divide the given Line AB into Two *Equal Parts* in the Point E, and at Right Angles. The *Angle* A E C on one Side thereof, being equal to the *Angle* C E B on the other Side.

PROB. I.

Upon any Point *(as* O*) taken in the* Right Line Q R, *to erect a* Perpendicular O S.

Practice. FIrſt, Open the Compaſſes to any ſmall Diſtance; and ſetting one Foot in the given Point O, with the other Foot make Marks on either Side of O, as at T and V.

2. Open the Compaſſes to any Diſtance, greater than the former; and ſetting one Foot in T, with the other deſcribe the Arch *b b*. —Alſo, with the ſame Diſtance, ſet one Foot in V, and with the other deſcribe the Arch *g g*, croſſing the former Arch *b b* in the Point S. *Fig.* VIII.

3. Draw the Line O S, and it will be *Perpendicular* to the given Line Q R.

PROB. III.

From the End *of a* Line X Z, *to erect a* Perpendicular Z A.

Fig. X. *Practice.* First, The Compaſſes being opened to any ſmall Diſtance, ſet one Foot in Z, and with the other deſcribe the Arch B C D, and upon it ſet the ſame Diſtance from B to C, and from C to D.

2. The Compaſſes ſtill continuing at the ſame Diſtance, ſet one Foot in C, and with the other deſcribe the Arch F D: Alſo ſet one Foot in D, and with the other deſcribe the Arch C E, cutting the Arch D F in A.

3. From Z draw the Line Z A, which will be *Perpendicular* to the Line X Z.

PROB. IV.

Another Way to erect a Perpendicular *upon the* End *of a* Line.

Fig. XI. *Practice.* First, With any ſmall Diſtance of the Compaſſes ſet one Foot in H, and with the other deſcribe the Arch I K, and ſet that ſame Diſtance from I to K.

2. Upon K (with the ſame Diſtance) deſcribe the Arch I L M N.

3. Upon this Arch ſet the ſame Diſtance from I to L, from L to M, and from M to N.

4. A Line drawn from H, through N, ſhall be *Perpendicular* to the Line G H.

A Third Way.

Fig. XII. 1. THE Compaſſes opened to any ſmall Diſtance, ſet one Foot in O, and pitch the other Foot down at Pleaſure, as at Q; and one Foot reſting upon Q, turn the other Foot about, till it croſs the Line P O in S, and alſo deſcribe the ſmall Arch *b a*.

2. Lay a Ruler from S to Q, and it will cut the Arch *b a* in R.

3. A Line drawn from O, through R, ſhall be a *Perpendicular* to P O.

PROB. V.

From a given Point *above, as* P, *to let a* Perpendicular *fall upon a given* Right Line *under it,* N O.

IN the Performance there are Two Varieties: For, (1.) The given Point may be ſcituate over (or about) the Middle; or over (or near) the End of the given Line

In the Firſt Caſe.

Practice. LET N O be the *Right Line* given; and let P be the *Point* above, from whence the *Perpendicular* is to be let fall.

1. Open the Compaſſes to a Diſtance greater than is the neareſt Diſtance between the Point above P, and the Line upon which the *Perpendicular* is to fall; and with that Diſtance, ſetting one Foot in P, with the other deſcribe the Arch of a Circle, which will cut the given Line in Two Points, R and S.

2. Divide the Space between R and S (by the firſt Probl.) into Two equal Parts in the Point Q, then will a Right Line, drawn from P to Q, be *Perpendicular* to the given Line N O. *Fig.*XIII.

Note, To avoid the dividing of the Space between R and S into Two equal Parts, to find the Point Q, (if you have room either above or below the given Line) you may ſet one Foot of the Compaſſes in S, and opening the other to any convenient Diſtance, deſcribe an Arch *y y*; and removing the Compaſſes to R, deſcribe the Arch *z z*, croſſing the former in A; and ſo, a Line drawn from A, through the given Point, ſhall be *Perpendicular* to the given Line N O.

In the Second Caſe.

Let V be the Point given, *Fig.*XIV.

Practice. FIrſt, From any Part of the given Line N O, as from T, draw a Right Line to the given Point V, which divide into Two equal Parts in X.

2. Set one Foot of the Compaſſes in X; and with the Diſtance X T, deſcribe the Semicircle V O T, cutting the given Line in O, ſo ſhall O be the Point in the Line N O; from which, if you draw a Line to V, it will be *Perpendicular* to N O.

PROB. VI.

Unto a given Right Line P Q, *to draw another* Right Line S T, *which ſhall be* Parallel *to it*; *and at the* Diſtance *of the given* Line M.

Fig. XV. *Practice.* First, Take in your Compaſſes the Length of the given Line of Diſtance M.

2. Set one Foot in O, (near to one end of the give Line P Q,) and with the other deſcribe the Arch *a a*: Alſo, upon the Point L, (near the other end of the given Line) deſcribe the Arch *b b*; then,

3. By the Convexity (or Tops) of thoſe Two Arches, if you draw a *Right Line* S T, it will be *Parallel* to the given Line P Q, and at the *Diſtance* of the *Line* M.

PROB. VII.

To a given Line V W, *to draw another* Right Line Y A, *which ſhall paſs through a given Point* R, *and* Parallel *to* V W.

Fig. XVI. *Practice.* First, Set one Foot of the Compaſſes in the given Point R, and with the other deſcribe the Arch *c c*, ſo that it may only touch the given *Line* V W; and with the ſame Diſtance, ſet one Foot in any Point of the *Line* V W, as at X, and with the other deſcribe the Arch *d d*.

2. A Line drawn through the given Point R, by the Convexity of the Arch *d d*, as the Line Y R A, that Line ſhall be a *Parallel* to the *Line* V W.

PROB. VIII.

A ſecond Way to draw a Line Parallel *to a given* Line A B, *which ſhall paſs through a given Point* P.

Fig. XVII. *Practice.* First, With the Diſtance P A, upon the Point B, deſcribe the Arch *e e*.

2. With the Diſtance A B, upon P, deſcribe the Arch *f f*, croſſing the Arch *e e* in the Point C.

3. A *Right Line* drawn from the given Point P, through C, ſhall be *Parallel* to the given Line A B.

PROB. IX.

To make an Angle D F E, *equal to a given* Angle A B C.

Practice. First, Upon the Angular Point B, at any Diſtance, deſcribe an Arch *g g*. Then, *Fig.* XVIII.

2. Having drawn another Line, as F E, upon the end F (with the ſame Diſtance) deſcribe the Arch *h h*.

3. Take the Diſtance *g g* in your Compaſſes, and ſet it from *h* to *h*: Then,

4. A Line drawn from F through *h*, as F *h* D, ſhall make the *Angle* D F E equal to the *Angle* A B C.

PROB. X.

To divide an Angle G H K *into Two* Equal Parts.

Practice. First, Upon the Angular Point H (with any Diſtance of the Compaſſes) deſcribe an Arch cutting the Two Sides, containing the Angle, in the Points *h* and *s*. *Fig.* XIX.

2. The Compaſſes opened to the ſame (or any other) Diſtance, ſet one Foot in *h*, and with the other deſcribe the Arch *k k*; and (with the ſame Diſtance) one Foot ſet in *s*, deſcribe the Arch *i i*, croſſing *k k* in the Point L.

3. Join H L, and ſo is the *Angle* G H K divided into Two *Equal Parts*, by the Line H L.

PROB. XI.

How to divide a given Right Line M N, *into any Number of* Equal Parts: *Suppoſe* Five.

Practice. First, From one end of the given Line, as N, draw the Line N O at Pleaſure, making the Angle O N M. *Fig.* XX.

2. Upon the Point M (by Prob. IX.) make the Angle N M P equal to the Angle O N M; or (by Prob. VIII.) through the Point M, draw the Line M P Parallel to the Line O N.

3. With any ſmall Diſtance of the Compaſſes, one Foot being ſet in N, run over *Four* of thoſe Diſtances upon the Line N O, at the Points 1, 2, 3, 4.—Do the like upon the Line M P: Then,

4. If you draw the obſcure Lines 1 4, 2 3, 3 2, and 4 1, they will croſs the given Line M N in the Points *d*, *c*, *b*, *a*, dividing it into *Five* Equal Parts, as was required.

PROB. XII.

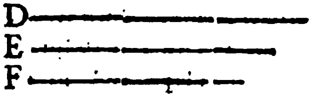
To make an Equilateral Triangle A B C, *whoſe* Sides *ſhall be Equal to a given* Line O.

Fig. XXI. *Practice.* MAke the Side B C equal to the given Line O, and with the ſame Diſtance of the Compaſſes, ſetting one Foot in B, with the other deſcribe the Arch *m m*; and ſet alſo on C, deſcribe the Arch *l l*, croſſing the former Arches in A: Then join A B and A C, ſo have you conſtituted a *Triangle* A B C, whoſe Three Sides are ſeverally equal to the given Line O.

PROB. XIII.

To make a Triangle G H K, *whoſe* Three Sides *ſhall be Equal to the* Three *given* Right Lines D, E, F.

Fig. XXII. *Practice.* FIrſt, Make the Line G H equal to the given Line D.



2. Take the given Line E in your Compaſſes, and ſetting one Foot in H, with the other deſcribe the Arch *n n*.

2. Take the Line F in your Compaſſes; and ſetting one Foot in G, with the other deſcribe an Arch *o o*, croſſing the former Arch in the Point K.

4. Join G K and H K; ſo ſhall you have conſtituted a *Triangle*; whoſe Three Sides G H, H K, and G K, are equal to the Three given Lines D E and F.

PROB. XIV.

To make a Geometrical Square B C D E, *whoſe Sides ſhall be equal to the given* Line A.

Fig. XXIII. *Practice.* FIrſt, Make the Line B C for one Side of the *Square*, equal to the given Line A, and on one End thereof, as

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Fig. XXIII. P

ts on C (by Prob. IV.) erect the Perpendicular C D, equal alſo to he Line A.

2. With the ſame Diſtance of A, ſet one Foot in B, and with he other deſcribe the Arch *a a*, and on D, and deſcribe the Arch *b b*, croſſing *a a* in the Point E.

3. Join B E and D E, which will conſtitute the *Geometrical Square* B C D E.

PROB. XV.

To make a Parallelogram *(or* Long Square*)* G H I K, *whoſe* Length *and* Breadth *ſhall be equal to Two given* Lines E *and* F.

Practice. First, Make the Line G H equal to the given Line F, and on the end H, (by Prob. IV.) erect the Perpendicular H I, equal to the given Line E. *Fig.* XXIV.

2. With the Diſtance of the Line F, ſet one Foot in G, and with the other deſcribe the Arch *c c*; and with the Diſtance of the Line F, with one Foot in I, deſcribe the Arch *d d*, croſſing *c* in K.

3. Join I K and G K, and ſo have you formed the *Parallelogram* GHI K, whoſe *Sides* are equal to the given Lines E and F.

PROB. XVI.

To make a Rhombus, M N O P, *whoſe Four* Sides *ſhall be equal to a given* Line L.

Practice. First, Make M N, (for one Side of the *Rhombus*) equal to the given Line L, and with that Length, ſet one Foot of the Compaſſes in N, and with the other deſcribe the *Arch* M O P Q. *Fig.* XXV.

2. Set the ſame Diſtance upon this *Arch*, from N to O, and from O to P.

3. Join M O, O P, and P N, ſo ſhall you have conſtituted a *Rhombus*, whoſe Four Sides are all equal to the given Line L.

PROB. XVII.

To make a Rhomboyades C D E F, *whoſe* Sides *ſhall be equal to Two given* Lines A *and* B; *and the* Acute Angles *thereof* C *and* E, *equal to a given* Angle Z.

Fig. XXVI. *Practice.* First, Make the Side of the *Rhomboyades* C D equal to the Line B.

2. On the end C, (by Prob. IX.) make an Angle F C D, equal to the given Angle Z; and the Side C F equal to the given Line A.

3. Upon the Point F, (with the Length of the given Line B) deſcribe an Arch *h h*; and upon D, (with the Diſtance A) deſcribe the Arch *g g*, croſſing *h h* in the Point E.

4. Join F E and D E, and ſo have you conſtituted a *Rhomboyades*, whoſe Sides are equal to the Lines A and B, and its *Acute Angles* C and E, equal to the given *Angle* Z.

PROB. XVIII.

To make a Trapezia H I K L, *(or Figure of Four unequal Sides) which ſhall have one* Angle *at* I, *equal to a given* Angle G; *and the Four* Sides *equal to Four (poſſible)* Right Lines *given*, viz. *to the* Lines O, D, E, F.

Fig. XXVII. *Practice.* First, Make the Line H I (for one of the Sides of the *Trapezia*) equal to one of the given Lines, as O.

2. Upon the Point I (by Prob. IX) make an *Angle* H I K equal to the given *Angle* G, making the *Side* K I equal to the given Line D.

3. Take another of the given Lines in your Compaſſes (as the Line E) and ſetting one Foot upon H, with the other deſcribe the Arch *k k*; alſo take the fourth given Line F, and ſetting one Foot of the Compaſſes in K, with the other deſcribe the Arch *m m*, croſſing *k k* in the Point L.

4. Join H L and K L; ſo ſhall you have conſtituted a *Trapezia*, whoſe Four *Sides* are equal to the Four given *Lines* O, D, E, F; and one of the *Angles*, *viz.* I, equal to the given Angle G.

PROB. XIX.

To divide a Circle A F C G, *into any Number of* Equal Parts; *not exceeding* Ten.

Practice. FIrſt, Deſcribe any Circle, and croſs it with Two Diameters A C and F G, croſſing each other at Right Angles (by Prob. I.) in the Centre E. *Fig.* XXVIII.

2. Make A B and A D, equal to E F, and join B D; ſo is B D the *Third Part* of the *Circle*.

3. Join A and F; ſo will A F be the *Fourth Part*.

4. Upon H, and Diſtance H F, deſcribe the Arch F I, and join F I; which F I is the *Fifth Part*.

5. E F, E G, E A, E C, either of them, are the *Sixth Part*.

6. H D or H B are the *Seventh Part*.

7. Draw a Line from the Centre E through the Point M, extending it to K; join K A, which be the *Eight Part*.

8. Divide the Arch D A B into *Three Equal Parts* at S, and join S D, which will be the *Ninth Part*.

9. E I is the *Tenth Part*.

PROB. XX.

To any Three Points *given*, A, B, C, *(which are not in one Right Line,) to find a* Centre O, *upon which a* Circle *may be deſcribed, which ſhall paſs through all the given Points* A, B, C.

Practice. FIrſt, Set one Foot of Compaſſes in one of the given Points, as in A, and extend the other Foot to another of the given Points, as to B, and on A, with the Diſtance A B, deſcribe an Arch of a *Circle* G F D. *Fig.* XXIX.

2. The Compaſſes open ſtill to the ſame Diſtance, ſet one Foot in B, and with the other Foot croſs the former Arch in the Points D and E, and through them draw the Right Line D E.

3. Set one Foot of the Compaſſes in the third given Point C, (being ſtill open to the former Diſtance) and with the other Foot croſs the Arch firſt drawn in the Points F and G, through which Points draw the Right Line F G, which will cut the Line D E in the Point O, which is a *Centre*; on which,

4. If you ſet one Foot of the Compaſſes, and extend the other to any of the Three given Points, the Circle ſo deſcribed ſhall paſs through all of them.

PROB. XXI.

Two Points X *and* Y, *within the* Circle ABD *being given, how to find the* Centre *of the* Arch *of a* Great Circle A X Y L, *which ſhall paſs through thoſe Two given* Points X *and* Y.

Fig. XXX.

Definition. A *Great Circle* of the *Sphere*, is ſuch a *Circle* as divideth the *Sphere* or *Globe* into *Two Equal Parts*; and ſo the *Arch* of a *Great Circle* deſcribed upon a *Plain*, is ſuch an *Arch* as divideth the *Periferie* or *Circumference* of the *Fundamental* or *Primitive Circle*, within which it is deſcribed, into *Two Equal Parts:*

Practice. LET GBHD be a *Primitive Circle* given, whoſe Centre is C, and let the Two *Points* within the ſame (through which the *Arch* of the *Great Circle* is to paſs) be X and Y.

Firſt, Through one of the given *Points*, as X, and the *Centre* C, draw a Right Line XCE, extending it infinitely towards F.

Secondly, Upon this Line, from the Centre C, erect the Perpendicular CB, and from B, through X, draw BXG; and from G, through C, draw the Diametre GCH.

Thirdly, Through the Points B and H, draw a Right Line, extending it till it cut the Line XCF in R, ſo have you found a *Third Point*, *viz.* R, through which the *Arch* of the *Great Circle* muſt paſs: And now, having *Three Points*, X Y and R, you may through them (by the laſt Problem) draw the Arch AXYLR, whoſe *Centre* will be at K; and this Arch doth divide the *Primitive Circle* into Two equal Parts in the Points A and L: And that it doth ſo is evident; for that the Right Line drawn from A to L, doth paſs through the *Centre* C.

A Compendium.

It will often fall out that the third Point R will fall very remote from the *Centre* C of the *Primitive Circle*, notwithſtanding (in all Caſes) the Work may be performed without finding it at all. For, Having found the Points E and G, as before, take the Diſtance E B, and ſet it upon the *Circle* from G to F, and from F let fall a Perpendicular to C E, as F O, extending it (if Need be) infinitely towards M; for in ſome Point of that Line will the Centre be. And to find that Point,

Divide the Line ſuppoſed between the Two given Points, X and Y, into Two Equal Parts at Right Angles, and that Line extended will cut the Line O M in the Centre, as here in K, as before.

PROB. XXII.

About a Triangle D E F, *to deſcribe a* Circle.

Practice. First, (by *Prob.* I.) Divide any Two of the Sides of the *Triangle*, as D E and D F, each into Two equal Parts, at Right Angles in the Points G and H; through which Points the Lines *k k* and *m m* being drawn at Right Angles, will croſs each other in the Point ⊙, which will be the *Centre* of the *Circle*. Fig. XXXI.

PROB. XXIII.

Within a Triangle G H K, *to* Inſcribe *a* Circle.

Practice. Divide any Two of the *Angles* of the given *Triangle*, as the *Angles* at G and H into Two equal Parts, (by *Prob.* X.) by the Lines H O and G P, croſſing each other in the *Point* ⊙, for that *Point* ſhall be the *Centre* upon which the *Greateſt Circle* that the *Triangle* is capable to receive muſt be deſcribed. Fig. XXXII.

GEOMETRICAL
THEOREMS.

I. *If a* Right Line *do fall upon* Two Parallel Right Lines, *it maketh the Alternate* Angles *Equal.*

Fig. XXXIII. So the Right Line P Q, falling upon the Two Parallel Right Lines L M and N O, doth make the Alternate Angles Equal. As the Angle P R M, equal to the Angle N S Q; and P R L, equal to Q S O; *Elem.* L. 1. P. 29.

II. *If divers* Right Lines, *be cut by divers other* Right Lines, *which are* Parallel *one to the other, the* Segments *are* Proportional.

Fig. XXXIV. Let the Two Lines R S and R T, be cut by the Four Parallel Lines V, X, Y, Z. I ſay then, the inner Segment R *a* and R *b*; as alſo *a* T and *b* S, are proportional one to the other: For if R *a* be one third Part of R T, R *b* ſhall be one Third of R S, *&c.* becauſe the Right Line *a* X, cutteth off one third Part of the Space Æ V T Z, and therefore it cutteth off a third Part from every Line drawn within that Space.

III. *If Two* Right Lines *be multiplied into one another, there is made of them a Right Angled* Parallelogram.

Fig. XXXV. Let the Two Sides to be multiplied be A 7 and B 9. If of the Lines A and B, a rectangled *Parallelogram* be made, it will be ſuch a Figure as C D E F: Or, if A 7 and B 9, be multiplied each by other, the Product will be 63, and ſo many little *Parallelograms* are there contained in the larger *Parallelogram* C D E F.

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IV. *If Two* Rectangled Figures, *be made of One of the Sides of that Figure, and of any Two* Segments *of the other Side of the ſame* Figure; *thoſe Two* Parallelograms *added together, ſhall be equal to that* Figure.

So if the *Parallelogram* G L I M be made of the Side B 9, and a Segment of the other Side; namely, 4, that *Parallelogram* ſhall produce 36, for 9 times 4 is 36. And if another *Parallelogram* ſhall be made of the Whole Side B 9, and the other Segment of the other Side A 7, namely, 3, they will produce the other *Parallelogram* L H M K 27, for 3 times 9 is 27: And theſe Two *Parallelograms* 36 and 27 added together, do make 63 equal to the firſt.

V. *If Four* Right Lines *be proportional, (that is, as the firſt is to the ſecond, ſo is the third to the fourth) the* Right-angled Figure *made of the Two* Means, *(or* Middle Terms) *ſhall be equal to the* Rectangled Figure, *made of the Two* Extream Terms.

Let there be Four Proportionals, as N 4, O 6, P 8, Q 12. I ſay, that the *Right angled Figure* R S T V, made of the Two *Means*, O 6, and P 8, ſhall be equal to the *Rectangled Figure* X Y Z Æ, made of the Two *Extreams*, N 4, and Q 12: For, as 6 times 8 is 48, ſo 4 times 12 is 48 alſo. And from hence will follow *Fig.* XXXVI.

CONSECTARY I.

If Four Right Lines *(or Numbers) be proportional; if Three of them be given, the* Fourth *is alſo given.* For,

The *Rectangle Figure* made of the Two *Means*, being divided by one of the *Extreams*, the Quotient ſhall be the other *Extream*. ——As the *Rectangle Figure*, made of the Two *Means* 6 and 8, is 48, this divided by 4, the *Leſſer Extream*, ſhall give you in the Quotient 12, for the *Greater Extream*.——Or, if the proportional Terms given had been (by Tranſpoſition) 4, 8, 6, 12, the *Rectangle* made of the *Means* 8 and 6, would be 48; which divided by 12, the *Greater Extream*, would give in the Quotient 4, the *Leſſer Extream*.

CONSECTARY II.

Hence alſo it followeth, that equal Rectangled Figures, have their Sides reciprocally proportional: That is,

Fig. XXXVI. As the *Leſſer Side* of the *Firſt Figure*, is to the *Leſſer Side of the Second Figure*; ſo is the *Greater Side* of the *Second*, to the *Greater Side* of the *Firſt*, & Contra. As in the *Equi-rectangular Figures* R S T V, and X Y Z Æ, appeareth: For,

So is
As R T : is to X Y : : Y Æ : to R S
6 : 4 : : So is 12 : 8
Or, As X Y : is to R T : : R S : to X Z
4 6 8 12

VI. *If Three* Lines *or* Numbers *be* proportional, (*that is, as the Firſt is to the Second, ſo ſhall the Second be to a Fourth*) *the* Square *made of the* Means, *is Equal to the* Oblong *made of the* Extreams.

As let the proportional Numbers be 4 : 8 : 8, the Proportion will be

As 4 : is to : 8 : : So is 8 : to 16.

∴ So the *Square* of the Two *Means* 8 is 64 : So alſo the *Oblong* made of the Two *Extreams* 4 in 16 is 64 alſo.

VII. *In a* Plain Triangle, *a* Line *drawn* Parallel *to the* Baſe, *cutteth the* Sides *thereof proportionally.*

Fig. XXXIV. As in the *Plain Triangle* R T S (in the Scheme of the Second hereof) if *c d* be Parallel to the *Baſe* T S, it cutteth off from the Side R S one third Part, and it cutteth from the Side R T one third Part alſo: And ſo they ſhall be proportional by the ſecond hereof. For,

As R T : to R S : : So is R *c* : to R *d*. And
As R *c* : to *c* T : : So is R *d* : to *d* S. Alſo,
As R *c* : to R *d* : : So is *c* T : to *d* S.

VIII. *If divers* Plain Triangles *be compared together, All* Equiangled Triangles *have the* Sides *about (or containing) the* Equal Angles *proportional: And the contrary*, Eucl. Lib. 6. P. 4.

For Illuſtration,

Fig. XXXVII. (1.) Let A B C and A D E be Two *Plain* and *Equiangled Triangles*: So that the Angles at B and D, at A and A, and alſo at C and E, be *Equal* one to another, each to its Correſpondent : I ſay, the *Sides* about the *Equal Angles* are *Proportional*: For,

1. As A B : is to B C : : So is A D : to D E.
2. As A B : is to A C : : So is A D : to D E.
3. As A C : is to C B : : So is A E : to E D.

DEMONSTRATION.

(2.) Becauſe the *Angles* B A C and D A E are equal, therefore if A B be applied to A D, then A C ſhall of Neceſſity fall in A E; and by ſuch Application ſhall ſuch a *Figure* be made.

In which *Figure*, becauſe that A B and A C do meet together, and alſo the *Angles* at B and D, and at C and E, are *Equal*; therefore the other *Sides* B C and D E ſhall be *Parallel*, *(by the firſt hereof)*. But in a plain *Triangle* a Right Line, *Parallel* to the *Baſe*, cutteth the *Sides* proportionally *(by the ſeventh hereof)*; therefore in the *Triangle* A D E, the Right Line B C being *Parallel* to the *Baſe* D E, cutteth the *Sides* A D and A E proportionally; and therefore it follows, that

As A B : is to A D : : So is A C : to A E.

(3.) Again, By the Point B let the Right Line B F be drawn Parallel to the *Baſe* A E, and it ſhall cut the other Two *Sides* D A and D E proportionally in the Points B and F, *(by the laſt hereof)* and the Proportion will be

As A B : A D : : F E : D E; or,
A B : A D : : B C : D E.

For F E and B C are *Equal*, *(by the ſecond hereof)*. And ſince it is that,

As A B : A D : : A C : A E,

And ſo B C to D E, they ſhall alſo be,

As A C : A E : : B C : D E.

For thoſe *Two Things*, which are *agreeable* to a *third Thing*, are *agreeable* one to another: Therefore it generally follows,

(1.) As A B : A D : : B C : D E
(2.) As A B : A D : : A C : A E
(3.) As A C : A E : : B C : D E

Or, by tranſpoſing the *Second* and *Third Terms* of the *Proportion*, thus;

(1.) As A B : B C : : A D : D E
(2.) As A B : A C : : A D : A E
(3.) As A B : B C : : A E : D E

And thus it is *demonſtrated*, that all *Plain Equiangled Triangles* (as theſe, A B C and A D E are) have their *Sides* comprehending their *Equal Angles*, proportional.

IX. *If ſeveral* Plain Triangles, *(how many ſoever) be compounded, and be cut by* Parallel Right Lines, *the* Inter-Segments *are* Proportional. *As thus:*

Fig. XXXVIII. If the Two *Triangles* H F I and I F K be compounded, and be cut with the *Parallel Lines* G L M, and H I K, their *Inter-Segments* are proportional. For,

As G L : H I : : L M : I K. Or,
As G L : L M : : H I : I K

(by the ſecond and ſeventh hereof) For the *Triangles* F G L and F H I are *Equiangled, (by the firſt hereof)* becauſe G L and H I are *Parallel:* Therefore,

As F L : F I : : G L : H I

(by the ſeventh hereof) but *(by the ſame ſeventh hereof)*

As F L : F I : : L M : I K.

And thoſe that are agreeable to a third, are alſo agreeable between themſelves. Therefore,

As G L : H I : : L M : I K.

X. *If any one* Side *of a* Plain Triangle *be continued, the outward* Angle *(made by that Continuation) is equal to the Two inward and oppoſite* Angles *of the ſame* Triangle.

Fig. XXXIX. If, in the *Plain Triangle* N O P, the *Side* N P be continued to Q, the outward *Angle* O P Q ſhall be equal to the Two inward *Angles*, O N P and N O P: For,

If from the Point P, a Right Line P R, be drawn *Parallel* to N O, the outward Angle O P Q ſhall be compounded of the Angles O P R, and R P Q; but the Angles R P Q and R P O, are equal to the Two inward Angles O N P and P O N; that is to ſay, the Angle R P Q, to the Angle O N P, and the Angle O P R, to the Angle N O P, (*by the firſt hereof*) becauſe of the the *Parallels* N O and P R; and therefore the *Angle* O P Q is equal to the Two inward and oppoſite *Angles*, O N P and N O P: *Which was to be demonſtrated.*

XI. *The* Three Angles *of every* Plain (*or* Right Lined) Triangle, *are equal to* Two Right Angles.

Fig. XXXIX. As in the *Plain Triangle* N O P (in the Diagram before of the tenth) I ſay, the Three *Angles* N O P, O P N and O N P, are together equal to *Two Right Angles*.

For the Angles (how many ſoever) *meeting in* One Point, *in one and the ſame* Right Line, *are equal to* Two Right Angles.

But the *Three Angles* N O P, O P N and O N P, are equal to the *Three Angles*, meeting in the Point P, upon the ſame *Right Line* N Q. For the *Angle* O P N is common to both, and the *Angles* R P Q and N O P (*by the laſt.*) Therefore, the *Three Angles* N O P, O P N and O N P, are *Equal* to *Two Right Angles*. And from hence will follow theſe

COROLLARIES.

1. That there can be but *One Right*, or *One Obtuſe*, *Angle*, in any *Plain Triangle*.
2. And if *One Angle* be *Right*, or *Obtuſe*, the other *Two* ſhall be *Acute*.
3. That the *Third Angle* of any *Plain Triangle* is the *Complement* of the other *Two* to *Two Right Angles*.
4. That if *Two Triangles* be *Equiangled* in any *Two* of their *Angles*, they are wholly *Equiangled*.

XII. *In every* Right-angled Plain Triangle, *The* Square *made of the* Side *which ſubtendeth (or is oppoſite to) the* Right Angle, *ſhall be* Equal *to both the* Squares, *which are made of the* Two Sides *which ſubtend the* Right Angle. Eucl. Lib. 1. Pr. 47.

Fig. XL. As in the *Plain Right-angled Triangle* S T V, *Right-angled* at T; I ſay, the *Sides* S T and T V, including the *Right Angle* S T V, are equal in *Power* to the *Hypotenuſe* V S; that is, the *Squares* of the *Sides* S T and T V; namely, the *Squares* S W T Æ, and V T B C, added together, are equal to the *Square* of the *Hypotenuſe* V S; to wit, the *Square* X Y S V.

For, if from the *Right Angle* at T, be let fall the *Perpendicular* T A Z, then out of the *Square* X Y V S is made *Two Rectangled Figures*, A S Y Z and A V X Z, the *Rectangle* (or *Oblong*) A S Y Z equal to the *Square* S W T Æ, and the *Rectangle* X Z V A, equal to the *Square* V T B C. ——And therefore the *Square* V X Y Z, compounded of thoſe *Two Oblongs*, is equal to the *Two Squares* V T B C and S T Æ W.

But that the *Two Oblongs* A Z Y S and A V X Z are equal to the *Squares* S W T Æ and V T B C, ſhall thus be proved.

If Three Right Lines be Proportional, (as by the ſixth hereof) *the Square of the* Means *is equal to the* Oblong *made of the* Two Extreams.

But the *Three Right Lines* S Y, (equal to S V) S T and S A, are *Proportional*; that is,

As S Y (= S V) : S T : : S T : S A:

Therefore the *Square* of S T is equal to the *Oblong* made of S Y (= S V) and S A; for the *Triangles* S T V and T A S are *Equiangled*, becauſe of the common *Angle* at S, and the *Two Right Angles* at T and A: Therefore *(by the eighth hereof)*

As S V : S T : : S T : S A.

In the ſame manner it is alſo proved, that the *Oblong* V X Z A is equal to the *Square* V T B C: For the *Triangles* T S A and A T V are *Equiangled*, becauſe of their common *Angle* at V, and the *Two Right Angles* at T and A. Therefore,

As S V : T V : : T V : A V.

And ſo *(by the ſixth hereof)* the *Square of* T V is equal to the *Oblong* Z X A V. *Which was to be demonſtrated.*

Hence it followeth: That,

If in a Right-angled Plain Triangle, *any* Two Sides *be given, the* Third *may be ſaid to be given alſo.*

Fig. XL. As if the *Two Sides* including the *Right Angle* T V 8, and T S 6 were given, their *Squares* 64 and 36 added together, make 100; the *Square Root* whereof is 10, for the *Third Side* (or *Hypotenuſe*) S V —— On the contrary, if the *Hypotenuſe* S V 10, and one of the containing *Sides* T S 6, be given; ſubſtract the *Square* of 6, *viz.* 36, from the *Square* of 10, *viz.* 100, the Remainder will be 64; the *Square Root* whereof is 8, for the other containing *Side* T V.

XIII. *In every* Plain Triangle (*as well* Right *as* Oblique angled) *the* Sides *are in* Proportion *one to the other, as are the* Sines *of the* Angles *oppoſite to thoſe* Sides, & contra.

DEMONSTRATION.

Fig. XLI. Let the *Triangle* A B C, be inſcribed in a *Circle*, and from the Centre D, draw the *Radii* D F, D E, D G; biſecting as well the *Peripheries* as the Subtenſes; and let there be alſo drawn the *Radius* D C. Now, becauſe the *Angle* at the Centre E D C is equal to the *Angle* in the *Peripherie* A B C, and C D F, equal to C A B (*by the twentieth of the third of Eucl.*) Therefore ſhall the Halves of the *Sides* be as *Sines*; and what Proportion the *Side* C A hath to the *Side* C B, the ſame ſhall the *Sine* H C, have to the *Sine* C I: For what Proportion the *Whole* hath to the *Whole*, the ſame ſhall the *Half* have to the *Half*. *Which was to be demonſtrated.*

And from hence follow theſe

CONSECTARIES.

I. *If the* Angles *of a* Triangle *be given, the* Reaſon *of the* Sides *is alſo given.*

II. *If* One Side *be given, beſides the* Angles, *both the other* Sides *are alſo given.*

III. *If* Two Sides *of a* Triangle *be given, with an* Angle *oppoſite to one of them, the* Angle *oppoſite to the other of them is alſo* Given.

Theorems Extraordinary.

I. If in a Circle Two Right Lines be inſcribed, cutting each other, the Rectangles of the Segments of each Line, are equal: And the Angle at the Point of Interſection, is meaſured by the half Sum of its intercepted Arches.

II. If to a Circle Two Right Lines be adſcribed from a Point without, the Rectangles of each Line from the Point aſſigned, to the Convex and Concave are equal: And the Angle at the aſſigned Point is meaſured by the half Difference of its intercepted Arches.

III. If in a Circle Three Right Lines ſhall be inſcribed, one of them cutting the other Two: Then the Rectangles of the Segments of each Line, ſo cut, are directed proportional to the Rectangles of the reſpective Segments of the Cutter.

IV. If a Plain Triangle be inſcribed in a Circle, the Angles are one half of what their oppoſite Sides do ſubtend; And if it hath one Right Angle, then the longeſt Side of that Triangle ſhall be the Diameter of the Circle.

V. If in a Circle, any Plain Triangle be inſcribed, and a Perpendicular be let fall upon one of the Sides, from the oppoſite Angular Point. Then, as the Perpendicular, to one of the adjacent Sides; ſo is the other adjacent Side, to the Diameter of the circumſcribing Circle.

The End of the Firſt Part.

JANUA

JANUA MATHEMATICA.
THE GATE TO THE Mathematical Sciences OPENED.

PART II.

OF TRIGONOMETRY.

WHEREIN
The Doctrine of the Dimenſion of
PLAIN and SPHERICAL TRIANGLES
is Succinctly Handled, Geometrically
Demonſtrated,

And { Arithmetically, Geometrically, Inſtrumentally, } Performed.

By *William Leybourn*, Philomathemat.

LONDON:
Printed *Anno Domini*, MDCCIV.

JANUA MATHEMATICA.

SECTION I.

Of Plain (*or* Right Lined) Triangles.

CHAP. I.

DEFINITIONS.

OF *Triangles* there are Two Kinds; *viz. Plain*, (or *Right-lined*) and *Spherical*, (or *Circular*.) Either of which do conſiſt of Six Parts; namely, of Three *Sides*, and as many *Angles*; but in this Place we ſhall only treat of the *Plain*.

I. *A Plain* (or Right Lined) *Triangle conſiſteth of Three Sides, and as many Angles:* And ſuch are the Two *Figures* C B A and C D B; in which, in the firſt *Figure* A B, B C and C A, are the Three *Sides* of the *Triangle* C B A; and C A B, A C B and C B A, are the Three *Angles* of the ſame *Triangle* C B A. *Fig.* I.

¶ And Note here] *That an Angle* (in any Caſe) *is always noted with Three Letters; the middlemoſt whereof repreſents the Angular Point.* So in the *Triangle* A B C, if I would expreſs the *Angle* at C, I would ſay, The *Angle* A C B; or, The *Angle* B C A.

Alſo in the ſecond *Figure* D B C, the Lines D C, D B and C B, are the Three *Sides* of the *Triangle* D B C; and the *Angles* B D C, C B D and B C D, are the *Angles* of the ſame *Triangle* D B C.

II. *Any*

Fig. I. II. *Any Two Sides of a Triangle, are called the Sides of that Angle contained by* (or comprehended between) *them:* So the *Sides* B C and C A are the *Sides* containing the *Angle* B C A.

III. *Every Side of a Triangle, is the ſubtending Side of that Angle which is oppoſite unto it.* As in the *Triangle* A B C, B C is the *ſubtending Side* of the *Angle* C A B; the *Side* B A ſubtends the *Angle* B C A, and the *Side* C A ſubtends the *Angle* C B A.

¶ And here Note again,] *That the greateſt Side, always, ſubtends the greateſt Angle, the leſſer Side the leſſer Angle, and equal Sides ſubtend equal Angles.*

IV. *The Meaſure of an Angle is the Arch of a Circle deſcribed upon the Angular Point, and is intercepted between the Two Sides containing the Angle,* (increaſing the Sides, if Need be). So in the *Triangle* A B C, the *Meaſure* of the *Angle* C B A, is *Arch* C F.

Fig. II. V. *Every Circle is divided into* 360 *Degrees, and every Degree into* 60 *Minutes,* (or rather into 100 or 1000 Parts, *&c.*) Which Degrees are ſo much the *Greater,* by how much the *Circle* is *Greater*; and thoſe *Arches* which contain the ſame Number of Degrees in equal *Circles,* are *Equal:* But in unequal *Circles* they are termed *Like-Arches.* So the *Arches* C F and D E are *Equal Arches,* they being equal Parts of the ſame *Circle* D H F K. But the *Arches* C F and O P are *Like-Arches:* For, as C F is 40 Parts (or Degrees) of the *Greater Circle* D E F C, ſo is O P 40 Degrees of the *Leſſer Circle* P O G.

VI. *A Quadrant* (or Quarter) *of a Circle, is an Arch of* 90 *Degrees.* As is the *Quadrant* (or Arch) H F.

VII. *The Complement of an Arch leſs than a Quadrant, is ſo much as that Arch wanteth of* 90 *Degrees.* So the *Complement* of the *Arch* C F 40 Degrees, is the *Arch* H C 50 Degrees.

VIII. *The Exceſs of an Arch Greater than a Quadrant, is ſo many Degrees as that Arch exceedeth* 90 *Degrees.* So the *Arch* D H C being 140 Degrees, is the *Arch* H C, 50 Degrees more than the *Quadrant* D H.

IX. *A Semicircle is an Arch of* 180 *Degrees.* As is the *Semicircle* D H F.

X. *The Complement of an Arch leſs than a Semicircle, to a Semicircle, is ſo much as that Arch wants of* 180 *Degrees.* So the Complement of the *Arch* D H C 140 Degrees, to the Semicircle D H E 180 Degrees, is the *Arch* C F 40 Degrees.

XI. *The oppoſite Angles made by the croſſing of Two Diameters in a Circle,* (or any Two other Right Lines croſſing each other) *are equal.* So the *Angles* C B F and C B E, (made by the Interſection of the Two *Diameters* D F and C E in the *Center* B) are equal. *Fig.* II.

XII. *An Angle is either Right or Oblique.*

XIII. *A Right Angle is that whoſe Meaſure is a Quadrant or* 90 *Deg.* So the *Angles* H B D, and H B F, are *Right Angles,* their *Meaſures* being the *Quadrants* D H and H F.

XIV. *All Oblique Angles are either Acute or Obtuſe.*

XV. *An Acute Angle, is that whoſe Meaſure is leſs than* 90 *Deg.* So the *Angles* H B C 50 Deg. and C B F 40, are Oblique *Acute Angles.*

XVI. *An Obtuſe Angle, is that whoſe Meaſure is more than a Quadrant or* 90 *Deg.* So the *Angle* D B C (conſiſting of 90 and 50 Deg. *viz.* of 140 Deg) is an Oblique *Obtuſe Angle.*

XVII. *The Complements of Angles are the ſame, as are the Complements of Arches.*

XVIII. *All Angles concurring (or meeting) together upon One Right Line, all of them being taken together, are equal to a Semicircle, or* 180 *Deg.* So the *Angle* D B H 90 Deg. H B C 50 Deg. and C B F 40 Deg. (made by the concurring, or meeting, of the Three Lines D B, H B and C B, upon the *Diameter* D F, in the *Center* B) are all of them *Equal* to the *Semicircle* D H C F, or 180 Deg.

XIX. *A Triangle hath ſome of its Sides Equal, or elſe they be all Unequal.*

XX. *A Triangle of ſome Equal Sides is either* Equicrural *or* Equilateral.

XXI. *An* Equicrural Triangle *is that which hath only Two Equal Sides.* And ſuch is the *Triangle* D B E; whoſe *Sides* B D and B E are Equal.

XXII. *An* Equicrural Triangle *is Equi-angled at the Baſe.* So in the *Equicrural Triangle* B D E, the *Angles* B D E and B E D, at the *Baſe* D E, are *Equal, viz.* each of them 70 Deg. for the *Angle* D B E being 40 Deg. that taken from 180 Deg. leaves 140 Deg. the half, whereof 70 Deg. is equal to the *Angle* B D E or D E B, and all the Three *Angles* equal to 180 Deg. or Two *Right Angles.*

This is *Demonſtrated* in the XIth *Theorem* hereof.

XXIII. *An*

Fig. II. XXIII. *An* Equilateral Triangle, *is that whoſe Sides are all Equal, and whoſe Angles contain* (each of them) 60 *Deg.* So the *Triangle* E B K hath its *Sides* B E, B K and E K, all of them equal; and the *Angles* E B K, E K B and K E B equal alſo, and each of them equal to 60 Deg. and conſequently all of them equal to 180 Deg.

XXIV. *A* Triangle *is either Right Angled or Oblique Angled.*

XXV. *A* Right-angled Triangle, *is that which hath one Right Angle.* And ſuch is the *Triangle* C A B, *Right-angled* at A.

XXVI. *An* Oblique-angled Triangle *is that which hath all its Angles Oblique.* And ſuch is the *Triangle* B C D.

XXVII. *An* Oblique-angled Triangle, *is either Acute-angled, or Obtuſe-angled.*

XXVIII. *An* Oblique Acute-angled Triangle *is that which hath all its Three Angles Acute.* And ſuch are the *Triangles* D B E, and E B K.

XXIX. *An* Oblique Obtuſe-angled Triangle *is that which hath One Obtuſe, and Two Acute Angles.* And ſuch is the *Triangle* D B C, whoſe *Angle* D B C is *Obtuſe*, and the *Angles* B D C and B C D *Acute.*

CHAP. II.

Of Right Lines, applied to a Circle.

FOraſmuch as the *Ratio* or *Proportion* of an *Arch Line* to a *Right Line*, is as yet unknown, yet it is abſolutely neceſſary that *Right Lines* be applied to a *Circle*, for the *Calculation* of *Triangles*, wherein *Arch Lines* come in Competition: For the *Angles* of *Plain* (or *Right-lined*) *Triangles* are meaſured by *Arches* of *Circles.*

Now, the *Right Lines* applied (or relating) to a *Circle*, are *Chords*, *Sines*, *Tangents*, *Secants* and *Verſed Sines.*

Fig. III. 1. A *Chord*, or *Subtenſe*, is a *Right Line*, joining the Extremities of an *Ark*, as A C is the *Chord* of the *Arks* A B C and A D C.

2. A *Right Sine*, which is ſingly called a *Sine*, is a *Right Line*, drawn from one end of an *Ark*, perpendicular to the Diameter drawn through to the other End: Or, it is half the *Chord* of twice the *Ark*; ſo A E is the *Right Sine* of the *Arks* A B and A D. The *Radius* (or *Sine* of 90 Deg.) is called the *Whole*

Fig. III.

Sine, and is the greateſt of all *Sines*: For the *Sine* of an *Ark* greater than a Quadrant, is leſs than the *Radius*; ſo F G is the whole *Sine* or *Radius*.

3. A *Verſed Sine* is the Segment of the *Radius* between the *Ark* and its *Right Sine*; ſo E B is the *Verſed Sine* of the *Ark* A B, and of the *Ark* A G D.

4. The *Secant* of an *Ark*, is a *Right Line*, drawn from the Center through one end of an *Ark*, till it meet with the *Tangent*: That is, a *Right Line* touching the Circle at the neareſt end of that Diameter which cuts the other end of the *Ark*. F M is the *Secant*, and B M the *Tangent*, of the *Ark* A B, or of A D.

5. The *Difference* of an *Ark* from a Quadrant, (or 90 Deg.) whether it be Greater or Leſs, is called the *Complement* of that *Ark*; ſo G A is the *Complement* of the *Arks* A B and A G D, and H A is the *Sine* of that *Complement*: G I the *Tangent* of that *Complement*: and F I the *Secant* of that *Complement*. All which (for Brevity) we write *Co-Sine*, *Co-Tangent*, *Co-Secant* of the *Ark*.

6. The Difference of an *Ark* from a *Semicircle* (or 180 Deg.) is called its *Supplement*; ſo the *Ark* A B is the *Supplement* of the *Ark* D G A, to a *Semicircle*.

7. That Part of the *Radius* which is between the Centre and its *Right Sine*, is equal to the *Co-Sine*. As F E is equal to H A, and F O is equal to the *Co-Sine* of the *Ark* D S.

8. If an *Ark* be Greater or Leſs than a Quadrant, the Sum or Difference, accordingly, of the *Radius* and *Co-Sine*, is equal to the *Verſed Sine*: For F D and H A together, are equal to D E, the *Verſed Sine* of the *Ark* D G A; and F B leſs by H A (or E F) is equal to E B, which is the *Verſed Sine* of the *Ark* A B.

CHAP. III.

Some Short Problems, *to make* Canons *of* Sines, Tangents *and* Secants.

PROB. I.

The Sine *of an* Ark *being given, how to find out the* Sine *of the* Complement.

B C *being given, to find* A C.

Fig. IV. BEcauſe the Triangle A C B is a Rectangle (by the Definition of a *Sine)* and the Sides A C, B C, are of the ſame Power as the *Hypotenuſe*; that is, the *Radius* A B: Therefore, if the *Square* of the *Sine* B C be ſubſtracted from the *Square* of the *Radius* A B, there remains the *Square* of A B, whoſe Side is the *Right Line* A C, the *Sine* ſought for.

PROB. II.

The Sine *of an* Ark *being given, and likewiſe the* Sine *of the* Complement, *to find the* Sine *of half the* Ark.

R Q *and* A Q *being given, to find* B O *or* R O.

Fig. V. As A B : is to B O, ſo is : : B O : to B G; therefore B O will be the *Square* Side of the Plain from the *Radius* A B and B G, the half *Verſed Sine*. For Q B, the *Verſed Sine* of the *Ark* B R, is given; becauſe A Q, the *Sine* of the Complement, and A B, the *Radius*, are given by the Suppoſition.

PROB. III.

The Sines *of* Two Arks, *and the* Sines *of the* Complements *being given, to find the* Sine *of the* Sum.

R Q, Q A, *and* S T, T A, *being given, to find* S P.

Fig. VI. As A R is to R Q, ſo is A T to T G, or C P.
As A R is to A Q, ſo is S T to S C.
B T and C P together make S P, the *Sine* of the Sum of Two *Arks*.

PROB.

Fig. VI.

PROB. IV.

The ſame being given, to find the Sine *of the Difference.*

R Q, Q A, *and* S P, P A, *are given, to find* S T.

As A Q is to Q R, ſo is A P to P O, from whence you may find O S. As A R is to A Q, ſo is O S to S T.

To theſe join the *Theorems.*

Theorem I. *The leaſt* Sines *are, almoſt, in the ſame* Ratio *as their* Arks.

This *Theorem* will be prov'd to be true hereafter in the continued Biſections; but the leaſt *Arks* are about one of the firſt Scruples, or leſs, and are almoſt in the ſame *Ratio* as their *Sines*, becauſe they are, almoſt, contiguous amongſt themſelves, and almoſt of the ſame Quantity, as it appears, tho' not to every Search, yet to a very profound one.

Theorem II. *If the ſame* Line *be cut into unequal Parts in Number, the Number of the Parts of the firſt Section, is to the Number of the ſecond, (reciprocally) as one Part of the ſecond Section is to one Part of the firſt Section.*

Divide the ſame *Line*, firſt into Four, then into Three Parts; then it will be as 4 to 3, ſo $\frac{1}{3}$ Part to $\frac{1}{4}$ reciprocally; the Reaſon is, becauſe 3 in $\frac{1}{3}$ makes 1, and 4 in $\frac{1}{4}$ makes 1; and becauſe the Products are equal, the Multiplicands will be reciprocally proportionable.

The Conſtruction *of the* Canon *of* Sines.

The *Sine* of the whole Quadrant is call'd the *Radius*, for it is the Semidiameter of a Circle. Set in the *Canon* a *Radius* of 100000 Parts, or 100000.00 for Neceſſity of Calculation; but for the better making of the *Canon* take a *Radius* 100000.00000 Parts; for by that Means, the Errors which oftentimes happen amongſt the Right Hand Figures may ſafely be corrected without any Prejudice to the *Canon.*

Fig. VI. Then biſect the Quadrant, and look for the *Sine* of the *Biſegment* by *Prob.* 2. and its *Co-ſine* by *Prob.* 1. then biſect this *Biſegment* again, and look for the *Sine* of the ſecond *Biſegment* by *Prob.* 2. and the *Co-ſine* by *Prob.* 1. then biſect this ſecond *Biſegment*, and look for its *Sine* or *Co-ſine* by *Prob.* 2. and 1. and then biſect the third *Biſegment*, &c. and continue biſecting 13 times, till you find a *Sine* of $\frac{1}{8192}$ Part of the whole Quadrant, as 'tis here ſet in the *Table*. Now we come to the leaſt *Arks*, where the Truth of the firſt *Theorem* is illuſtrated: For as the *Ark* of the Quadrant $\frac{1}{4096}$ is double to the *Ark* $\frac{1}{8192}$, ſo is its *Sine* almoſt to that *Sine*.

The Sine of the Quadrant.	100000.00000
The Sine of $\frac{1}{2}$ *of the Quad.*	70710 67811 +
The Sine of $\frac{1}{4}$ *of the Quad.*	38268.34323 +
The Sine of $\frac{1}{8}$ *Part of the Q.*	19509.03220 +
The Sine of $\frac{1}{16}$ *Part of the Q.*	9801.71403 +
$\frac{1}{32}$	
$\frac{1}{64}$	
$\frac{1}{128}$	
$\frac{1}{256}$	
$\frac{1}{512}$	
$\frac{1}{1024}$	
$\frac{1}{2048}$	
$\frac{1}{4096}$	
$\frac{1}{8192}$	

This leaſt *Sine* being thus found out, the *Sine* of one of the firſt Scruples, that is, of $\frac{1}{5400}$ part of the whole Quadrant, or of one hundredth part of a Degree; that is, of $\frac{1}{9000}$ part of the whole Quadrant is to be found. Therefore by the *ſecond Theorem*, as $\frac{5400}{9000}$ is to 8192, ſo the quantity of the firſt Part of this Diviſion is to the quantity of the firſt Part of that Diviſion, and by the firſt *Theorem*; ſo is the *Sine* of $\frac{1}{8192}$ Part, which you have in the Table at the *Sine* of $\left\{\begin{matrix}\frac{1}{60}\\ \frac{1}{100}\end{matrix}\right.$ part of Degree. *Fig.* VI.

Therefore the *Sine* of the firſt Minute, or of the firſt hundredth part thus form'd by *Prob.* 1. extract the *Sine* of the Complement, to wit, of the *Ark* 89 Deg. $\left\{\begin{matrix}\frac{59}{60}\\ \frac{99}{100}\end{matrix}\right.$ then by *Prob.* 3. find out the *Sine* of the ſecond Minute, and its *Co-ſine* by *Prob.* 1. and from thence you will find the *Sine* of the Sum of 2 Min. and 1 Min. that is 3 Min. by *Prob.* 3. and its *Co-ſine* by *Prob.* 1. and from the *Sine* and *Co-ſine* 2 Min. or from the *Sines* and *Co-ſines* 3 Min. and 1 Min. you may look for the 4th *Sine* by *Prob.* 3. and the *Sine* of the Complement by *Prob.* 1. Likewiſe from the *Sines* and *Co-ſines* of 2 Min. and 3 Min. or 4 Min. and 1 Min. you may find the 5th *Sine* and *Co-ſine* by *Prob.* 3. and 1. &c. even to $\left\{\begin{matrix}\frac{60}{60}\\ \frac{100}{100}\end{matrix}\right.$ or 1 Deg. and from the *Sine* of 1 Deg. you may by the ſame means find all the *Sines* of the 90 whole Deg. and from the *Sines* and *Co-ſines*, before found, of 60 ſingle Minutes, it will be eaſie by 3d *Prob.* taking the 4th *Prob.* too, when it will be uſeful, to extract the ſingle *Sines* of all the ſingle Minutes interſpers'd.

A Deduction of the Tangents *and* Secants *from the* Tables *of* Sines.

The *Tangents* are made thus:

As A C, the *Co-ſine*, is to C B the *Sine*; ſo is A E, the *Radius*, to E D the *Tangent*. *Fig.* VII.

But the *Secants* thus:

As A C, the *Co-ſine*, is to A B the *Radius*; ſo is A E, the *Radius*, to A D the *Secant*.

By this way the whole *Canons* of *Tangents* and *Secants* are extracted from the *Canon* of *Sines*.

Fig. VII. I paſs by all Rules of Calculation, for I don't undertake to make new Canons; foraſmuch as ſome of the moſt excellent Artiſts, by their Study and Induſtry have ſav'd me that Trouble, it ſufficiently anſwers my Deſign, if the Reaſon of *Syntax*, whatever it be, be only underſtood, and the Truth of the Numbers put in the *Canon* which the Propoſitions above abundantly prove.

CHAP. IV.

Of the Affections *of Right-lined* Triangles, *in order to the* Calculating (*or* Reſolving) *of them.*

1. A *Plain Triangle* is contained under Three Right Lines, and is either *Right-angled*, or *Oblique*.

2. In all *Plain Triangles*, Two *Angles* being given, the third is alſo given; and One *Angle* being given, the Sum of the other Two is given; becauſe, *The Three Angles together are equal to Two Right Angles.* By the 32 Pro. Euclide. Lib. I. and by the 11th of Sect. I. hereof. Therefore,

In a Plain Right-angled Triangle, One of the Acute Angles is the Complement of the other to 90 Deg.

3. In the Reſolution of *Plain Triangles*, the *Angles* only being given, the *Sides* cannot be found; but only the (*Reaſon*, or) Proportion of them: It is therefore neceſſary that one of the Sides be known.

4. In a *Right-angled Triangle*, Two Terms (beſides the Right Angle) will ſerve to find the third; ſo that One of the Terms be a Side.

5. In *Oblique-angled Plain Triangles* there muſt be Three Things given to find a fourth.

6. In *Right-angled Plain Triangles* there are *Seven Caſes*, and *Five* in *Oblique*: For the Solution of which, the Four following *AXIOMS* are ſufficient.

AXIOME I.

In a Right-angled Plain Triangle: *The Rectangle made of Radius, and one of the Sides containing the Right Angle, is equal to the Rectangle, made of the other containing Side, and the Tangent of the Angle thereunto adjacent.*

DEMONSTRATION.

Fig. VIII.

In the Right-angled Plain Triangle B E D, draw the Arch F E; then is B E Radius, and D the Tangent of the Angle at B: Make C A parallel to D E, then are the Triangles A B C and B D E like, becauſe of their Right Angles at A and E, and their common Angle at B. Therefore,

As B A : B E : : A C : E D.

And, B A in E D is equal to B E in A C.
That is, B A in *t* B is equal to Radius in A C.
Which which was to be demonſtrated.

AXIOME II.

In all Plain Triangles: *The Sides are proportional to the Sines of their oppoſite Angles.*

DEMONSTRATION.

Fig. IX.

In the Plain Oblique Triangle C B D, extend B C to F, making B F equal to D C, and deſcribe the Arches F G and C H; then are the Perpendiculars F E and C A, the Sines of the Angles at D and B: and the Triangles B E F and B A C are like, becauſe of their Right Angles at E and A, and their common Angle at B. Therefore,

As B C : C A : : B F : F E.

That is,

As B C : *s* D : : D C (equal to B F) : *s* B.

Which was to be demonſtrated.

AXIOME III.

In all Plain Triangles: *As the half Sum of the Sides, is to their half Difference; ſo is the Tangent of the half Sum of their oppoſite Angles, to the Tangent of their half Difference.*

Otherwiſe,

In every Plain Triangle: *As the Sum of the Two Sides, is to their Difference; ſo is the Tangent of the half Sum of the Oppoſite Angles, to the Tangent of half their Difference.*

DEMONSTRATION.

Fig. X. In the Oblique angular Triangle A B C, let the known Sides be B A and B C; and the Angle A B C, comprehended by them: ~~Where it~~ is *Obtuſe* in the *ſuperior*, but *Acute* in the *inferior*, *Diagram.*

Continue the Side A B to H; ſo that H B may be equal to B C, and join C and H,—and make B I equal to A B: Alſo, from the Points B and I, draw the Right Lines B D and I G, parallel to the Side A C.

Then ſhall the exterior Angle C B H be equal to the Two interior and oppoſite Angles *(by the 32th of the 1ſt Euclid.)* For the Angle C B D is equal to the Angle A C B; and the Angle D B H, to the Angle C A B.

(Moreover, from the Point B, let fall a Perpendicular B E, which ſhall biſect C H at the Point E; then making B E the Radius, upon B, deſcribe the Arch M E L. Therefore ſhall C E be the *Tangent* of half the *Sum* of the oppoſite *Angles*: And D E (to which F E is equal) the *Tangent* of half their *Difference.*

Now, becauſe A C, B D, and I G, are *Parallel*, therefore ſhall C D, D G, F H, be *Equal*: As alſo, D F and G H: Therefore I ſay,

By Equality of Proportion:

As A H, the Sum of the Two Sides,
Is to I H, their Difference;
So is C E, the Tangent of half the oppoſite Angles,
To D E, the Tangent of half their Difference.

Otherwiſe,

Fig. VIII.

Otherwiſe,

As the Sum of the Two Sides,
Is to the greater Side doubled;
So is the Tangent of half the Sum of the oppoſite Angles,
To the Sum of the Tangents of the half Sum, and half Difference of the Angles.

Otherwiſe,

As the Sum of the Two Sides,
Is to the leſſer Side doubled;
So is the Tangent of half the Sum of the oppoſite Angles,
To the Difference of the Tangents of the half Sum, and the half Difference of the Angles.

AXIOME IV.

In all Plain Triangles: *As the Baſe, is to the Sum of the other Sides; ſo is the Difference of thoſe Sides, to the Difference of the Segments of the Baſe.*

DEMONSTRATION.

In the Triangle B C D, let fall the Perpendicular C A; extend B C to F, and draw F G and D H. Fig. XI.

Then is B F equal to the *Sum* of the Sides C D and C B; and H B equal to the *Difference* of thoſe Sides; and G B is equal to the *Difference* between A B and A D, the Segments of D B, the Baſe: And the Triangles H D B and B G F are like; becauſe of their equal Angles at D and F; the Arch H G being the double Meaſure of them both: And their common Angle at B.

Therefore,

As B D : B F : : H B : G B.

That is,

As D B : F B (the Sum of the Sides) : : H B (the *Difference* of the Sides C B and C D) : G B (the *Difference* of the Segments of the Baſe.)

ADVERTISEMENT.

A Table to Reduce Sexagenary Minutes, to Centeſimal Parts; and the contrary.

Sexag. Minut.	*Cent. Parts.*	*Sexag. Minut.*	*Cent. Parts.*
1	.01.07	31	.51.67
2	.03.33	32	.53.33
3	.05.	33	.55.
4	.06.67	34	.56.67
5	.08.33	35	.58.33
6	.10.	36	.60.
7	.11.67	37	.61.67
8	.13.33	38	.63.33
6	.15.	39	.65.
10	.16.67	40	.66.67
11	.18.33	41	.68.33
12	.20.	42	.70.
13	.21.67	43	.71.67
14	.23.33	44	.73.33
15	.25.	45	.75.
16	.26.67	46	.76.67
17	.28.33	47	.78.33
18	.30.	48	.80.
19	.31.67	49	.81.67
20	.25.33	50	.83.33
21	.35.	51	.85.
22	.33.67	52	.86.67
23	.38.33	53	.88.33
24	.40.	54	.90.
25	.41.67	55	.91.67
26	.43.33	56	.93.33
27	.45.	57	.95.
28	.46.67	58	.96.67
29	.48.33	59	.98.33
30	.50.	60	1.00.00

WHereas in this *Part* of this *Book*, which treateth of Trigonometry, where the *Angles* of *Right-lined*, and the *Sides* and *Angles* of *Spherical Triangles* are meaſured by *Arches* of *Circles*; and thoſe *Arches* are uſually numbred (or accounted) by *Degrees*, *Minutes*, *Seconds*, &c. Of which one *Degree* contains 60 *Minutes*, One *Minute* 60 *Seconds*, &c. which is called the *Sexary Diviſion* of the *Degree*. Or otherwiſe, the *Degree* is ſuppoſed to be divided into 100 or 1000 *Parts*, which is called the *Centeſimal*, *Milleſimal*, &c. *Diviſion*, (and is the better of the Two in many Reſpects.)

Now, whereas in the following *Trigonometrical Calculations* of this *Book*, and alſo in the other *Parts* (which concern the *Doctrine of Triangles* applied to *Practice* in ſeveral *Parts* of the *Mathematicks*) I have, ſometimes, uſed the *Sexagenary*, and ſometimes the *Centeſimal*, Way of *Dividing*

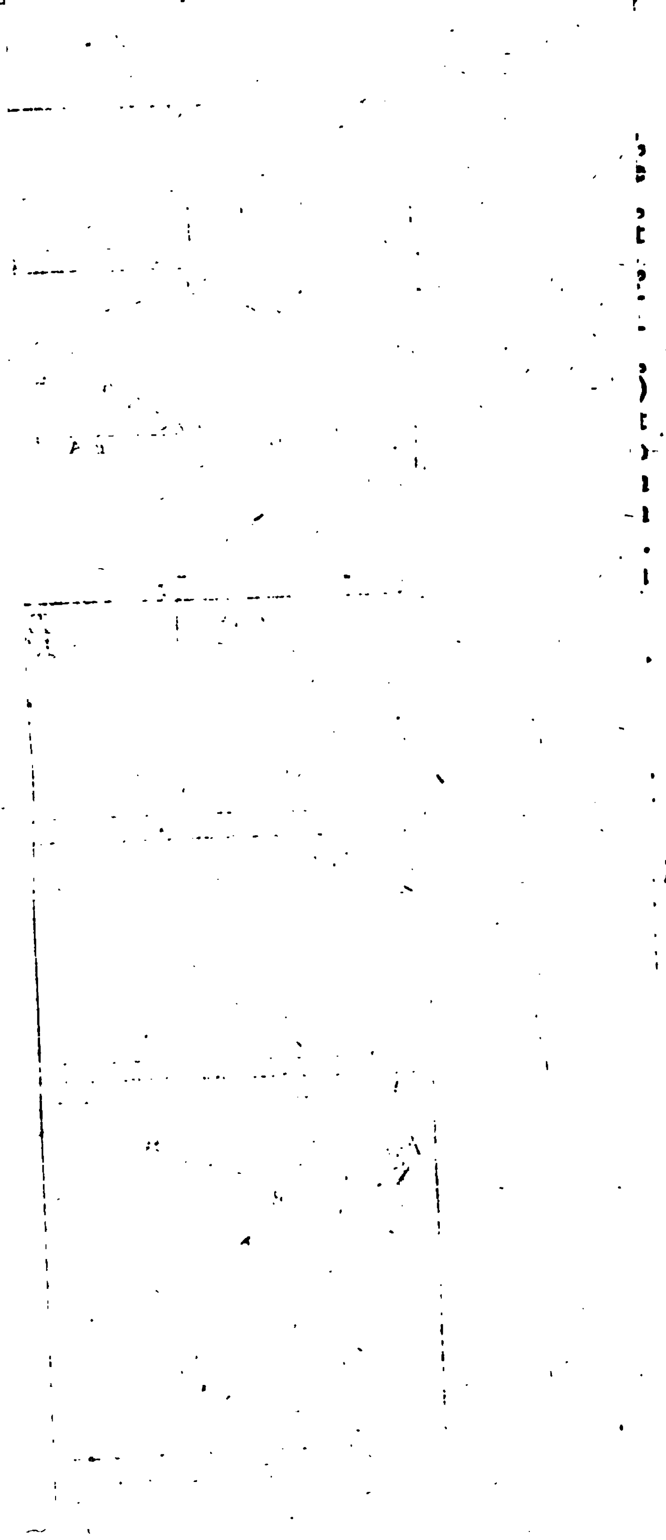

Fig. XII.

Diagram M.

Fig. XII.
Caſe I.

viding the *Degree:* (Not that I intended to make a *Difference*, (or rather a Confuſion, by intermixing them) but becauſe theſe ſeveral *Tractates* were not written at the ſame time, nor, at firſt, intended to be joined together, as here they are: But ſince it ſo is, I have here (in the beginning) inſerted a ſhort *Table*, by which *Sexagenary Minutes* are Reduced to *Centeſimal* or *Milleſimal Parts:* And alſo, *Centeſimal Parts* to *Minutes* and *Seconds:* So that when the *Reader* meets with either of them, in any *Part* of theſe *Tractates*, he need not be at any Stand) (or Demurr about it) but readily know, to which Account it belongs, for the Manner of writing them will diſcover it: So 23 Deg. 30 Min. is written as here; but the ſame written *Centeſimally*, thus, 23.5 Deg. Alſo 56 Deg. 43 Min. is written Sexagenarily, as here, but the ſame *Centeſimally*, thus, 56.71 Deg. or *Milleſimally*, thus, 56.716 Deg. And ſo of all others, as in the *Table*.

CHAP. V.

Of the Solution, Calculation, *or* Menſuration, *of* Right-angled Plain Triangles.

IN *Right-angled Plain Triangles*, I call thoſe *Sides*, which comprehend the *Right Angle*, the *Legs*; and the *Side* ſubtending the *Right Angle*, I call the *Hypotenuſe*; and the *Triangle* which I ſhall make uſe of in the *Reſolving* the Seven CASES following ſhall be that *Diagram* noted with M, in *Fig.* XII. Whoſe *Sides* and *Angles* are, *Fig.* XII. *Diagram* M.

Side	A B	235.00	Parts and Centeſms.
	B C	274.93	
	C A	142.72	
Angles	A	90.00	Deg. and Cent.
	B	31.27	
	C	58.73	

CASE I. *The* Legs A B *and* A C *given, to find the* Angle C. *Fig.* XII. *Caſe* I.

PROPORTION.

As Log. A C : Radius : : Log. A B : *t* C. [By *Axi.* I.

OPERATION.

As Log. C A, 142.72 Parts	2.1544848
Is to Rad. Tang. 45 Deg.	10.
So is Log. A B, 235.00 Parts	12.3710678
To Tangent C, 58.73 Deg.	10.2165830
Whoſe Compl. 31.27 Deg. is the Angle at B.	

Fig. XII. *Caſe* II. CASE II. *The* Angles C *and* B, *and the* Leg. A B *given*; *To find the* Leg. A C.

PROPORTION.

As Rad. : Leg. A B : : *t* B : Log. A C. [By *Axi.* I.

OPERATION.

As Radius, Tang. 45 Deg.	10.
To Log. A B, 235.00 Parts	2.3710678
So Tang. B, 31.27 Deg.	9.7883979
To Log. A C, 142.72 Parts	12.1544657

Fig. XII. *Caſe* III. CASE III. *The* Hypotenuſe B C, *and the* Leg. A B *given*; *To find the Angle* C.

PROPORTION.

As Leg. B C : Rad. : : Leg. A B. *s* C. [By *Axi.* II.

OPERATION.

As Hypo. B C, 274.93	2.4392537
Is to Radius, Sine 90 Deg.	10.
So is Leg. A B, 235.00	12.3710678
To Sine C, 58.73 Deg.	3.9318141

Fig. XII. *Caſe* IV. CASE IV. *The* Hypotenuſe B C, *and the* Angles B *and* C, *given*; *To find the* Leg. A B.

PROPORTION.

As Radius : Hyp. B C : : *s* C, : Leg. A B. [By *Axi.* II.

OPE-

OPERATION.

As Radius, Sine 90 Deg.	10.
To Hypotenuſe B C, 274.93 Parts	2.4392537
So is Sine C, 58.73 Deg.	9.9218293
To the Leg. B A, 235.00 Parts	12.3710830

CASE V. *The* Angles B *and* C, *and the* Leg. A C, *given*; *To find the* Hypotenuſe B C. Fig. XII. *Caſe* V.

PROPORTION.

As *s* B : Leg. A C : : Radius : Hypotenuſe B C. [By *Axi.* II.

OPERATION.

As Sine B, 31.27 Deg.	9.7152273
Is to Leg. A C, 142.72 Parts	12.1544848
So Radius, Sine 90 Deg.	10.
To the Hypotenuſe B C, 274.93 Parts	2.4392575

CASE VI. *The* Hypotenuſe B C, *and the* Leg. A C, *given*; *To find the other* Leg. A B. Fig. XII. *Caſe* VI.

PROPORTIONS.

1.) As Leg. B C : Rad. : : Leg. A C : *s* B. [By *Caſe* I.

2.) As *t* B : Leg. A C : : Rad. : Leg. A B. [By *Caſe* I.

OPERATIONS.

1.) As Hypotenuſe B C, 274.93 Parts	2.4392537
To the Radius, Sine 90 Deg.	10.
So is the Leg. A C, 142.72 Parts	12.1544848
To the Sine B, 31.27 Deg.	9.7152311
2.) As the Tangent of B, 31.27 Deg.	9.7833980
Is to the Leg. C A, 142.72 Parts	12.1544848
So is Radius; Tang. 45 Deg.	10.
To the Leg. A B, 235.00 Parts	2.3700868

CASE

Fig. XII. *Caſe* VII. *CASE* VII. *The two* Legs, B A *and* C A, *given*; *To find the* Hypotenuſe B C.

PROPORTIONS.

1.) As Leg. A B : Rad. : : Leg. A C : *t* B. [By *Caſe* I.

2.) As *s* B : Leg. A C : : Rad. : Leg. B C. [By *Caſe* V.

OPERATIONS.

1.) As the Leg. A B, 235.00 Parts,	2.3710678
Is to Radius, Tang. 45 Deg.	10.
So is the Leg. A C, 142.72 Parts	12.1544848
To Tangent B, 31.27 Deg.	9.7834170
2.) As Sine B, 31.27 Deg.	9.7151857
Is to Leg. A C, 142.72 Parts	12.1544848
So is Rad. Sine 90 Deg.	10.
To the Hypotenuſe B C, 274.93 Parts	2 4392991

The End of the Seven Caſes of Right-angled Plain Triangles.

CHAP. VI.

Of the Solution (*or* Menſuration) *of* Oblique-angled Plain Triangles.

Fig. XII. *Diagram* S. THE *Triangle* which I ſhall make uſe of in the Solution of the Five CASES of *Oblique-angled Triangles*, ſhall be that noted with S; whoſe *Sides* and *Angles* are

Side	B D,	1270	Parts.
	C D,	865	
	B C,	632	

Angle	C,	115.18	Degrees and Cent.
	B,	38.05	
	D,	26.77	

ASE I. *Two* Sides, D C, *and* C B, *with the* Angle *at* D, *oppoſite to* C B, *given*; *To find the* Angle *at* B, *oppoſite to* C D. *Fig.* XII. *Caſe* I.

PROPORTION.

s Log. C B : *s* D : : Log. C D, : *s* B. [By *Axiom* II.

ut here it muſt be known whether the *Angle* ſought be *Acute* or *Obtuſe*.

OPERATION.

s Log. C B, 632 Parts	2.800717
s to the Sine of ∠ D, 26.77 Deg.	9.653608
o is the Logar. of C D, 865 Parts	2.937016
	12.590624
To the Sine of ∠ B, 38.05 Deg.	9.789907

ASE II. *Two* Sides, D C, *and* B C, *with the* Angle C, *comprehended by them, given*; *To find the other* Angles B *or* D. *Fig.* XII *Caſe* II.

The given *Angle* C being above 90 Deg. *viz.* 115.18 Deg. ubſtract it from 180 Deg. the Remainder is 64.82 Deg. hich muſt be made uſe of in the Calculation: And then, This the

PROPORTION.

s half the Sum of the given Sides D C and B C,
Is to half the Difference of thoſe Sides;
o is the Tangent of half the Sum of the Angles at B and D,
To the Tangent of half the Difference of thoſe Angles.
By *Axiom* III. Then,

To the half Sum of theſe Angles D and B, if you *Add* the half ifference thus found; you will have the *Greater Angle*; but eing ſubſtracted, it will give you the *Leſſer Angle*.

O P E-

OPERATION.

	Parts.
The Side C D, is	865.0
C B, is	632.0
Their Sum	1497.0
The half Sum	748.5
Their Difference	233.0
The half Difference	116.5
The Sum of the Angles at B and D	64.82 Deg.
Their Difference	11.82 Deg.
The half Sum	32.41 Deg.
The half Difference	5.64 Deg.

Being thus prepared, ſay,

As the Log. of the half Sum of C B and C D, 748.5	2.8741918
Is to the Log. of half their Difference, 116.5	2.0663259
So is the Tang. of half the Sum of B and D, 32.41	9.8026808
	11.8690067
To the Tangent of 5.64 Deg.	8.9948149

This 5.64 Deg. is half the Difference between the Angle B and D,— Which added to the half Sum of thoſe Angles 32.41 Deg. gives 38.05 Deg. for the *Greater Angle* at B. But being ſubſtracted therefrom, it leaves 26.77 Deg. for the *Leſſer Angle* at D.

Fig. XII. *Caſe* III. *CASE* III. *Two* Angles C *and* D, *with the* Side C B, *oppoſite to* D, *given*; *To find either of the other* Angles.

PROPORTION.

As *s* D : Log. C B : : *s* C : Log. D B. [By *Axiom* II.

OPERATION.

As the Sine of the Angle at D, 26.77 Deg.	9.6536081
To the Logar of the Side C B, 632 Parts;	2.3007171
So is the Sine of the Angle C, 115.18 Deg. (or 64.82)	9.9566369
	12.7573540
To the Log. of the Side D B, 1270 Parts	3.1037549

CASE

C *A* S E IV. *Two Sides,* D B *and* C B, *with the Angle* B, *comprehended by them, being given; To find the Side* C B. Fig. XII. *Caſe* IV.

1.) Find the Angle C, by the Third *Axiome.*

2.) Find the Side C B, by the Second *Axiome.*

C *A* S E V. *The Three Sides* D C, C B, *and* D B, *being given; To find the Angle at* D. Fig. XII. *Caſe* V.

To the reſolving of this Caſe, Two Operations are required. The Firſt, To find the Segments of the Baſe D G and G B, by theſe

PROPORTIONS.

(1.) As the Log. of the Baſe D B,
Is to the Log. of the Sum of the Sides C D and C B:
So is the Log. of the Difference of thoſe Sides D E,
To the Log. of D G. By *Axiome* 4.
And G D ſubſtracted from D B, ſhall be equal to G B, and half G B, is equal to A G or A B. Then,

(1.) As Log. D C,
Is to Radius:
So is Log. D A,
To the Sine of the Angle D C A. By *Axiome* 2.
And A G and A B together, are equal to G B; and therefore, the Angle at B, may be found in the ſame manner.

OPERATION.

The Sum of the Sides C D and C B, is	1497
Their Difference is	.233
The Baſe B D is	1270

Being thus prepared, ſay,

As the Log. of the Baſe D B, 1270	3.103804
Is to the Log. of the Sum of C D and C B, 1497	3.175222
So is the Log. of the Difference of C D and C B, 233	12.367356
	5.542578
To the Log. of the Segment D G 274.65	2.438774

Fig. VI. Subſtract D G 274.65, from the whole Baſe D B 1270, the Remainder will be G B 995.35; the half whereof, 497.675, is equal to A G or A B.

Then,

As the Logar. of D C, 865,	2.9370161
Is to the Radius, Sine 90 Deg.	10.
So is the Log. of A D, 772.32	12.8877973
To the Sine of D C A, 63.23 Deg.	9.9507812

Whoſe Complement to 90 Deg. is 26.77 Deg. for the Angle C D B.

In the ſame manner; may the Angle at B be found, by, firſt, finding the Angle A C B, which you will find to be 51.95 Deg. Whoſe Complement to 90 Deg. 38.05 Deg. is the Quantity of the Angle at B.

The End of the Five Caſes of Oblique-angled Plain Triangles.

JANUA

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JANUA MATHEMATICA.

SECTION II.

Of Spherical Triangles.

CHAP. I.

Definitions *and* Theorems, *neceſſary to the right underſtanding of the* Doctrine *of the* Dimenſion *of* Spherical Triangles.

A *Spherical Triangle* (as well as a *Plain*, or *Right-lined Triangle*) conſiſteth of *Six Parts*; namely, of *Three Sides*, and as many *Angles*: The *Affections* whereof ſhall be demonſtrated from theſe Twenty *Theorems* following.

I. *The Three* Sides *of every* Spherical Triangle *are the* Arches *of Three* Great Circles *of the* Sphere, *every one of them being leſs than a* Semicircle, *or conſiſting of fewer Degrees than* 180, *which is the* Meaſure *of a* Semicircle. *By the 9th of* Sect. II.

II. *A* Great Circle *of the* Sphere, *is ſuch a Circle as divideth the whole* Sphere, *or* Globe, *into Two Equal Parts or* Hemiſpheres; *and ſo is in all Parts diſtant from the* Poles *thereof, by a* Quadrant *or 90 Deg. of the* Great Circle.

III. *If one* Great Circle *of the* Sphere, *do paſs by the* Poles *of another* Great Circle, *thoſe Two* Great Circles *do interſect, or cut, each other at* Right Angles: *And the contrary.*

Thus, Let A E C be a *Great Circle* of the *Sphere*; (and let it represent the *Horizon* of any Place) whoſe *Poles* let be B and D, (the *Zenith* and *Nadir* of the ſame Place, equi-diſtant from the *Horizon* A E C 90 Deg.) by which *Poles* B, and D, let another *Great Circle* paſs; namely, B E D, one of the *Colures* (or any *Azimuth*, or *Vertical Circle*). Now, I ſay, that the *Great Circle* Fig. XIII.

*Fig.*XIII. B E D, cutteth the *Great Circle* A E C, at *Right Angles* in the Points E and F. For, if upon the *Pole* E or F another *Great Circle* A B C D be deſcribed, it is manifeſt, that A B, B C, C D, and D A, ſhall be the *Meaſures* of the *Angles* at E and F: *(by the 4th of Sect. II.)* But the *Arches* A B, B C, C D, and D A, are *Quadrants (by the 3d hereof)* and therefore, the *Angles* at E and F, are *Right Angles (by the 13th of Sect. II.) Which was to be demonſtrated.*

IV. *The* Meaſure *of a* Spherical Angle, *Is the Arch of a* Great Circle, *deſcribed upon the Angular Point; and intercepted between the Two* Sides, *they being continued out till they be* Quadrants: *(by the Firſt hereof.)*

Fig. XIV. So, In the *Spherical Triangle* A B C, the *Meaſure* of the *Spherical Angle* at A, is not the *Arch* B C, but the *Arch* E C, intercepted between the Two *Sides* A B and A C, continued till they be *Quadrants*; that is, to the Points E and D; becauſe the *Arch* B C is not deſcribed upon the angular Point A, but the *Arch* E D is; *(by the 1ſt hereof)* and therefore, the *Arch* B C, cannot be the *Meaſure* of the *Angle* B A C, *(by the 4th of Sect. II.)*

V. *The* Sides *of a* Spherical Triangle, *being continued till they meet together, do make Two* Semicircles, *and at their Interſection, do comprehend an* Angle, Equal *and* Oppoſite *to the* Firſt Angle.

Fig. XV. Thus, In the *Triangle* A B C, the *Sides* A B and A C, of the *Angle* B A C, being continued to D, do make the *Semicircles* A B D and A C D, which do comprehend the *Angle* B D C, equal to the *Angle* B A C, becauſe the ſame *Arch* G H (being diſtant from A and D 90 Deg.) meaſureth both thoſe *Angles*, *(by the 4th hereof.)*

VI. *Every* Spherical Triangle, *hath from every* Angle *thereof, another* Triangle *oppoſite thereunto, whoſe* Baſe *and* Angle *oppoſite to the* Baſe, *are the ſame; and the other Parts of that* Triangle, *are the* Complements *of the Parts of the other* Triangle.

Thus, the *Triangle* A B C, hath another *Triangle* B D C, oppoſite thereunto; whoſe *Baſe* B C, and *Angle* B D C, oppoſite thereunto, are the ſame, *(by the 5th hereof.)* And the *Sides* B D and C D, are the *Complements* of the *Sides* A B and A C, to a *Semicircle:*

circle: And the *Angles* D B C, and D C B, are the *Complements* of the *Angles* A B C, and A C B, to Two *Right Angles*, or 180 Deg. *(by the 18th of Sect. II.)* Fig. XV.

VII. *The* Sides *of a* Spherical Triangle *may be changed into* Angles, *and the contrary*; *the* Complement *of the* Greateſt Side, *or* Greateſt Angle, *to a* Semicircle, *being taken for the* Greateſt Side, *or* Greateſt Angle.

Thus *(in the Figure of Theorem IV. hereof)* the *Spherical Triangle* A B C, obtuſe-angled at B: Let D E be the Meaſure of the *Angle* at A, and let F G be the Meaſure of the Acute *Angle* at B, (being the *Complement* of the Obtuſe *Angle* at B, the *Greateſt Angle* in the *Triangle*:) And let H I be the Meaſure of the *Angle* at C: Now, Fig. XIV.

K L	is equal to	D E	becauſe	K D	and	L E	are Quadrants, and their common Complement is	L D
L M		F G		L G		F M		L
K M		H I		K I		M H		K H.

Therefore, the *Sides* of the *Triangle* K L M are equal to the *Angles* of the *Triangle* A B C; taking for the *Greateſt Angle* A B C, the *Complement* thereof F B G.

It may alſo be *demonſtrated*, That the *Sides* of the *Triangle* A B C are equal to the *Angles* of the *Triangle* K L M: *(by the Converſe of the former.)* For, the Side

A B	is equal to	O P	the Meaſure of the Angle	M I K	which is the Complement of the Obtuſe Angle M K L.	—
B C		F H		L M K		
A C		D I		D K I		

—For that	A D	and	C I	are Quadrants, and their common Complement is	C D
	A P		O B		A O
	B F		C H		C F.

Therefore, The *Sides* may be turned into *Angles*, and the *contrary*. *Which was to be demonſtrated.*

VIII. *A Right-angled* Spherical Triangle *hath One* Right Angle, *or more than One.*

IX. Suppoſe the *Angles* at A and D, *viz.* B A C and B D C, to be *Right Angles.* Then, Fig. XV.

The Triangle	B A C	hath	One Right Angle, and Two Acute,
	B D C		Two Obtuſe Angles, and One Right,
	C D E		One Obtuſe, and Two Acute.

X. A

Fig. XV. X. *A* Right-angled Spherical Triangle, *hath, from the* Right Angle, *a* Right-angled Triangle *oppoſite thereunto, with Two* Obtuſe Angles, *& contra.*

As in the *Triangles* B A C, and B D C.

XI. *The* Sides *of a* Right-angled Spherical Triangle, *with Two* Acute Angles, *are either of them leſs than* Quadrants.

As the Sides of the *Triangle* B A C, are all of them, leſs than *Quadrants.*

XII. *The Two Sides of a* Right-angled Spherical Triangle, *with Two* Obtuſe Angles, *are* greater *than* Quadrants; *and the third* Side, *is* leſs *than a* Quadrant.

As in the *Triangle* B C D, *Obtuſe-angled* at B and C; the *Sides* D B and D C are *greater* than *Quadrants*; and the *Side* B C *leſſer.*

XIII. *A* Right-angled Spherical Triangle, *with Two* Acute Angles, *hath* (*from the* Acute Angle) *oppoſite to it, a* Right-angled Spherical Triangle, *with One* Acute, *and One* Obtuſe, Angle.

As in the *Right-angled Spherical Triangle* E D F, having the *Angles* at E and F *Acute*, is oppoſite to the *Right-angled Triangle* C D E, with the *Acute Angle* E C D, and the *Obtuſe* C E D.

XIV. *The* Sides *ſubtending the* Right Angles *of a* Spherical Triangle, *having divers* Right Angles, *are* Quadrants.

As in the *Triangle* A G H, If the Two *Great Circles* A G and A H, do cut the *Great Circle* G H at *Right Angles* in the Points G and H; then A is the *Pole* of the *Great Circle* G H (*by the third hereof*) and A G and A H are *Quadrants* (*by the ſecond hereof.*) But if the *Angle* at A be alſo a *Right Angle*, then G H is alſo a *Quadrant*, (*by the 13th of Sect. II. and by the 4th hereof.*)

XV. *A* Spherical Triangle, *having divers* Right Angles, *hath either Two or Three* Right Angles; *and ſo of the* Sides, *it hath Two or Three of them* Quadrants.

As if the *Angle* at A be put for a *Right Angle*, the *Spherical Triangle* A G H ſhall have the *Three Right Angles* at A, G and H;

and therefore the *Three Sides*, A G, A H and H G, ſhall be *Quadrants*. ---But, If you put the *Angle* at A for an *Acute Angle*, then the *Spherical Triangle* A G H ſhall have *Two Right Angles* at G and H; and thereupon the Two oppoſite *Sides*, A G and A H, *Quadrants*. *Fig.* XV.

XVI. *If the third* Angle *of a* Spherical Triangle, *having Two* Right Angles, *be* Acute; *the third* Side *is* leſs *than a* Quadrant; *but if* Obtuſe, *then it is* greater *than a* Quadrant.

As in the *Spherical Triangle* G H I, *Acute-angled* at G, the *third Side* H I is *leſs* than a *Quadrant*. ---But in the *Spherical Triangle* A G I, *Obtuſe-angled* at G, the *third Side* A I is *more* than a *Quadrant*.

XVII. *An* Oblique Spherical Triangle, *conſiſteth ſimply of* Acute *or* Obtuſe Angles, *or of both of them*.

XVIII. *A* Spherical Triangle, *with Two* Obtuſe Angles, *and One* Acute Angle; *is oppoſite to a* Spherical Triangle, *ſimply* Acute-angled, *& contra*.

As if the *Angles* at A and D, be ſuppoſed *Acute*; then the *Triangle* B C D, with *Two Obtuſe Angles* at B and C, and *One Acute Angle* at D, is oppoſite to the ſimple *Acute-angled Triangle* A B C.

XIX. *A* Spherical Triangle, *with Two* Acute Angles, *and One* Obtuſe; *is oppoſite to a* Spherical Triangle, *ſimply* Obtuſe-angled, *& contra*.

As if the *Angles* at A and D, be ſuppoſed *Obtuſe*; then the *Triangle* A B C, with *Two Acute Angles* at B and C, and *One Obtuſe Angle* at A, is oppoſite to the ſimply *Obtuſe-angled Triangle* B D C.

XX. *The* Three Angles *of every* Spherical Triangle, *are* greater *than Two* Right Angles.

This is evident in *Spherical Triangles*, having more *Right*, or *Obtuſe Angles*, than *One*: But in *Acute-angled Triangles*, it may be thus *Demonſtrated*.

DEMONSTRATION.

Fig. XVI. In the *Right-angled Spherical Triangle* A B C, *Right-angled* at C, and *Acute-angled* at A and B.

$$\text{The Meaſure of the Acute Angle}\left\{\begin{matrix}BAC\\ ABC \text{ or } BDE\end{matrix}\right\}\text{is the Arch}\left\{\begin{matrix}EF\\ HI\end{matrix}\right.$$

(*by the fourth hereof.*) But the *Arches* E F and E D together, are equal to a *Quadrant*; therefore the *Arches* F E and H I, added together, are more than a *Quadrant*; and conſequently, the *Angles* anſwering to thoſe *Arches*, namely, the *Angles* B A C and A B C together, are more than a *Quadrant*. But the *Angle* A C B is a *Right Angle*, by the *Propoſition*: Therefore, in the *Spherical Triangle* A B C, conſiſting of *Two Acute Angles*, the *Three Angles* together are *greater* than *Two Right Angles*.

Fig. XVII. Again, In the *Acute-angled Triangle* K L M

$$\text{The Meaſure of the Acute Angle}\left\{\begin{matrix}KLM\\ MKL\\ LMK\end{matrix}\right\}\text{is the Arch}\left\{\begin{matrix}NO\\ VX\\ RQ\end{matrix}\right.$$

But theſe *Three Arches* N O, V X, and R Q together, are more than *Two Quadrants*: For P Q, and P V, (being the *Complements* of the *Two Arches*, Q R and V X) added together, are *leſs* than the *Arch* N O, by the *Propoſition*: Therefore the *Arch* N O, being the *Meaſure* of the *third Angle*, is more than the *Complements* of the other *Two Angles* added together; and conſequently, the *third Angle* is *greater* than the *Complements* of the other *Two Angles*. And therefore, In *Acute-angled Spherical Triangles*, the *Three Angles* are *greater* than *Two Right Angles*. *Which was to be demonſtrated.*

CHAP. II.

Such Affections of Great Circles *of the* Sphere, *as relate to the Solution of* Spherical Triangles.

1. A *Great Circle* of the *Sphere*, Is ſuch a Circle, as divideth the whole Body of the *Globe* or *Sphere*, into Two Equal Parts.

2. A *Spherical Triangle* is that part of the *Superficies* of the Globe, as lyes between the Arches of Three Great Circles of the Sphere interſecting one another. Fig. XVII.

3. A *Spheric Angle* is the ſame with the mutual Aperture or Inclination of the Plains of ſuch Two Great Circles which conſtitute the Angle.

4. When one Circle falls upon another Circle, or when the Arches of Two Great Circles interſect each other, the Sum of the Angles made thereby is equal to Two Right Angles: And the Vertical Angles made thereby are mutually Equal.

5. In all *Spherical Triangles*, the *Greater Angle* is always oppos'd to the *Greater Side*.

6. An *Iſoſceles Triangle*, hath its Two Angles at the Baſe mutually *Equal*; and, on the contrary, if a *Triangle* hath Two *Angles Equal*, it hath *Two Sides* alſo *Equal*.

7. Two *Triangles* mutually *Equilateral*, are alſo *Equiangular* one to the other.

8. If there be Two *Triangles*, and in each one Angle, and the Two Sides including it, reſpectively equal: Or, if One Side, and the Two Angles adjacent, be ſeverally equal, then are thoſe Two *Triangles* equal.

9. An *Arch* of a *Great Circle*, is the ſhorteſt Diſtance between Two Points on the Surface of a Globe: And ſo, any Two Sides of a *Spherical Triangle* taken together, are *Greater* than the *Third*.

10. All *Great Circles* cut each other into Two equal Parts; for their *common Section* is a Diameter of the *Sphere*, and conſequently the Two *Sections* of the Peripheries of Two *Great Circles* are at a Semicircle's Diſtance. Hence it follows, That

11. Every *Side* of a *Spherical Triangle*, is leſs than a *Semicircle*. So D B is leſs than the *Semicircle* D B C or D A C. Fig. XVIII.

12. The oppoſite *Angles* at the *Sections* of Two *Circles*, are *Equal*; as the Angle at D, is equal to that at C; for the ſame *Plains* conſtitute both *Angles*.

13. In any *Spherical Triangle*, if the Sum of the *Legs* containing an Angle be *Greater*, *Equal* to, or *Leſſer* than a *Semicircle*, the internal *Angle* at the *Baſe*, is (accordingly) *Greater*, *Equal* to, or *Leſſer* than the outward oppoſite; and conſequently, the Sum of the Two internal Angles at the Baſe, are *Greater*, *Equal* to, or *Leſſer* than Two Right Angles.

Fig. XVIII.

DEMONSTRATION.

If D B and B A together be *Greater*, *Equal* or *Leſſer* than D C, then B A is *Greater*, *Equal* to, or *Leſſer* than B C; and therefore the Angles at C and D are *Greater*, *Equal* or *Leſſer* than the Angle B A C; and the Angles B D A and D A B, *Greater*, *Equal* to, or *Leſſer* than the Angles B A C and D A B, equal to Two *Right Angles*.

COROLLARY.

In an *Iſoſcheles Triangle*, if one of the *Equal Legs* be *Greater*, *Equal* to, or *Leſſer* than a *Quadrant*, the Angle at the *Baſe* is *Greater*, *Equal* to, or *Leſſer* than a *Right Angle*.

14. The *Sum* of the *Three Sides* of a *Triangle* is leſs than a *Whole Circle*, or 360 Deg. For B A is *Leſs* than B C and A C. Therefore D B, D A and B A together, are *Leſſer* than D B C and D A C.

15. If from the Point of an Angle, as a *Pole*, you deſcribe a *Great Circle*; or, if you deſcribe a Circle at 90 Deg. on the angular Point, the Ark of that Circle ſo deſcribed, which is intercepted between the *Legs* of the *Angle*, is the *Meaſure* of that *Angle*.

Fig. XIX. 16. The *Poles* of the Sides of any *Triangle* G H D, conſtitute another *Triangle n x m*, which we may call *Supplemental* to the *Triangle* G H D, for the *Supplements* of the *Angles* and *Sides* of the *Triangle n x m* are equal to the *Sides* and *Angles* of the *Triangle* G H D.

DEMONSTRATION.

From the Points G, H, D, as *Poles*, deſcribe Three Great Circles, *x* A Y, R T *m n*, *x* B *n* Z; then is Y *m* equal to a Quadrant, and equal to A *x*, becauſe *m* is the Pole of H G Y, and *x* or E, the Pole of G A; therefore *m x*, equal to A Y, equal to the Supplement of C A, equal to the *Angle* H G D; and the Quadrant Z *n*, equal to B X; therefore *n x*, equal to B Z, equal to the Supplement of the *Angle* H D G, and *n* T, equal to a Quadrant, equal to *m* R; therefore *n m*, equal to T R, equal to the Supplement of the *Angle* D H G.

Now that the *Triangle n* E *m*, conſtituted between the Three next *Poles*, hath its Three *Sides* and *Angles*, equal to the *Angles* and *Sides* of the *Triangle* G H D, ſave that the Greateſt Side *n m* is the Supplement of the Greateſt *Angle* H, and the *Angle* E, of the Side G D.

17. Any *Angle* of a *Triangle*, with the Difference of the other Two, is *Leſſer* than Two *Right Angles*: For $x\,n$ is *Leſſer* than $x\,m$ and $n\,m$, that is, Two *Right Angles*, wanting D, is *Leſſer* than Two *Right Angles*, wanting G, and Two *Right Angles*, wanting H. Therefore, G and H wanting D, is leſs than Two *Right Angles*. Fig.XIX.

18. If Two *Triangles* are mutually *Equiangular*, they are alſo mutually *Equilateral*; for, becauſe they are *Equiangular*, their *Supplemental Triangles* are *Equilateral* (*by the* 16*th*) and therefore *Equiangular* (*by the* 7*th*). And therefore the propoſed *Triangles* are *Equilateral* (*by the* 16*th*.)

19. The *Three Angles* of every *Spherical Triangle*, are *Greater* than *Two Right Angles*, and *Leſſer* than *Six Right Angles*. For, $n\,x$ and $x\,m$ and $n\,m$ together, are *Leſſer* than *Four Right Angles*, (*by the* 14*th*.) that is, *Six Right Angles*, wanting D, and G, and H, leſſer than *Four Right Angles*: That is, *Two Right Angles* are leſſer than D, and G, and H. Alſo, the *Sum* of the *Internal-angles* is leſs than the *Sum* of the *Internal* and *External Angles* taken together, for both of them make but *Six Right Angles*.

20. Of ſeveral *Arches* of *Great Circles* falling from the ſame Point of the *Spheres* Surface, on another Circle, the Greateſt is that which paſſeth through the *Pole* of the *Circle*; and the next to this, is *Greater* than that which is farther off. For ſuppoſe P the *Pole* of the Circle C ϖ D, and ϖ the *Pole* of D P C; then is A D *Greater* than A B, A B *Greater* than A E, A E *Greater* than A C: And the *Ark* B ϖ C *Greater* than the *Ark* B P, and B P *Greater* than B D. Fig. XX.

21. A *Great Circle* paſſing through the *Poles* of another *Great Circle*, cuts it at *Right Angles*; And on the contrary, If it cut it at *Right Angles*, it paſſeth through its *Poles*: The *Angle* P B O is equal to a *Right Angle*, equal to P G D, equal to P D B, equal to ϖ A C.

22. In an *Oblique-angled Triangle*, if the *Angles* at the *Baſe* are *like*, or of the *ſame Kinds*; that is, both *Acute*, or both *Obtuſe*, the *Perpendicular* falls *Within* the *Triangle*, and the *Quadrantal Arch* without. But if they be unlike, the *Perpendicular* falls *Without*; and the *Quadrantal Arch Within* the *Triangle*. For the *Triangle* A E F hath the *Angles* at E and F *Acute*, and the *Perpendicular* A C falls *Within*, and the *Quadrantal Arch* A ϖ *Without*. Alſo, the *Triangle* B A G hath the *Angles* at B and G, *Obtuſe*, and the *Perpendicular* A D *Within*, and the *Quadrantal*

Fig. XX. *Arch* A ϖ *Without*. But the *Triangle* B A E hath the *Angles* at B and E of *different Kinds*; and the *Perpendicular* A C falls *Without*, and the *Quadrantal Arch* A ϖ *Within*, the *Triangle.*

CHAP. III.

Of the Menſuration, or Solution, of Right-angled Spherical Triangles.

*Fig.*XXI. *Diagram* A. I. IN a *Right-angled Spherical Triangle*, there are (beſides the *Right Angle*) Five other *Parts*; whereof thoſe *Three* which are more remote from the *Right Angle*, the Lord *Nepeir* changeth into their *Complements:* ——As in this Triangle A B C, Right-angled at A: For the *Three Remote Parts*, to wit, the *Angles* B and C, and the *Side* C B, he takes their *Complements:* Theſe *Three Complements*, with the *Sides* C A and B A, do make *Five Parts:* Which, by an artificial Term, he calls

CIRCULAR.

Viz. The
- Side A B,
- Side A C,
- Complement of the Angle at B,
- Complement of the Angle at C,
- Complement of the Side B C.

But the *Right Angle* at A is ſet aſide from being any of the *Circular Points.*

II. In the Solution of a *Right-angled Spherical Triangles*, there are always *Two* other *Parts*, or *Terms*, given (beſides the *Right Angle*) to find out a *Fourth.*

III. Theſe *Three Terms* (namely, the *Two* that are *Given*, and the *Third* which is *Required*) muſt be firſt looked upon according to their *Circular Parts.*

IV. Of which, *One* is named the *Middle* (or *Mean*) *Part*; the other *Two* are called the *Extream Parts*, borrowing their Appellation from the Scituation of the *Terms* themſelves: For, of *Three Terms*, *One* muſt (of neceſſity) be in the *Middle*, and the other *Two* in the *Extreams:* Therefore the *Circular*

Part

Fig.XXI.

Fig. XX

Fig. XXI
Diagram
A.

Part of the *Middle Term*, is called the *Middle Part*, and the *Circular Parts* of the *Extream Terms* are called the *Extream Parts*. Fig.XXI.

V. But the *Extream Parts* may be twofold, either *Conjunct*, or *Disjunct*: For thoſe *Three Terms* (beſides the *Right Angle*) do come in queſtion, according as the *Two Extreams* are from either *Part* immediately *joined* to a third *Mean*, or are *diſ-joined* from the ſame, by a *Side* or *Angle* interpoſed on both *Sides*: So are their *Circular Parts* named, *Extreams Conjunct*, or *Extreams Disjunct*.

VI. But of thoſe *Three Terms* which may fall in queſtion, we will ſubject all their Varieties in their *Circular Parts*, according as every one of them ought (in reſpect of each other) to be called, The *Middle Part*; and which, The *Extreams Conjunct* or *Disjunct*, as in this following *Synopſis* is fully demonſtrated.

The Analyſis or Synopſis.

	Middle Part.	*Extreams Conjunct.*	*Extreams Disjunct.*
If the	Side A B	Side A C and Comp. B	Com. B C and Com. C
	Side A C	Side A B and Comp. C	Com. B C and Com. B
	Compl. B	Side A B and Comp. B C	Side A C and Com. C
	Compl. C	Side A C and Comp. B C	Side A B and Com. B
	Compl. B C	Comp. B and Comp. C	Side A B & Side A C.

Which *Synopſis* is thus to be *Read* and *Underſtood*: *Example* of the *Firſt Line*. If the *Side* A B be the *Middle Part*, then is the *Side* A C, and *Comp. of the Angle* B, the *Extreams Conjunct*: And the *Comp. of the Side* B C, and *Comp. of the Angle* C, the *Extreams Disjunct*. And ſo of all the reſt.

And here it is to be noted, That the Sides A B *and* A C, *are ſuppoſed to be joined together, (as one entire Part,) becauſe the* Right Angle *at* A, *is not reckoned amongſt the* Circular Parts.

VII. Therefore, In the *Reſolution* of a *Right-angled Spherical Triangle*, to know the *Mean*, and *Extream Parts*, you muſt obſerve, That

1. If *One* of the *Three Terms* (which, beſides the *Right Angle*, come in queſtion) doth ſtand *alone* by it*ſelf*, ſevered from the

Fig.XXI. the other *Two* on both *Sides*; (as the *Side* B C, from the *Sides* C A and B A, by the *Angles* B and C interpoſed) that ſhall be the *Middle Term*; and ſo its *Circular Part* ſhall be called the *Middle Part*; and the other *Two Circular Parts* are the *Extreams Disjunct*. But,

2. If the *Three Terms* do immediately adhere together, the *Middle Term* doth eaſily ſhew the *Middle Part*, and the *Extream Terms*, the *Extream Parts Conjunct*.

Theſe Things being all rightly underſtood, the whole *Trigonometry* of *Sphericals* will be abſolved by this *One Propoſition*; which therefore we will call *Catholick* or *Univerſal*.

Propoſition Univerſal.

The Sine *of the* Middle Part *and the* Radius, *are* Reciprocally Proportional, *with the* Tangents *of the* Extream Parts Conjunct; *and with the* Co-ſines (*or* Sines *Complements*) *of the* Extreams Disjunct. *That is,*

As the *Radius*,
To the *Tangent* of one of the *Extreams Conjunct*;
So is the *Tangent* of the other *Extream Conjunct*,
To the *Sine* of the *Middle Part*: *& contra.*

Then alſo,

As the *Radius*,
To the *Co-ſine* of one of the *Extreams Disjunct*;
So is the *Co-ſine* of the other *Extream Disjunct*,
To the *Sine* of the *Middle Part*: *& contra.*

COROLLARY.

I. If the *Middle Part* be Sought, the *Radius* ſhall be in the *Firſt Place* of the *Proportion*: But if one of the *Extream Parts* be Sought, then the other *Extream* ſhall be in the *Firſt Place*. Of the *Second* and *Third Places*, it mattereth nothing how they be diſpoſed.

II. If the *Extreams* (in any *Proportion*) be *Diſtinct* from the *Middle Part*, the *Proportion* will be performed by *Sines* only: But if the *Extreams* be *Conjunct* to the *Middle Part*, it muſt be performed by *Sines* and *Tangents* jointly. *Fig.*XXI.

The *Demonſtration* of the *Univerſal Propoſition* is obvious enough: For, where the *Extream Parts* are *Disjunct*, the *Proportions* differ nothing from the common ones: And in the *Extreams Conjunct*, where it is commonly ſaid,

As *Radius*, to the *Tangent*,

We here ſay,

As the *Co-Tangent*, to the *Radius*.

And likewiſe *Inverſly* and *Contrarily*; which is plainly the ſame thing: Becauſe,

The Radius *is a* Mean Proportional, *between the* Tangent *of an* Arch, *and the* Tangent Complement *of the ſame* Arch.

Note, That when a *Complement* in any *Proportion* doth chance to concur with a *Complement* in the *Circular Parts*, you muſt then (always) take the *Sine* it ſelf, or the *Tangent* it ſelf; inſtead of the *Co-ſine*, or *Co-Tangent*, in the *Circular Parts:* Becauſe the *Co-ſine* of the *Co-ſine*, is the *Sine* it ſelf, and the *Co-tangent* of the *Co-tangent*, is the *Tangent* it ſelf.
As is the *Sixth Caſe* following; where C B and C A are given, and the *Angle* at B is required: Here C A is the *Middle Part*, and C B, and B, are the *Extreams Disjunct:* Wherefore (by the ſecond Part of the foregoing *Corollary*) the *Proportion* will be performed by *Sines* only.——And (by this laſt) becauſe the *Two Extreams*, CB, and B, fall upon *Complements* in the *Circular Parts*; therefore, inſtead of *Co-ſine* B C, and *Co-ſine* B, you muſt ſay *Sine* B C, and *Sine* B.

Theſe Things premiſed, we will exemplifie in the Solution of *Right-angled Spherical Triangles* in all the Caſes thereof.

CHAP. IV.

The Analogies, *or* Proportions, *for the* Solution *of the ſeveral* Caſes *of* Right-angled Spherical Triangles, *by the* Univerſal Propoſition.

*Fig.*XXI. *Diagram* B. FOR the Performance hereof, I ſhall make uſe of this *Right-angled Spherical Triangle* A B C, *Right-angled* at A, the Quantities of whoſe *Sides* and *Angles* are adfixed to their reſpective *Circular Parts* in the Diagram noted with B, in Fig. XXI. And alſo in this *Table*, both in *Sexagenary Degrees* and *Minutes*; and in *Decimal Parts* alſo.

	D.	*M.*	*Cent.*
B C ——	66	30	66.50
C A ——	51	30	51.50
B A ——	50	10	50.16
∠ B ——	58	35	58.58
∠ C ——	56	52	56.85

And in every *Caſe* I ſhall diſtinguiſh the Two *Given Terms*, (beſides the *Right Angle*, which is, always the *third*) by marking the *Sides* or *Angles Given*, by a ſhort Stroak, (|), and the *Term Required*, I ſhall mark with (∘.) All which are to be ſeen in Figure XXI.

The XVI *Caſes* of *Right-angled Spherical Triangle*, Reſolved,

The *Hypotenuſe* B C, and the *Angle* at C, given; To find

*Fig.*XXI. *Caſe* I. *CASE* I. The *Oppoſite Side* A B, the *Middle Part.*

As Radius 90 Deg.	10 0000000
To Sine C, 56 Deg. 52 Min.	9.9229334
So Sine B C, 66 Deg. 30 Min.	9.9623978
To Sine B A, 50 Deg. 10 Min.	19.8853312

CASE II. *The* Side Adjacent A C, Extream Conjunct. *Fig.* XXI. *Caſe* II.

As Co-tangent B C, 23 Deg. 30 Min.	9.6383019
To Radius, 90 Deg.	10.
So Co-ſine C, 33 Deg. 8 Min.	19.7376611
To Tangent C A, 51 Deg. 30 Min.	10.0993592

CASE III. *The other* Angle B, Extream Conjunct. *Caſe* III.

As Co-Tangent C, 33 Deg. 8 Min.	9.8147277
To Radius, 90 Deg.	10.
So Co-ſine B C, 23 Deg. 30 Min.	19.6006997
To Co-Tangent B, 31 Deg. 25 Min.	9.7859720

The Hypotenuſe B C, *and* Side A C, *given, to find*

CASE IV. *The* Oppoſite Angle *at* B, Ex. Diſj. *Caſe* IV.

As Sine B C, 66 Deg. 30 Min.	9.9623978
To Radius, 90 Deg.	10.
So Sine C A, 51 Deg. 30 Min.	19.8935444
To Sine B, 58 Deg. 35 Min.	9.9311466

CASE V. *The* Adjacent Angle C, Middle Part. *Caſe* V.

As Radius, 90 Deg.	10.
To Tangent C A, 51 Deg. 30 Min.	10.0993948
So Co-tangent B C, 23 Deg. 30 Min.	9.6383019
To Co-ſine C, 33 Deg. 8 Min.	19.7376967

CASE VI. *The* other Side A B, Extream Diſjunct. *Caſe* VI.

As Co-Sine C A, 38 Deg. 30 Min.	9.7941496
To Radius, 90 Deg.	10.
So Co-ſine B C, 23 Deg. 30 Min.	19.6006997
To Co-ſine B A 39 Deg. 50 Min.	9.8065501

Fig. XXI. *The* Side A C, *and the* Angle *oppoſite thereto* B, *being Given; To find*

Caſe VII. *CASE* VII. *The other* Side B A, Middle Part.

As Radius, 90 Deg.	10.
To Tangent C A, 51 Deg. 30 Min.	10.0993948
So Co-Tangent B, 31 Deg. 25 Min.	9.7859004
To Sine B A, 50 Deg. 10 Min.	19.8852952

Caſe VIII. *CASE* VIII. *The* other Angle *at* C, Extream Disjunct.

As Co-ſine C A, 38 Deg. 30 Min.	9.7941496
To Radius, 90 Deg.	10.
So Co-ſine B, 31 Deg. 25 Min.	19.7170526
To Sine C, 56 Deg. 52 Min.	9.9229030

Caſe IX. *CASE* IX. *The* Hypotenuſe B C, Extream Disjunct.

As Sine B, 58 Deg. 35 Min.	9.9311522
To Radius, 90 Deg.	10.
So Sine C A, 51 Deg. 30 Min.	19.8935444
To Sine B C, 66 Deg. 30 Min.	9.9623922

The Side C A, *and the* Angle C, *adjacent thereto, given; To find*

Caſe X. *CASE* X. *The* other Side A B, Extream Conjunct.

As Co-tangent C, 33 Deg. 8 Min.	9.8147277
To Radius, 90 Deg. 8 Min.	10.
So Sine C A, 51 Deg. 30 Min.	19.8935444
To Tangent B A, 50 Deg. 10 Min.	10.0788167

Caſe XI. *CASE* XI. *The* other Angle B: Middle Part.

As Radius, 90 Deg.	10.
To Sine C, 56 Deg. 52 Min.	9.9229334
So Co-ſine C A, 38 Deg. 30 Min.	9.7941496
To Co-ſine B, 31 Deg. 25 Min.	19.7170830

CASE XII. *The Hypotenuſe* B C, Extream Conjunct. *Fig.* XXI. *Caſe* XII.

As Tangent C A, 51 Deg. 30 Min.	10.0993948
To Radius, 90 Deg.	10.
So Co-ſine C, 33 Deg. 8 Min.	19.7376611
To Co-tangent B C, 23 Deg. 30 Min.	9.6382663

The Two Sides, A C and A B, *given*; *To find*

CASE XIII. Either Angle, *as* C; Extream Conjunct. *Caſe* XIII.

As Tangent B A, 50 Deg. 10 Min.	10.0787534
To Radius, 90 Deg.	10.
So Sine C A 51 Deg. 30 Min.	19.8935444
To Co-tangent C, 33 Deg. 8 Min.	9.8147910

CASE XIV. *The* Hypotenuſe C B: Middle Part. *Caſe* XIV.

As Radius, 90 Deg.	10.
To Co-ſine A C, 38 Deg. 30 Min.	9.7941496
So Co-ſine B A, 39 Deg. 50 Min.	9.8065575
To Co-ſine C B, 23 Deg. 30 Min.	19.6007071

The Two Angles B *and* C, *given*; *To find*

CASE XV. *Either of the* Sides, *as* A C: Extream Disjunct. *Caſe* XV.

As Sine C, 56 Deg. 52 Min.	9.9229334
To Radius, 90 Deg.	10.
So is Co-ſine B, 31 Deg. 25 Min.	19.7170526
To Co-ſine C A, 38 Deg. 30 Min.	9.7941192

CASE XVI. *The* Hypotenuſe B C: Middle Part. *Caſe* XVI.

As the Radius, 90 Deg.	10.
To Co-tangent C, 33 Deg. 8 Min.	9.8147277
So Co-tangent B, 31 Deg. 25 Min.	9.7859004
To Co-ſine B C, 23 Deg. 30 Min.	19.6006281

Fig.XXI. *Note*, Theſe are the *Proportions* anſwerable to the *Univerſal Propoſition*, yet may (many of them) be varied; ſo that the *Radius* may be brought into the *firſt Place*, and that by the latter part of the foregoing *Corollary*: Which ſays, — *Radius is a Mean Proportional between the Tangent of an Arch, and the Tangent Complement of the ſame Arch.* —So that (in the XIIIth *CASE*) where it is ſaid,

As the *Tangent* B A, Is to *Radius*:
So is *Sine* C A, To *Co-tangent* C.

It is all one, as if you ſhould ſay,

As *Radius*, 90 Deg.	10.
To the *Co-tangent* B A, 39 Deg. 50 Min.	9.9222466
So the *Sine* of C A, 51 Deg. 30 Min.	9.8935444
To the *Co-tangent* of C, 33 Deg. 3 Min.	19.8157910

The like Courſe may be taken in the *Second*, *Third*, *Tenth* and *Twelfth* CASES.

And thus you have the whole *Doctrine* of the *Dimenſion* of *Right-angled Spherical Triangles*, performed by Help of this one *Catholick Propoſition.*

CHAP. IV.

Some Prænotions concerning Oblique-angled Spherical Triangles, *in order to the Solution of them.*

IN *Oblique-angled Spherical Triangles* there are XII *Caſes*, Ten of which may be reſolved by the *Univerſal Propoſition*; but then the *Oblique Triangle* muſt be reduced into Two *Right-angled Triangles* by help of a *Perpendicular* let fall, ſometimes *within*, ſometimes *without*, the *Triangle*: And to know whether it fall *within* or *without*, the ſubſequent *Rules* are to be obſerved.

RULE I. *If the* Angles *at the Baſe of the* Triangle *be both of the ſame Affection; that is, both Acute or Obtuſe; the* Perpendicular *let fall from the* Vertical Angle *ſhall fall within: But if of different Affections, without.*

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Fig. XXII.

As in the *Oblique angled Triangle* A B C, whoſe *Angles* at B and C are both *Acute*, the *Perpendicular* A D ſhall fall within the *Triangle*: For, if it fall not within, it muſt be the ſame with one of the Sides, or elſe it muſt fall without the *Triangle*: If it be the ſame with either of the Sides, then the *Angle* at B or C muſt be a *Right Angle*; which is contrary to the Propoſition: If it fall without the *Triangle*, as ſuppoſe at E, then the *Angle* A E B ſhall be a *Right Angle*: But the *Angle* A B E is *Obtuſe*; for it is the Complement of the *Acute Angle* A B C, and therefore the Side A E is greater than a Quadrant; And the *Angle* A C E being *Acute*, A E ſhall be alſo leſs than a Quadrant: But, that the ſame Side ſhould be both *More* and *Leſs* than a Quadrant, is abſurd: And therefore, in this *Caſe*, the *Perpendicular* ſhall fall within the *Triangle*.

Fig. XXII.

But, In the *Triangle* A E B, *Obtuſe-angled* at B, and *Acute* at E; the *Perpendicular* A D ſhall fall without the *Triangle* upon the Side E B, contrived: Or, if otherwiſe, it muſt be the ſame with one of the Sides, or fall within the *Triangle*: It cannot be the ſame with either of the Sides, for then the *Angle* at B or E ſhould be a *Right Angle*: And it cannot fall within the *Triangle*, becauſe then the *Angles* at B and E muſt either be both *Obtuſe*, or both *Acute*, as hath been already proved. If therefore the *Angles* at the Baſe be of different *Affections*, the *Perpendicular* ſhall fall without; as was to be proved.

However this *Perpendicular* falleth, it muſt be always *oppoſite* to a *known Angle*; and for better Direction herein, take this *General Rule*.

RULE II. *Let your* Perpendicular *fall from the End of a* Side *given, and adjacent to an* Angle *given.*

As in this *Triangle* A B C, if there were given the Side A B, and the *Angle* at A; by the former and this *Rule*, the *Perpendicular* muſt fall from B upon the Side A C.

Fig. XXIII.

But if there were given the Side A C, and the *Angle* at A, the *Perpendicular* muſt fall from C, upon the Side A B, continued to E,——And to know whether the Side upon which the *Perpendicular* ſhall fall, muſt be continued or not, is no more than to ask whether the *Perpendicular* muſt fall *within* or *without* the *Triangle*. But, if the former Directions be not ſufficient, the Calculation will determine it. For,

RULE

Fig. XXIII. RULE III. *If the Ark found at the firſt Operation (whether it be of a* Side *or an* Angle*) be more than the* Arch *given, the* Perpendicular *ſhall fall without; if leſs, within the* Triangle.

And this will plainly appear in the Solution of the following *Caſes*.

General Rules to be obſerved in the Second Operation of the Solution of Oblique-angled Spherical Triangles, *when they are reduced into* Two Right-angled Triangles.

After the firſt Operation, whereby either the Segments of the Baſe, or the *Angle* at the *Cathetus*, or *Perpendicular*, is found; a diligent Care being had to the Addition or Subſtraction of them: The ſecond Operation will be performed by one of the Four following *Rules*.

Fig. XXIV. *RULE* I. The Sines of the Complement of the Hypotenuſes, to the Sines of the Complement of the Baſes, are in direct Proportion. So,

cs BA : *cs* DA :: *cs* BC : *cs* DC.

RULE II. The Sines of the Baſes, to the Tangents of the *Angles* at the Baſe, are in Reciprocal Proportion: So,

s BA : *s* DA :: *ct* B : *ct* D :: *t* B : *t* D.

RULE III. The Sines of the Complement of the *Angles* at the Baſe; to the Sines of the Complement of the *Angles* at the *Cathetus* (or Perpendicular) are in direct Proportion: So,

s BCA : *s* DCA :: *cs* B : *cs* B : *cs* D.

RULE IV. The Tangents of the Hypotenuſes, to the Sines of the Complement of the *Angles* at the *Cathetus* (or Perpendicular) are in Reciprocal Proportion: So,

cs BCA : *cs* DCA :: *ct* BC : *ct* DC :: *t* DC : *t* BC.

CHAP. V.

The Solution of Oblique-angled Spherical Triangles: *By letting fall a Perpendicular, whereby the* Oblique Triangle *is Reduced into Two* Right-angled.

CASE I. *In the* Oblique-angled Spherical Triangle A B C, *there is given, the Two* Sides A B *and* B C, *with the* Angle C, *oppoſite to* B A; *to find the* Angle *at* B: If it be alſo known whether the enquired *Angle* be *Acute* or *Obtuſe.*

Analogy.

As *s* B A : *s* B C : : *s* C : *s* A. Fig. XXV.

For by the Univerſal Propoſition,

(1.) As Rad. : *s* A B : : *s* A : *s* B D.
(2.) As Rad. : *s* B C : : *s* C : *s* B D.

Therefore,

As *s* A B : *s* B C : : *s* C : *s* A.

CASE II. *Two* Angles, A *and* C, *and the* Side B C, *oppoſite to the* Angle A, *given*; *To find the* Side B A: If it be known whether the enquired *Side* be more or leſs than a Quadrant.

Analogy.

As *s* A : *s* C : : *s* B C : *s* B A.

CASE III. *In the* Oblique-angled Spherical Triangle A C D, *there is given, the Two* Sides A D *and* A C, *with the* Angle D A C, *contained by them*: *To find the third* Side D C.

In this *Caſe* the Perpendicular may fall from the Extremity of either *Side*, but oppoſite to the *Angle* given. Fig. XXVI.

Analogies.

(1.) As *c t.* A C : Rad. : : *c s.* D A C : *t.* A B.
And A D — A B = B D in *Triangle* I.
But A D + A B = B D in *Triangle* II.
(2.) As *c s.* A B : *c s.* A C : : *Ra.* : *c s.* B C.
(3.) As *Ra.* : *c s.* B C : : *c s.* B D : *c s.* D C.

Therefore,

Fig. XXVI. As *cs.* AB : *cs.* AC :: *cs.* BD : *cs.* DC.

CASE IV. *Two* Sides AC *and* DC, *with the* Angle ADC, *opposite to* AC *given*; *To find the third* Side AD.

In this *Case*, Let the Perpendicular fall from the Concourse of the given *Sides*, upon the *Side* enquired, continued, if need be.

Analogies.

(1.) As *ct.* CD : Rad. :: *cs.* ADC : *t.* BD.
(2.) As *cs.* BD : *cs.* CD :: *Rad.* : *cs.* *CB.*
(3.) As *cs.* *CB* : *Rad.* :: *cs* AC : *cs* AB.

Therefore,

cs. CD : *cs.* BD :: *cs.* AC : *cs.* AB.

$$\left.\begin{array}{l}\text{And, } BD + AB = AD \text{ in the first}\\ \text{And, } BD - AB = AD \text{ in the second}\end{array}\right\}\textit{Triangle.}$$

CASE V. *Two* Sides CA *and* DA, *and their contained Angle* CDA *given*; *To find one of the other* Angles.

In this *Case* the Perpendicular may fall from the Extremity of either of the *Sides*, opposite to the *Angle* given.

Analogies.

(1.) As *ct.* AC : Rad. :: *cs.* DAC : *t.* AB.

$$\left.\begin{array}{l}\text{And } AD - AB = BD \text{ in the first}\\ \text{And } AD + AB = BD \text{ in the second}\end{array}\right\}\textit{Triangle.}$$

(2.) As *ct.* CAD : *s.* AB : *Rad.* : *t.* BC.
As *t.* *BC.* *Rad.* :: *s.* BD : *ct.* ADC.

Therefore,

As *s* AB : *ct.* CAD :: *s* BD : *ct.* ADC.

CASE VI. *Two* Angles, ADC *and* CAD, *and the Side* AC, *opposite to one of them being given*; *To find the* Side *between them.* If it be known whether the *Side* sought, or the *Side* opposite to the other given *Angle*, be *Acute* or *Obtuse*.

In this *Case*, Let the Perpendicular fall from the Extremity of the given *Side*, on the *Side* enquired, continued, if need be.

Analogies.

(1.) As *c t.* AC : Rad. :: *c s.* DAC : *t.* AB.
(2.) As *c t.* CAD : *s.* AB :: *Rad.* : *t. BC.*
(3.) As *Rad.* : *t. B*C :: *c t.* ADC : *s.* BD.

Fig. XXVI.

Therefore,

As *c t.* CAD : *s.* AB :: *c t.* ADC : *s.* BD.
And, AB + BD = AD in the 1ſt } *Triangle.*
But, DB — AB = AD in the 2d } *Triangle.*

CASE VII. *Two* Angles DAC *and* ADC, *and the* Side AC, *oppoſite to one of them,* (viz. D) *being given*; *To find the other* Angle *at* C. If it be known, whether the *Angle* enquired, or the *Side* oppoſite to the other given *Angle*, be *Acute* or *Obtuſe*.

Fig. XXVI. *Caſe* VII.

In this *Caſe*, Let the *Perpendicular* fall from the *Angle* enquired.

Analogies.

(1.) As *c t.* CAD : Rad. :: *c s.* AC : *c t.* ACB.
(2.) As *s.* ABC : *c s.* CAB :: *Rad.* : *c s. B*C.
(3.) As *c s. B*C : *Rad.* :: *c s.* BDC : *s.* BCD.

Therefore,

As *c s.* CAB : *s.* ACB :: *c s.* BDC : *s.* BCD.
And, ACB + BCD = ACD in the 1ſt } *Triangle.*
But, BCD — ACB = ACD in the 2d } *Triangle.*

CASE VIII. *Two* Angles, ACD *and* CAD, *with the* Side *between them,* AC, *being given*; *To find the third* Angle D.

Fig. XXVI. *Caſe* VIII.

In this *Caſe*, Let the *Perpendicular* fall from the Extremity of the given *Side*, and oppoſite to the *Angle* enquired.

Analogies.

(1.) As *c t.* CAB : Rad. :: *c s.* AC : *c t.* ACB.
And, ACD — ACB = BCD in the 1ſt } *Triangle.*
But, ACD + ACB = BCD in the 2d } *Triangle.*
(2.) As *s.* ACB : *c s.* CAB :: *Rad.* : *c s. B*C.
(3.) As *Rad.* : *c s. B*C :: *s.* BCD : *c s.* CDB.

Therefore,

As *s.* A C B : *c s.* C A B : : *s.* B C D : *c s.* C D B.

Fig. XXVI. *Caſe* IX. *C A S E* IX. *Two* Sides, A C *and* C D, *with the* Angle A D C, *oppoſite to one of them*, (viz. A C) *being given*; *To find their contained* Angle A C D. If it be known, whether the enquired *Angle*, or the *Angle* oppoſite to the other given *Side*, be *Acute* or *Obtuſe*.

In this *Caſe*, Let the *Perpendicular* fall from the *Angle* enquired.

Analogies.

(1.) As *ct.* C D B : Rad. : : *c s.* C D : *ct.* B C D.
(2.) As *ct.* C D : *c s.* B C D : : *Rad.* : *t. B C*
(3.) As *Rad.* : *t. B C* : : *ct.* A C : *c s.* A C B.

Therefore,

As *c t.* C D : *c s.* B C D : : *ct.* A C : *c s.* A C B.

$$\left.\begin{array}{l}\text{And } BCD + ACB = ACD \text{ in the 1ſt}\\ \text{But } BCD - ACB = ACD \text{ in the 2d}\end{array}\right\}\textit{Triangle.}$$

Fig. XXVI. *Caſe* X. *C A S E* X. *Two* Angles, D A C *and* A C D, *with the* Side *between them*, A C, *being given*; *To find either of the other* Sides, D C *or* A D.

In this *Caſe*, Let the *Perpendicular* fall from the Concourſe of the *Side* given and ſought, on the third *Side*, continued, if need be.

Analogies.

(1.) As *c t.* C A B : Rad. : : *c s.* A C : *c t.* A C B.

$$\left.\begin{array}{l}\text{And, } ACD - ACB = BCD \text{ in the 1ſt}\\ \text{But, } ACD + ACB = BCD \text{ in the 2d}\end{array}\right\}\textit{Triangle.}$$

(2.) As *c t.* A C : *c s.* A C B : : *Rad.* : *t. B C*
(3.) As *t. B C* : *Rad.* : : *c s.* B C D : *c t.* C D.

Therefore,

As *c s.* B C A : *ct.* A C : : *c s.* B C D : *ct.* C D.

C A S E

CASE XI. *The Three* Sides *being given* ; *To find an* Angle. *Fig.* XXVI. *Caſe* XI.

For the reſolving of this *Problem*, there muſt be ſome Preparation made; for that the *Univerſal Propoſition* of the Lord *Nepier*'s is not, of it ſelf, ſufficient for the Solution of this or the following *Caſe*. And therefore, the ſaid Lord *Nepier*'s, to bring theſe *Caſes* within ſome Compaſs of this his *Univerſal Propoſition*, he firſt finds the Difference of the Segments of that *Side* ; which being made the Baſe of the *Triangle*, is divided into Two Parts, by letting fall of a *Perpendicular* ; and by help of this

ANALOGY.

As the Tangent of half the Baſe,
Is to the Tangent of half the Sum of the other Two *Sides* ;
So is the Tangent of half the Difference of thoſe *Sides*,
To the Tangent of half the Difference of the Segments of the Baſe.

Thus then,

In the *Oblique-angled Spherical Triangle* A C D, there is given, the Two *Sides* (or *Legs*) A C and C D, together with the Baſe A D; to find the *Angle* C A D.

Analogies.

As the Tangent of half A D,
Is to the Tangent of half A C and A D ;
So is the Tangent of half the Difference of A C and A D,
To the Tangent of half A E. *Fig.* XXVII.
And half A D + half A E = A B in the 1ſt
But half A E — half A D = B E or B D in the 2d } *Triangle.*

Hence, to find the *Angle* at A.

As Rad. : *c t.* A C : : *t.* A B : *c s.* C A B.

CASE XII. *The Three* Angles *being given* ; *To find a* Side. *Fig.* XXVIII. *Caſe* XII.

This Caſe is but the Converſe of the laſt beforegoing, and is to be Solved after the ſame manner: If ſo be that we convert the *Angles* into *Sides*. Which, how to perform, is ſhewed in the 12th *Caſe* of the following *Tract* of the *Solution of Oblique-angled Spherical Triangles, without letting fall a Perpendicular*.

Fig. XXVII. And for the *Proportions* or *Analogies* for the Reſolving of this *Caſe* there are alſo Two Ways propoſed, to which I referr you.

CHAP. VI.

The foregoing XII Caſes, *of* Oblique-angled Spherical Triangles, *Geometrically Demonſtrated, and Reſolved, without letting fall a Perpendicular.*

FOR the Performance of what is here promiſed, theſe Six *Theorems* following, being demonſtrated, will clear.

THEOREM I.

In any Spherical Triangle, whether Right or Oblique-angled.

The Sines of the Angles are Proportional, to the Sines of their oppoſite Sides: & contra.

This is demonſtrated in *Sect.* 1. *Chap.* 4. of *Right-lined Triangles*, and is the ſame in *Spherical Triangles*, whether *Right* or *Oblique Angular.*

THEOREM II.

In all Oblique-angled Spherical Triangles, whoſe Three Sides together are leſs than a Semicircle, or 180 Degrees.

As the Sine of half the Sum of the Angles at the Baſe,
Is to the Sine of the half Difference of thoſe Angles;
So is the Tangent of half the Baſe,
To the Tangent of half the Difference of the Sides.

And alſo,

As the Co-ſine of half the Sum of the Angles at the Baſe,
Is to the Co-ſine of half the Difference of thoſe Angles;
So is the Tangent of half the Baſe,
To the Tangent of half the Sum of the Sides.

DE-

DEMONSTRATION.

I. It is already proved; That, Fig. XXVIII.
The Sine of the Sum of the Angles B *and* E,
Is to the Sine of the Difference of those Angles B *and* E;
As the Tangent of half the Baſe B E,
Is to the Tangent of half B C.

And multiplying the latter part of this Proportion, by the Tangent of half the Baſe B E: It is,

As the Sine of the Sum of the Angles B *and* E,
Is to the Sine of the Difference of those Angles B *and* E;
So is the Square of the Tangent of half B E,
To the Rectangle made of the Tangent of half B E, *and the Tangent of half* B C.

But, the Rectangle made of the Tangent of half B E, and the Tangent of half B C, is equal to the Rectangle made of the Tangent of the half Sum, and half Difference, of the Sides B A and A E: Therefore,

As the Sine of the Sum of the Angles B *and* E,
Is to the Sine of the Difference of the Angles B *and* E;
So is the Square of the Tangent of half B E,
To the Rectangle made of the Tangent of the half Sum, and half Difference of the Sides B A *and* A E.

And the former Part of this Proportion, being multiplied by the Sine of the Sum of the Angles B and E: It is,

As the Square of the Sine of the Sum of the Angles B *and* E,
Is to the Rectangles made of the Sines of the Sum, and Difference of those Angles;
So is the Square of the Tangent of half B E,
To the Rectangle made of the Tangents of the half Sum, and half Difference of the Sides B A *and* A E.

But, the Rectangle made of the Sines of the Sum and Difference of the Angles, is equal to the Rectangle made of the Sum and Difference of the Sines.

And therefore,

As the Square of the Sine of the Sum of the Angles B and E,

Is

Fig. XXVIII. Is to the Rectangle made of the Sum and Difference of the Sines;
So is the Square of the Tangent of half B E,
To the Rectangle made of the Tangent of the half Sum, and half Difference of the Sides B A and A E.

II. *As the Difference of the Sines of* B *and* E,
Is to the Sum of the Sines of the Angles B *and* E;
So is the Tangent of the half Difference of the Sides,
To the Tangent of the half Sum of the Sides B A *and* A E.

And multiplying the former Part of this Proportion, by the Difference of the Sines of the Angles; and the latter Part thereof, by the Tangent of half the Difference of the Sides A B and A E: It will be,

As the Square of the Difference of the Sines of B and E,
Is to the Rectangle made of the Sum and Difference of the Sines of B *and* E;
So is the Square of the Tangent of the half Difference of B A and A E,
To the Rectangle made of the Tangents of the half Sum, and half Difference of the Sides.

But, by the Firſt Section of this Demonſtration it is proved, That,

As the Square of the Sine of the Sum of B and E,
Is to the Rectangle made of the Sum and Difference of the Sines;
So is the Square of the Tangent of half B E,
To the Rectangle made of the Tangents of the half Sum, and half Difference of the Sides.

Therefore,

As the Square of the Sine of the Sum of B and E,
Is to the Square of the Difference of the Sine of B and E;
So is the Square of the Tangent of half B E,
To the Square of the Tangent of half the Difference of the Sides.

And alſo,

As the Sine of the Sum of the Angles B *and* E,
Is to the Difference of their Sines;
So is the Tangent of half the Baſe B E,
To the Tangent of half the Difference of the Sides B A *and* A E.

But,

Fig. XXVIII.

As the Sine of the half Sum of B and E,
Is to the Sine of their half Difference;
So is the Sine of the Sum,
To the Difference of the Sines.

And therefore,

As the Sine of the half Sum of the Angles B and E,
Is to the Sine of the half Difference of B and E;
So is the Tangent of half B E,
To the Tangent of half the Difference of the Sides B A and A E, which is the firſt Part of the *Propoſition*.

III. Having already proved,—That the Sum of the Sines of the Angles B and E, is to the Difference of the Sines of thoſe Angles; as the Tangent of the half Sum of the Sides, is to the Tangent of their half Difference. Therefore,
If you multiply the former Part of this Proportion, by the Sum of the Sines of the Angles of B and E; and the latter Part thereof by the Tangent of the half Sum of the Sides of A B and A E: Then it will be,

As the Square of the Sum of the Sines of B and E,
Is to the Rectangle made of the Sum and Difference of the Sines;
So is the Square of the Tangent of half the Sum of the Sides;
To the Rectangle made of the Tangent of the half Sum, and half Difference of the Sides.

But,

As the Square of the Sine of the Sum of B and E,
Is to the Rectangle made of the Sum and Difference of the Sines;
So is the Square of the Tangent of half B E,
To the Rectangle made of the Tangent of the half Sum, and half Difference of the Sides B A *and* A E.

Therefore,

As the Square of the Sine of the Sum of B and E,
Is to the Square of the Sum of the Sines of thoſe Angles;
So is the Square of the Tangent of half B E,
To the Square of the Tangent of the half Sum of the Sides B A and A E.

And,

Fig. XXVIII.

And,

As the Sine of the Sum of the Angles B *and* E,
Is to the Sum of the Sines of the Angles B *and* E;
So is the Tangent of half the Baſe B E,
To the Tangent of the half Sum of the Sides A B and A E.

But,

As the Co-ſine of the Sum of B and E,
Is to the Co-ſine of the Difference of the Angles B and E;
So is the Sine of the Sum of the Angles B *and* E,
To the Sum of the Sines of the ſaid Angles.

And therefore,

As the Co-ſine of the Sum of the Angles B and E,
Is to the Co-ſine of the Difference of the Angles B and E;
So is the Tangent of half the Baſe B E,
To the Tangent of the half Sum of the Sides A B and A E.
Which was to be Demonſtrated.

THEOREM III.

In all Spherical Triangles:
As the Difference of the Verſed Sines, of the Sum and Difference of any Two Sides (including an Angle,)
Is to the Diameter;
So is the Difference between the Verſed Side of the Third Side, and the Verſed Sine of the Difference of the other Two Sides,
To the Verſed Sine of the Angle comprehended by the ſaid Two Sides.

DEMONSTRATION.

Fig. XXIX. Let the Sides of the Triangle Z S P be known, and let the Vertical Angle be S Z P: Then ſhall Z S, the one Side, be equal to Z R, and P R equal to their Sum; and P B the Verſed Sine of P R, and P C, is the Difference of the Sides Z S and Z P, and the Verſed Sine of P C, is P M.

Now then, M B is the Difference between B P, the Verſed Sine of P R, the Sum of the Sides, and P M the Verſed Sine of P C, the Difference of the Sides.

M H is the Difference between P H the Verſed Sine of P S, and P M the Verſed Sine of P C the Difference of the Sides: Q V is the Diameter, and O V the Verſed Sine of P Z S the Angle Sought: And the Right Lines N C, K L, and R G, being Parallel, by the Work, their Inter-ſegments M B and R C, and alſo M H and S C, are proportional. *Fig.* XXIX.

And therefore,

M B : M H : : R C : S C;
R C : S C : : Q V : O V;

Therefore, M B : M H : : Q V : O V,
Or, M B : M H : : half Q V : half O V.

Which was to be Demonſtrated.

THEOREM IV.

In all Spherical Triangles;

As the Rectangle of the Sines of the Sides containing the Angle enquired,
Is to the Square of the Radius;
So is the Difference between the Verſed Sine of the Baſe, and the Verſed Sine of the Difference of the other Two Sides;
To the Verſed Sine of the Angle ſought.

DEMONSTRATION.

Let the Sides of Triangle A E K be given, and let the Angle at A be enquired; and from O, let fall the Perpendicular O B. *Fig.* XXX.

Now then, O E being the Difference of the Sides A E and A K, equal to A O; the Right Line O Q is the Right Sine thereof, and E Q the Verſed Sine. In like manner, S M is the Right Sine, and E M the Verſed Sine of E S; that is, of the Baſe E K; and M Q or O B, is the Difference of thoſe Verſed Sines.

O K, is the Verſed Sine of the Angle O A K, in the Meaſure of the Parallel O F; and D X is the Verſed Sine of the ſame Angle in the meaſure of a Great Circle, whoſe Diameter is H D. Now then, becauſe of their like Arches, it ſhall be

As D C : D X : : O L : O K:

And becauſe N E and O K are Parallel, as alſo E C and O B, the Angles B O K and C E N are equal: And the Triangles E C N and O K B like; and therefore the Sides N E, E C, O B, M Q and O K, are Proportional: And it will be

Fig. XXX.

As L O : D C : : O K : D X,
N E : E C : : O B : O K.

It will alſo be.

As L O * N E : D C * E C : : O K * O B : D X * O K :

Or rejecting the common Altitude O K, it will be

As L O * N E : D C * E C : : O B : D X,

The Verſed Sine of the Angle ſought. *Which was to be Demonſtrated.*

THEOREM V.

In all Spherical Triangles,

As the Rectangle of the Sines of the Sides containing the enquired Angle,
Is to the Square of the Radius;
So is the Rectangle of the Sines of the half Sum, and half Difference of the Baſe, and Difference of the Legs,
To the Rectangle made of the Radius, and half the Verſed Sine of the Angle enquired.

DEMONSTRATION.

Fig. XXXI.

It is already proved by the laſt *Theorem*, That, As L O * N E : to the Square of the Radius: So is O B : D X.

And therefore alſo,

L O * N E : Sq. of Rad. : : half O B : half D X.

And now in the following Diagram, O E G H, the Sum of E S and O E, is the double Meaſure of the Angle B S O; and the Arch O S is the Difference between the Baſe E S and E O, the Difference of the Sides A K and A E; And,

As R : half O S : : O H : half O B. And,
Half O S, and half O H, is equal to O B * Rad.

Therefore,

L O * N E : Sq. Rad. : : half O B * Rad. : half D X * Rad.

And alſo, Fig. XXXI.

L O * N E : Sq. R. : : half O S * half O H * : half D X * R.
Which was to be Demonſtrated.

THEOREM VI.

In all Spherical Triangles,

As the Rectangle of the Sines of the Sides containing the enquired Angle,
Is to the Square of Radius;
So is the Rectangle of the Sines of the half Sum, and half Difference of the Baſe, and Difference of the Legs,
To the Square of the Sine of half the Angle enquired.

DEMONSTRATION.

It is already proved by the laſt *Theorem*, That Fig. XXXI.
L O * N E : Sp. R. : . half O S * half O H : half D X * R.

But the Rectangle made of Radius, and half the Verſed Sine of an Arch, is equal to the Square of the Sine of half the Arch:

As in the foregoing Diagram; let the Arch given be D T, then is D X the Verſed Sine of that Arch; and D F the Right Sine of half the Arch; and the Triangles D F R and D T X are like.

Therefore,

D R : D F : : half D T (= D F) : half D X:
And, D R * half D X, is equal to the Square of D F.

Therefore,

L O * N A : Rq. : : half O S * half O H : the Square of D F.
Which was to be Demonſtrated.

CHAP. VII.

The Solution of the Twelve Caſes of Oblique-angled Spherical Triangles, *without any Regard had to a Perpendicular let fall, whereby to reduce it into Two* Right-angled Triangles.

Fig. XXXII. AND the *Spherical Triangle* which I ſhall make uſe of is that noted with Z S P, whoſe *Sides* and *Angles*, both in Sexagenary Degrees and Minutes, and Decimal Parts alſo, are as in this Table is expreſſed.

	De.	*Mi.*	*Centeſ.*
S P —	42	04	42.07
Z S —	30	00	30.00
P Z —	24	04	24.06
∠ Z —	104	00	104.00
∠ S —	36	08	36.13
∠ P —	46	18	46.30

Fig. XXXII. Caſe I. CASE I. *Two Sides,* Z S *and* Z P, *with the Angle* P, *oppoſite to one of them, being given; To find the Angle* S, *oppoſite to the other.*

Analogy.

As the Sine of Z S, 30 Deg. *Co-Ar.*	0.3010299
Is to the Angle P, 46 Deg. 18 Min.	9.8591186
So is the Sine of Z P, 24 Deg. 4 Min.	9.6104465
To the Sine of S, 36 Deg. 8 Min.	~~1~~9.7705850

Fig. XXXII. Caſe II. CASE II. *Two Angles,* S *and* P, *with the Side* Z P, *oppoſite to one of them, being given; To find the Side* Z S, *oppoſite to the other*

Analogy.

As the Sine of the Angle S, 36 Deg. 8 Min. *Co-Ar.*	0.2293936
Is to the Sine of Z P, 24 Deg. 4 Min.	9.6104465
So is the Sine of the Angle P, 46 Deg. 18 Min.	9.8591180
To the Sine of Z S, 30 Deg.	~~1~~9.6989581

CASE

Fig.
XXXII.
Caſe III.

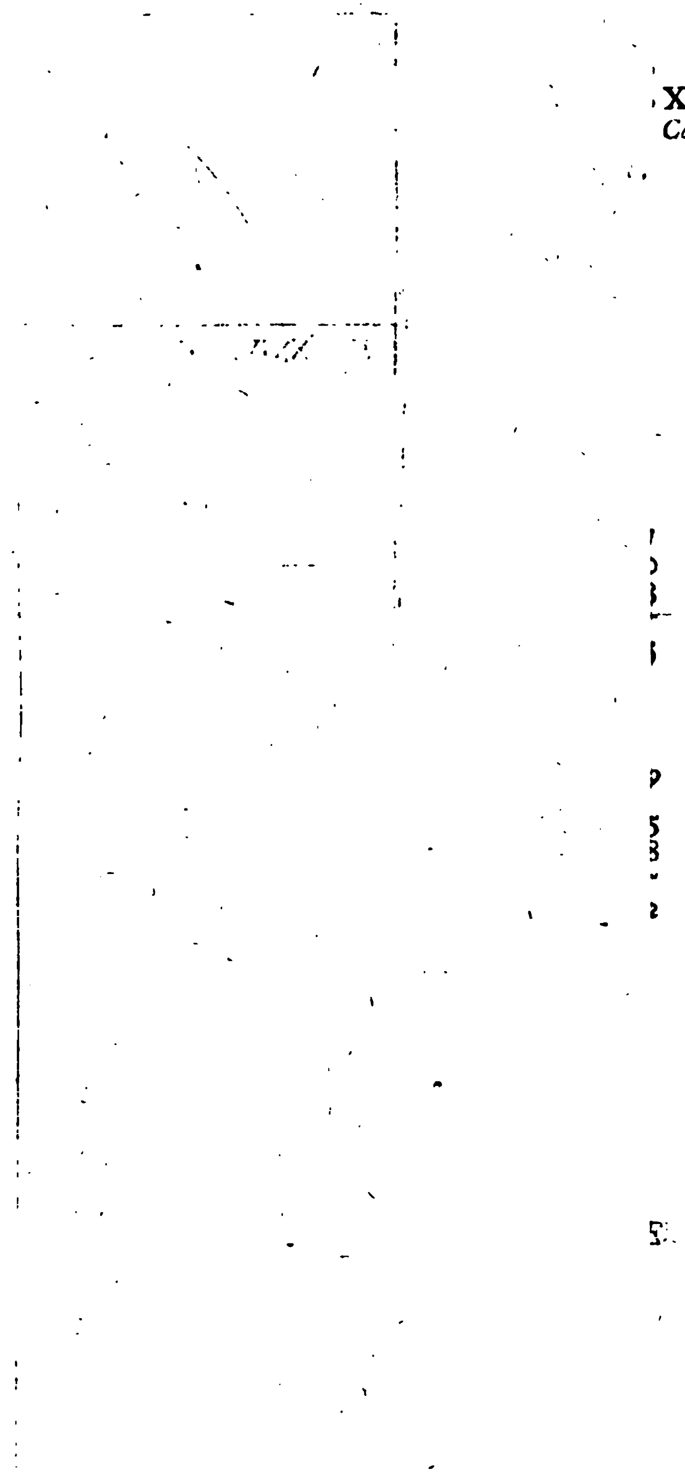


Fig.
XXXII.

Fig.
XXXII.
Caſe I.

Fig.
XXXII.
Caſe II.

CASE III. *Two Sides,* ZP *and* ZS, *with the Angle* SZP, *contained between them, being given; To find the other Angles,* S *and* P.

Fig. XXXII. *Caſe* III.

		d.	′.
Sides	ZS	30	00
	ZP	24	04
	Sum	54	04
	Difference	5	56
Half Sum		27	02
Half Difference		2	58
Half the Angle Z.		52	00

Analogy.

First Operation.

As the Sine of half the Sum of the Sides 27 d. 2′ *Co-Ar.*	2.3424577
Is to the Sine of half the Difference of the Sides 2.58	8.7139520
So is the Co-tangent of half Z, 52 d.	9.8928098
To the Tangent of half the Difference of the ∢'s S and P, 5 d. 5 m.	x8.9492195

Second Operation.

As Co-ſine of half the Sum of the Sides 27 d. 2 m. *Co-Ar.*	0.0502479
To Co-ſine of half their Difference 2 d. 58 m.	9.9994175
So Co-tangent of half ∢ Z, 52 d.	9.8928098
To Tangent of half the Sum of the ∢'s S and P 41.31′.	x9.9424752

	d.	m.
Which half Sum of the Angles S and P	41	13
Added to half Difference of the Angles	5	05
Gives the Quantity of the greater ∢ P	46	18
And ſubſtracted, gives the leſſer ∢ S.	36	08

CASE

Fig. XXXII. *Caſe* IV.

CASE IV. *Two Angles,* S *and* P, *and their contained Side* SP, *being given; To find the other Two Sides,* ZS *and* ZP.

Angles	S	36	08
	P	46	10
Sum		82	18
Difference		10	02
Half Sum		41	09
Half Difference		5	1

Analogy.

Firſt Operation.

As Sine of half the Sum of the Angles S and P 41 d. 9 m. *Co-Ar.*	0.1817525
Is to the Sine of half their Difference 5 d. 1 m.	8.9402960
So is the Tangent of half the Side S P 21 d. 5 m.	9.5856859
To the Tangent of half the Diff. of Z S and Z P 2 d. 55 m.	~~1~~8.7077344

Second Operation.

As Co-ſine of half the Sum of the Angles S and P 41 d. 9 m. *Co-Ar.*	0.1232111
To Co-ſine of half their Difference 5 d. 1 m.	9.9983331
So is the Tang. of half the Side S P 21 d. 5 m.	9.5863624
To Tan. of half the Sum of Z S and Z P 27 d. 2 m.	~~1~~9.7076060

	d.	m.
To this half Sum of Z S and Z P	27	02
Add half their Difference	2	55
Their Sum is the Greater Side Z S	29	57
And ſubſtracted gives the Leſſer Z P	24	07

CASE

CASE V. *Two Sides* ZS, *and* ZP; *with the Angle* P, *oppoſite to one of them, being Given; To find the third Side* SP.

Fig. XXXII. *Caſe* V.

Firſt Operation.

As the Sine of the Side ZS, 30 d. *Co-Ar.*	0.3010299
Is to the Sine of the Angle P, 46 d. 18 m.	9.8591185
So is the Sine of ths Side ZP, 24 d. 4 m.	9,6104465
To the Sine of the Angle at S, 36 d. 8 m.	x9.7705950

Then,

The Sides are				The Angles are	
ZS	30	00	P	46	18
ZP	24	04	S	36	08
Their Sum	54	04		82	26
Their Difference	5	56		10	10
Half Sum	27	02		41	13
Half Difference	2	58		5	05

Second Operation.

As the Sine of the half Difference of the Angles S and P, 5 deg. 5 min. *Co-Ar.*	1.0525439
Is to the Sine of half the Sum of thoſe Angles 41 d. 13 m.	9.8188250
So is the Tangent of half the Difference of the Sides ZS and ZP, 2 deg. 58 min.	8.7139520
To the Tangent of half SP, 21 deg. 4 min.	x9.5853209

Whoſe double is 42 deg. 9 min. for SP.

CASE VI. *Two Angles,* S *and* P, *with the Side* SZ, *oppoſite to one of them, being given; To find the third Angle* Z.

Fig. XXXI. *Caſe* VI.

Firſt Operation.

As the Sine of the Angle at P, 46 d. 18 m. *Co-Ar.*	0.1408813
Is to the Sine of the Side ZS, 30 deg.	9.6989700
So is the Sine of the Angle at S, 36 deg. 8 min.	9.7700063
To the Sine of the Side ZP, 24 deg. 4 min.	x9.6098576

Then,

Fig. XXXII.

Then,

The Sides				The Angles	
Z S	30	00	S	36	08
Z P	24	04	P	46	18
Their Sum	54	04		82	26
Their Difference	5	56		10	1
Half Sum	27	02		41	1
Half Difference	2	58		5	0

Second Operation.

As the Sine of half the Difference of the Sides Z S and Z P, 2 d. 58 m. *Co-Ar.* } 1.286047

Is to the Sine of half the Sum of the Sides, 41 d. 13 m. 9.81882

So is the Tangent of half the Difference of the Angles S and P, 5 d. 5 m. } 8.94916

To the Co-tangent of half the Angle Z, 52 deg. 10.05404

Whoſe Double, 104 deg. is the Angle Z required.

Fig. XXXII. *Caſe* VII.

CASE VII. *Two Angles*, S *and* P, *and a Side* Z S, *oppoſit* *One of them, being given; To find the Side between them*, S

Firſt Operation.

As the Sine of the Angle at P, 46 deg. 18 min. *Co-Ar.* 0.14095

Is to the Sine of the Side Z S, 30 deg. 9.6989

So is the Sine of the Angle at S, 36 deg. 8 min. 9.7706

To the Sine of the Side Z P, 24 deg. 4 min. 19.0105

Then,

The Sides				The Angles
Z S	30	00	S	36
Z P	24	04	P	46
Their Sum	54	04		82
Their Difference	5	56		10
Half Sum	27	02		41
Half Difference	2	58		5

S

Second Operation.

the Sine of half the Difference of the Angles S and P, 5 deg. 5 min. *Co-Ar.* 1.0525439
Is to the Sine of half their Sum, 41 deg. 13 min. 9.8188250
is the Tangent of half the Difference of the Sides Z S and Z P, 2 deg. 58 min. 8.7145345
To the Tangent of half the enquired Side, 21 deg. 4 ½ min. x9.5859034
ıe Double whereof is 42 deg. 9 min. for the Side S P.

Fig. XXXI. Case VIII.

ASE VIII. *Two Sides,* Z S *and* Z P, *and the Angle* P, *opposite to the Side* Z S, *being given; To find the Angle* Z, *contained between the Two given Sides.*

First Operation.

the Sine of Z S, 30 deg. *Co-Ar.* 0.3010299
Is to the Sine of the Angle P, 46 deg. 18 min. 9.8591186
is the Sine of Z P, 24 deg. 4 min. 9.6104465
To the Sine of the Angle at S, 36 deg. 8 min. x9.7705950

Then,

The Sides				The Angles	
Z S	30	00	S	36	08
Z P	24	04	P	46	18
Their Sum	54	04		82	26
Their Difference	5	56		10	10
Half Sum	27	02		41	13
Half Difference	2	58		5	05

Second Operation.

he Sine of half the Difference of the Sides Z S and Z P, 2 deg. 58 min. *Co-Ar.* 1.2860479
s to the Sine of half their Sum, 27 deg. 2 min. 9.6575423
is the Tangent of half the Difference of the Angles S and P, 5 deg. 5. min. 8.9491675
'o the Co-tangent of half Z, 52 deg. x9.8927577
The Double whereof, 140 deg. is the Angle at Z required.

Fig. XXXI. *Caſe* IX. CASE IX. *Two Sides* Z S *and* Z P, *with the Angle* Z, *comprehended between them; To find the third Side* S P.

You muſt firſt find the Two other Angles S and P (by the 3d *Caſe*) and then you may find the Side S P (by the 1ſt *Caſe* hereof.)

And ſo,

		d.	m.
The Angle	S	36	08
will be found to be	P	46	18
And the Enquired Side S P		42	39

This *Problem* may be otherwiſe reſolved at Two Operations, by a *Problem* next following the Twelve *Caſes*.

Fig. XXXI. *Caſe* X. CASE X. *Two Angles,* S *and* P, *with the Side between them,* S P *being given; To find the third Angle at* Z.

This Caſe muſt be reſolved in the ſame manner as the foregoing, it being but the Converſe thereof: By finding firſt the Two Sides Z S and Z P, (by the 4th *Caſe*) and then the Angle Z (by the 1ſt *Caſe*.) So you ſhall find

		d.	m.
The Side	Z S	30	00
to contain	Z P	24	04
And the Enquired Angle Z		104	00

Fig. XXXI. *Caſe* XI. CASE XI. *Three Sides,* Z S, Z P *and* S P, *being given; To find the Angle at* Z, *oppoſite to (the Baſe)* S P.

Note, I call that Side the Baſe (which ever it be), that is oppoſite to the Enquired Angle. Then,

First Operation.

As the Radius, or Sine of 90 deg.	10.
Is to the Sine of the Side Z S, 30 deg.	9.6989700
So is the Sine of the Side Z P, 24 deg. 4 min.	9.6104465
To a fourth Sine, *viz.* 11 deg. 46 min.	19.3094165

Fig. XXXI.
Caſe XL.

Then,

		d.	m.
The Sides are	Z P	24	04
	Z S	30	00
	S P	42	09
Their Sum		96	13
Half Sum		48	06½
The Baſe S P		42	09 Subſt.
Difference		5	57½

Second Operation.

As the Sine of the fourth Sine before found, 11 deg. 46 min. *Co-Ar.*	0.6905835
Is to the Sine of the half Sum, 48 deg. 6½ min.	9.8718115
So is the Sine of Difference 5 deg. 57½	9.0162186
To a ſeventh Sine.	19.5786136
Half this ſeventh Sine is	9.7893068

Which is the Sine of 38 deg. whoſe Complement 52 deg. is half the enquired Angle at Z, *viz.* 104 deg.

Another Way to reſolve this Caſe.

Take half the Difference between the Two Sides that contain the required Angle; and add it to half the Baſe, and likewiſe ſubſtract it from the ſame, noting the Sum and Difference. Then,

The Arithmetical Complements of the Sines of the Sides comprehending the enquired Angle, added to the Sines of the Sum and Difference before found, half the Sum of them ſhall be the Sine of half the Angle required.

		d.	m.
The Side	Z S is	30	00
	Z P	24	04
Their Difference is		5	56
The half Difference is		2	58
Which added to half S P		21	04½
The Sum is		24	02½
And ſubſtracted, the Difference is		18	6

Then,

The Sine of the Side Z S, 30 deg. *Ar. Co.*	0.3013299
The Sine of the Side Z P, 24 deg. 4 min. *Ar. Co.*	0.3895535
The Sine of the Sum, 24 deg. 2 ½ min.	9.6190219
The Sine of the Difference, 18 deg. 6 ½ min.	9.4924864
The Sum	19.7933917
The half whereof is the Sine of 52 deg. 1 min. half of the Angle at Z.	9.8966958

CASE XII. *Three Angles* Z, P *and* S, *being given*; *To find any of the Sides.*

This is the Converſe of the former, and is to be reſolved after the ſame manner: But firſt, the Angles muſt be turned into Sides as followeth: For,

The Two Leſſer Angles, S and P, are always equal unto Two Sides of another Triangle, comprehended by the Arches of Three Great Circles drawn from their Poles, and the Complement of the Greater Angle Z to a Semicircle (or 180 deg.) muſt be taken for the third Side. Therefore, in this Triangle thus converted, you ſhall by the preceding 11th Caſe find an Angle; that Angle ſo found, ſhall be one of the Three Sides enquired.

Fig. XXXIII. As in the Triangle A C D, the Poles of thoſe Arches are H, R and Q; which connected, make the Triangle H R Q the Sides of the former Triangle, being equal to the Angles of the latter; taking for one of them the Complement of the Greater Angle to a Semicircle.

So A D is equal to the Angle at H; whoſe Meaſure is the Arch E N.

D C is equal to the Angle Q; whoſe Meaſure is the Arch M P.

And A C is equal to the Complement of the Angle H R Q; whoſe Meaſure is G L.

Therefore, if the Angles A, D and C, be given, the Sides Q R, Q H and R H, are likewiſe given.

If therefore we reſolve the Triangle H R Q by the Directions of the 11th Caſe beforegoing, the Angle ſo found ſhall be the Side required.

CHAP. VIII.

A Problem and Theorem, by Way of Supplement.

In any Spherical Triangle, Two Sides, D B and B C, with the Angle B, included between them being given; To find the other Angles, B and C, by Two Proportions.

THIS Problem was invented by the Lord *Nepier*, and is celebrated, not leſs for its Subtilty, than Uſefulneſs, in reſolving this *Caſe*, without letting fall a Perpendicular, or any Ambiguity. Fig. XXXIV.

ANALOGIES.

1. As the Sine of half the Sum of the Sides including the Angle given,
Is to the Sine of half the Difference of thoſe Sides;
So is the Co-tangent of half the Vertical Angle,
To the Tangent of half the Difference of the Two unknown Angles.

Then,

2. As the Co-ſine of half the Sum of the Two given Sides,
Is to the Co-Sine of half the Difference of thoſe Sides,
So is the Co-tangent of half the Vertical Angle,
To the Tangent of half the Sum of the Angles.

The half Difference before found, being *added* to this *half Sum* now found, gives the *Greater*, and *ſubſtracted* therefrom, gives the *Leſſer* of the Two unknown *Angles*.

DEMONSTRATION.

Suppoſe A, E, G, Poles of the Sides D B, D C, B C, therefore the Arch A E is equal to the Angle D; the Arch E G equal to the Angle C; the Arch A G equal to 180 deg. wanting the Angle B. Suppoſe the Ark E O, equal to the Ark E G, equal to the Ark E P. Then if the Points G A O E P be Stereographically projected, the Right Line A E will be equal to Tangent of half the Angle at D; and A G, equal to the Co-tangent of half B; and A O, equal to the Tangent of half the Difference of the Angles; and A P, equal to the Tangent of half the Sum of the Angles; and O G P will be a Semicircle, deſcribed from its Pole E, through G, whoſe

Fig. XXXIV. whoſe Center ſuppoſe, n. Then take $B\lambda$, equal to BC, equal to BL; and draw the Diameter $BmA\beta$, and DK, parallel to Bm, parallel to δH. Therefore, DB is equal to δB; and mK, equal to mH; and $F\lambda$, equal to DL. Draw $\lambda\omega$ perpendicular to $D\delta$, then the Triangle $\lambda\delta y$ is equiangular to the Triangle DLy; and DLK, equiangular to the Triangle $\lambda\delta\omega$; therefore, $Ly : \delta y :: DL : \delta\lambda$: And, as $DL : \delta\lambda :: LK : \delta\omega$; and therefore $L\delta$, parallel to $K\omega$. But the Points $\delta\lambda H\omega$ are in a Circle, whoſe Diameter is $\delta\lambda$: Therefore the Angle $\lambda\omega H$, is equal to (the Angle $\lambda\delta H$, equal to the Angle $L\delta D$, equal to) $K\omega y$. Therefore the Angle $K\omega H$, equal to $y\omega\lambda$, equal to a Right Angle: Therefore, $mH = m\omega = mK$. Alſo, $\lambda K : LK :: (\omega\lambda : \omega R ::)$ Tangent of the Angle $D\lambda\delta$: the Tangent of the Angle $D\delta L$; that is,

As the Sine of the *Sum* of the Sides,
Is to the Sine of the *Difference* of the Sides;
So is the Tangent of half the *Sum* of the Sides,
To the Tangent of half the *Difference* of the Sides.

Therefore,

As the Sine of the *Sum* of the *Angles*,
Is to the *Sine* of the *Difference* of the *Angles*;
So is the *Tangent* of half the *Sum* of the *Angles*,
To the *Tangent* of half the *Difference* of the *Angles*.

But,

The *Sine* of the *Sum* of the Sides,
Is to the *Sine* of the *Difference* of the Sides,
(as the Sine of the Sum of the Angles : Sine of the *Difference* of the Angle;)
So is the *Tangent* of half the Sum of the Angles,
To the *Tangent* of half the *Difference* of the Angles.

That is, As $\lambda K : \lambda H :: AP : AO$.

Therefore,

$$\tfrac{1}{2}\lambda K + \tfrac{1}{2}\lambda H : \tfrac{1}{2}\lambda K - \tfrac{1}{2}\lambda H :: \tfrac{1}{2}AP + \tfrac{1}{2}AO : \tfrac{1}{2}AP - \tfrac{1}{2}AO.$$

That is, $\lambda m : m\omega :: An : nG$.

But alſo, the Angle $m\lambda\omega$, is equal to the Angle GAn; therefore (by 7 *Pr.* 6 *Lib. Eucl.*) the Triangles $\lambda m\omega$, and AnG, are equiangular; and therefore the Triangles $\lambda H\omega$ and AOG; and alſo the Triangles $\lambda K\omega$; and APG, are equiangular. Therefore,

Fig. XXXIV.

$\lambda\omega : \lambda H :: AG : GO$, and $\lambda\omega : \lambda K :: AG : AP$.
~~But, $D\lambda : \lambda\omega :: DL\ (LK =) : \lambda H$,~~
And $\delta\lambda : \lambda H :: AG : AO$, and $\lambda\omega : \lambda K :: AG : AP$.
But $D\lambda : \lambda\omega :: DL\ (LK =) : \lambda H$; and $\delta\lambda : \lambda\omega :: \delta L\ (LH) : \lambda K$.

Therefore,

$D\lambda : DL :: (\lambda\omega : \lambda H ::)\ AG : AO$.
And $\delta\lambda : \delta L :: (\lambda\omega : \lambda K ::)\ AG : AP$.

That is,

As the Sine of half the *Sum* of the Sides,
Is to the Sine of half the *Difference* of the Sides;
So is the Co-tangent of half the *Vertical Angle*,
To the Tangent of half the *Difference* of the *Angles*.

And,

As the Co-ſine of half the Sum of the Sides,
Is to the Co-ſine of half the *Difference* of the Sides;
So is the Co-tangent of half the *Vertical Angle*,
To the Tangent of half the *Sum* of the Angle.

COROLLARIES.

In any Spherical Triangle,

I. Tan. $\frac{1}{2}$ Baſe : Tan. $\frac{1}{2}$ Sum of the Sides : : Tan. : $\frac{1}{2}$ Differ. of the Sides : Tan. $\frac{1}{2}$ Differ. of the Segments of the Baſe AG, made by a perpendicular Arch falling thereon from E.

$$AG : AO :: AP : Ad.$$

II. Tan. $\frac{1}{2}$ Baſe : Tan. $\frac{1}{2}$ Sum Sides : : Co-ſine. $\frac{1}{2}$ Sun ∠'s : Co-ſine $\frac{1}{2}$ Diff. ∠'s. $AG : AP :: \delta\lambda : \delta L$: For the Angle $EAG =$ the Arch DB, and the Angle $AGE =$ to the Arch BC.

III. Tan. $\frac{1}{2}$ Baſe : *t*. $\frac{1}{2}$ Sum Sides : : *s*. $\frac{1}{2}$ Sum of Ang. : *s*. $\frac{1}{2}$ Diff. ∠s.

$$AG : AO :: D\lambda : DL.$$

IV. If

Fig. XXXIV. IV. If the Angle E G A be ſuppoſed Right, then $AP \times AO = AGq$. that is, In a Right-angled Spherical Triangle, E G A, the Tang. $\frac{1}{2}$ Hypot. $\times \frac{1}{2}$ Per. $\times$ Tan. $\frac{1}{2}$ Hypot. $-\frac{1}{2}$ Per. is $=$ the Square of the Tangent of half the Baſe.

CHAP. IX:

Some Problems *in Plain Triangles, which come not within the Limits of the Twelve foregoing Caſes.*

PROBLEM I.

In the Oblique-angled Triangle C B D, *there is given the Angle at* C, (116.20 deg.) *the Side* D B, *oppoſite thereunto* (1270 *Foot*:) *And the Two Sides*, D C *and* C B, *in One Sum*, (1496 *Foot*:) *To find the other Angles* D *and* B, *and the Sides ſeverally.*

CONSTRUCTION.

Fig. XXXV. EXtend the Side of the Triangle D C, to A, making C A equal to C B; and draw the Line B A, ſo ſhall you have conſtituted a new Triangle A O B, in which you have given, (1.) The Side A D, equal to the Sum of the Sides D C and C B, (1497.) And (2.) the Side D B (1270.) And (3.) being you have the Angle D C B, (116 d. 12 m.) you have alſo the Angle B C A in the other Triangle, (63 80 d.) the Complement thereof to a Semicircle, or 180 deg.

Then in the Triangle A C B, having the Angle at (63.80) you have alſo the Sum of the other Two Angles, C A B and A B C, equal to the given Angle D C B, (116.20) the half whereof is the Angle C A B (58.10 d.) to which the Angle A B C is equal, becauſe the Sides C A and C B ſubtending thoſe Angles, are equal: And now in the Triangle D A B you have given, (1.) the Side D A, 1497. (2.) The Side D B, 1270. And (3.) the Angle, D A B (oppoſite to D B) (58.10 d.) by which you may find the whole Angle D B A (by *Caſe* I. of Oblique Triangles: For,

As Log. D B : *s.* C A B :: Log. D A : *s.* A B D.
1270 . 58.10 d. 1497. 95.53 d.

Fig.
XV.

Fig.
XVI.

Fig.
XXXI

Fig.
XXX

From which, the Angle C B A 58.10 d. being ſubſtracted, there will remain 37.43 d. for the Angle C B D, and then the Angle C D B will be 26.37 d. the Remainder of the other Two, C and B, to 180 d. *Fig.* XXXV.

And for the Sides, they may be found by the firſt Caſe of Oblique Triangles, thus:

As *s* C : Log. D B : : *s* B : Log. D C.
116.20 d. 1270 37.43 d. 865

As *s* B : Log. D C : : *s* D : Log. C B.
37.43 d. 865 26.37 d. 632

PROB. II.

In the Oblique-angled Triangle C D B *there is given the Angle at* C, *(*116.20 *d.) The Side* D B, *oppoſite thereunto (*1270 *Foot.) And the Difference of the other Two Sides* C D *and* C B, (233 *Foot.) To find the Two unknown Sides ſeverally; and the Angles at* D *and* B.

CONSTRUCTION.

IN the Given Triangle, make C A equal to C B, and draw the Line B A; ſo is the Given Triangle C D B, reduced into Two Oblique-angled Triangles C A B and A D B; in which laſt is given the Two Sides, A D and D B, but never an Angle; and in the other, only the Angle at C, and never a Side. *Fig.* XXXVI.

But (by Conſtruction) the Side C A being made equal to the Side C B, the Angles C A B and C B A are equal, and the Sum of them, equal to the Complement of the Angle at C, (116.20 d.) to 180 d. that is, to 63.80 d. the half whereof 31.90 d. is equal to the Angle C A B, and the Complement thereof to 180 d. is 148.10 d. equal to the Angle D A B.

And now in the Triangle D A B, there is given, the Sides A D and D B, and the Angle at A, to find the other Angles, by *Caſe* I. and II. of Oblique Triangles, Thus,

As Log. D B : *s* A : : Log. A D : *s*. A B D.
1270 148.10 d. 233 5.53 d.

Which added to the Angle C B A, before found, 31.90 d. the Sum will be 37.43 d. for the Angle C B D.

Fig. XXXVI.

Then,

As s D C B	: Log. D B	: : s C B D	: Log. C D
116.20 d.	1270	37.43 d.	865

And,

As s D C B	: Log. D B	: : s C D B	: Log. C B
116.20 d.	1270	26.37 d.	632

P R O B. III.

In the Right-angled Plain Triangle A B C, *there is given, the Baſe* A B, *(*28 *Foot.) And the Hypotenuſe* C B, *and Perpendicular* C A, *in one Sum,* (viz. 56 *Foot) to find the other Sides and Angles ſeverally.*

Fig. XXXVII. IN the given Triangle A B C, extend the Perpendicular A C to D, making C D equal to C B; ſo ſhall you have conſtituted a new Triangle D A B; in which there is given, (1.) The Right Angle at A. (2.) The Baſe A B 28 Foot. (3.) The Side A D, equal to the Sum of the Sides A C and C B, by which you may find the Angles A D B, and A B D, *(by Caſe I. of R. Ang. Tri.)*

As Log. A B	: Rad.	: : Log. D A	: t D B A
28	t. 45.00 d.	56	63.43 d.

Whoſe Complement 26.57 d. is the Angle A D B, to which the Angle C B D is equal, becauſe the Sides C B and C D, ſubtending them, are equal by Conſtruction. Then from the Angle D B A 63.43 d. before found, you ſubſtract the Angle D B C, 26.57 d. laſt found, the Remainder 36.86 d. is equal to the Angle C B A, and the Complement thereof, 53.14 d. to the Angle C A B.

And then,

As s A C B	: Log. A B	: : s C B A	: Log. C A
53.14 d.	28	36.86 d.	21
	And ſo : :	Rad.	: Log. C B.
		90 d.	56

Otherwiſe, by this Theorem. Fig. XXXVII.

If from the Square of the Sum of the Baſe and Perpendicular D A 56 (3136) *you ſubſtract the Square of the Baſe* A B 28, (784) *and divide the Remainder* (2352) *by double the Sum of the Hypotenuſe and Perpendicular* 56, (viz. 112), *the Quotient* (21) *will give the Perpendicular: Which ſubſtracted from the Sum* (56), *leaves* (35) *for the Hypotenuſe.*

PROB. IV.

The Three Sides of a Right-lined Triangle, *being given*; *To find the* Area.

RULE.

ADD the Three Sides of the given Triangle together, and take the half thereof: From which half Sum, ſubtract each Side of the Triangle ſeverally, and note the ſeveral *Differences*: Then multiply the *Half Sum*, by any one of the *Differences*, and that *Product* multiply by another of the *Differences*, and that *Product* multiply by the third *Difference*: The *Square Root* of this third *Product*, ſhall be the *Area* of the given Triangle.

Example.

In the *Triangle* A B C, whoſe Three Sides are A B 20, A C 34, and B C 42: Their *Sum* is 96, the half whereof is 48: From which ſubtract the ſeveral Sides, 20, 34, and 42, and there will remain theſe *Differences*, 28, 14, 6.——Then, multiply 48 (the half Sum) by 28 (the firſt Difference) the Product will be 1344: Which multiplied by 14 (the ſecond Difference (the Product will be 18816: And this Product multiplied by 6 (the third Difference) produceth 112896: The *Square Root* whereof is 336, for the *Area* of the Triangle A B C. Fig. XXXVIII.

By Logarithms.

To the Logarithm of half the *Sum* of the *Sides*, add the *Logarithms* of the ſeveral *Differences* of the *Sides* from the half *Sum*: Half the *Sum* of thoſe *Logarithms* ſhall be the Logarithm of the *Area* of the Triangle.

Example.

Fig. XXXVIII.

The half *Sum* of the Sides		48	1.6812412
The Difference of	A B	28	1.4471580
	A C	14	1.1461280
	B C	6	0.7781512
whoſe Logarithm is			
	Their Sum		5.0526784
	The Half Sum		2.5263392

Which is the Logarithm of 336, the *Area* of the *Triangle.*

PROB. V.

By the Three Sides *given; To find the Point in the longer* Side, *where a* Perpendicular *ſhall fall, and the Length of that* Perpendicular.

RULE.

TAKE the *Sum* and *Difference* of the Two *Sides* containing the *Angle* from whence the *Perpendicular* is to fall, and multiply them together, and divide the Product by the third *Side*, upon which the *Perpendicular* is to fall, the Quotient added to the third *Side*, or ſubſtracted from it, ſhall be the Double of the *Greater* or *Leſſer Segment* on either *Side* of the *Perpendicular.*

Example.

Fig. XXXVIII. In the former *Triangle* A B C, the *Sum* of the *Sides* A B and A C, is 54, and their Difference is 14; which multiplied together, produce 756; which divided by the *Side* B C 42, the Quotient is 18; which added to the *Side* B C 42, gives 60; the half whereof 30, is the *Greater Segment* of the *Side* B C; or the Quotient 18, ſubſtracted from the *Side* BC, leaves 24; the half whereof 12, is the *Leſſer Segment* B D, where the *Perpendicular* is to fall.

By Logarithms.

To the *Sum* of the *Logarithms* of the *Sum* and *Difference* of the *Sides* containing the *Angle* from whence the *Perpendicular* is to fall; ſubſtract the *Logarithm* of the *Side* upon which it is to fall. The Remainder ſhall be the *Logarithm* of a Number; which added to, or ſubſtracted from the *Side* on which the *Perpendicular* is to fall, ſhall be the Double of the *Greater* or *Leſſer Segments* of the *Side* on which the *Perpendicular* is to fall.

Example.

Example.

Fig. XXXVIII.

The Sum of A B and A C is 54	1.7323937
The Difference of A B and A C is 14	1.1461281
Their Sum	2.8785218
The *Logarithm* of the third *Side* B C 42, ſubſtract	1.6232493
Remains	1.2552725

Which is the *Logarithm* of 18. Which 18 added to B C 42, makes 60; the half whereof 30, is the *Greater Segment* C D; or 18 ſubſtracted from B C 42, leaves 24; the half whereof 12, is the *Leſſer Segment* B D.

For the Length of the *Perpendicular.*

R U L E.

Multiply the *Sum* of A B and B D 32, by the *Difference* of A B and B D, 8; the Product will be 256, whoſe *Square Root* is 16; for the Length of the *Perpendicular* A D.

By Logarithms.

Half the *Sum* of the *Logarithms* of 32 and 8, the *Sum* and *Difference* of A B and B D is the *Logarithm* of the *Perpendicular.*

The *Logar.* of the Sum of A B and B D 32,	1.50515
The *Logar.* of the *Difference* of A B and A D 8.	0 90309
Their *Sum*	2.40824
The half	1.20412

Which is *Logarithm* of 16, the Length of the *Perpendicular* A D.

P R O B. VI.

The Baſe B A *(or longeſt* Side) *of a* Plain Triangle B A D, *being* 47.8 P. *and a* Perpendicular D C, *let fall from the* Angle *oppoſite to that* Side 17.33 P. *being given; To find the* Area *of that* Triangle.

TO the *Logarithm* of half the given *Side*, add the *Logarithm* of the *Perpendicular*; the *Sum* of thoſe *Logarithms* ſhall be the *Logarithm* of the *Area* of that *Triangle.* Fig. XXXIX.

Fig. XXXIX.
The *Baſe* B A, is 47.8 —
The half *Baſe* 23.9 — *Log.*—1.3783978
The *Perpendicular* D C 17.33— *Log.*—1.2387986
The *Logarithm* of 414.2 2.6171965
Which is the *Area* of the *Triangle.*

After the ſame manner, if the *Baſe* of a *Triangle* were 42, and the *Perpendicular* 16, the *Area* will be found to be 336.

PROB. VII.

There is an Oblique-angled Plain Triangle A B C, *one of whoſe* Angles *at the Baſe* B C *is* 53.48 *d. And the* Sum *of the* Two Sides *oppoſite to the Baſe* B C, viz. A B *and* A C, *is* 129, *and the Baſe* C B, *is longer than the longer ſide* A C, *by* 16.8. *The reſpective* Sides *and* Angles *of the* Triangle *are required.*

CONSTRUCTION.

Fig. XL. DRaw a *Right Line* B C, at Liberty, for the Baſe; and upon one end thereof, at B, make an *Angle* A B C, to contain 53.84 d. Then the *Sum* of the Two *Sides* A B and A C being together 120, break it into any Two equal or unequal Parts, as into 43.2 and 76.8, then (by the *Propoſition*) muſt the Baſe B C, be 93.6, which is greater than A C by 16.8.—So in the *Oblique-angled Triangle* A B C, you have given, (1.) The *Side* A B 43.2. (2.) The *Side* A C 76.8. (3.) The *Angle* oppoſite thereto, A B C 53 d. 50 m. by which you may find the *Angle* A C B. For,

As A C 76.8 : Is to *s* A B C 53.84 d.
So is A B 43.2 : To *s* A C B 27.02 d.

PROB. VIII.

Of the Menſuration of the Area *of a* Spherical Triangle.

Lemma I. *The Lunary Superficies of the Hemiſphericks, are as the* Angles *of the ſame Superficies.*

Fig. XLI. THE Proof of this *Lemma* (amongſt many other Ways) may be this:

Let the Meridian *Semicircle* A E D be imagined to be equally moved over the Longitude of the *Equator* B E C, upon the *Poles* A and D. Therefore the *Angles* on the other ſide of A and D (to

Fig. XLI. (to wit, F and G) ſhall be as the Times. The Superficies alſo, K and M, ſhall be as the Times; therefore F ſhall be to G, as the Superficies K, to the Superficies M. And G unto M, as the Superficies M, to the Superficies K, &c. howſoever they can be taken. For,

Thoſe Things that agree to a Third,
Agree among themſelves.

COROLLARY.

Therefore, by Compoſition,

$$\text{As } F + G : G :: K + M : M,$$
$$\text{Or, As } F + G : F :: R + M : K.$$

That is,

As the Two *Right Angles* are unto G,
So is half the *Spherical Superficies*, to M.
And, As Two *Right Angles*, are to F;
So is half the *Spherical Superficies*, to K.

Lemma II. *The Triangle* G, *is equal to the Triangle* H, *becauſe the Angles and Sides of one, are equal to the Angles and Sides of the other: To wit*, A = D, B = E, C = F. *Alſo*, L = O, M = P, N = Q. *Therefore they are Congruous and Equal.* Fig. XLII.

THEOREM.

The Exceſs of the Three Angles, over and above Two Right Angles, divided by 720, *ſhews what the Area of the Triangle, is, in reſpect of the whole Spherick.*

$$\text{For by } \textit{Lemma} \text{ I.} \begin{cases} 180 : A :: \frac{1}{2}\,\text{Sph.} : G + R \\ 180 : B :: \frac{1}{2}\,\text{Sph.} : G + S \\ 180 : C :: \frac{1}{2}\,\text{Sph.} : G + T = H + T. \end{cases}$$

(by the Second Lemma.)

Therefore, As 180 : A + B + C :: half the Spherick : to 3 G + R + S + T, (*by* 24 *El.* 5 *Ev.*)

As 180 : A + B + C — 180 ::
:: So half the Spherick : 3 G + R + S + T—half the Sph.
But G + R + S + T is equal to half the Spherick;

Therefore,

Fig. XLII. Therefore, $3\,G + R + S + T -$ half the Sph. $= 2\,G$.
So that, As $180 : A + B + C - 180 ::$
$::$ So is half the Spherick : to $2\,G$.

And the Antecedent Terms being Quadropled, it ſhall be,

As $720. : A + B + C - 180 :: 2$ Sphericks : $2\,G$.
And ſo the Spherick to G. Therefore,

$\left.\dfrac{A + B + C - 180}{720}\right\}$ ſhews what part the *Triangle* is of the whole Spherick.

Theſe Things are likewiſe true in all *Spherical Polygons*, of what *Ordinate* Figure ſoever they be; or *In-ordinate*, ſo all the *Angles* be given. And the Reaſon is, becauſe all *Polygons* may be reſolved into *Triangles*. Therefore, this *Rule* ſhall hold in theſe *Multangles* alſo.

RULE.

Multiply 180 d. by the Number of the *Angles*; ſubduct the Product out of the Aggregate of all the *Angles* increaſed by 360 d. The Reſidue divided by 720 d. gives the *Area* of the *Polygon*.

Of the Completion of a Solid Body.

FRom the foregoing Menſuration of the *Area* of a *Spherical Triangle*, this Fruit ariſeth.

If the Radius of the Sphere be 100000.00, the *Side* of an inſcribed *Icoſaedrum* ſhall be 105146.22, equal to the Subtenſe of 63 deg. 26 min. 10 ſec. Therefore the *Plain Equilateral Triangle* F E O (in the *Icoſaedrum*) anſwers to the *Equilateral Spherical Triangle* in the *Sphere*; whoſe Three *Spherical Triangles* are connected in the *Plain Angles*, in the ſame Points, F, E, O.

And the *Sides* of this *Spherical Triangle* are ſeparately taken 63 deg. 26 min. 10 ſec. to wit, becauſe their Subtenſes F E, E O, O F, in the *Plain Triangle*, are equal to one another.

Fig. XLIII. Let fall now the *Perpendicular* E P, the *Spherical Triangle* E P O ſhall be Rectangled; where, over and above the *Right Angle* at P, are given, E O and P O, equal to half E O; wherefore the *Vertical Angle* P E O ſhall be 36 deg. juſt, and the whole *Angle* at E, 72 deg. And the *Sum* of the Three equal *Angles*, E, F, O, ſhall be 216 deg. from whence taking Two *Right Angles*,

Angles, equal to 180 Deg. there remains 36 Deg. Therefore, the *Triangle* E F O is $\frac{36}{720}$, of the whole *Spherick*; that is $\frac{1}{20}$ Part: And this moſt truly, for 20 *Pylamides* F E O C, fill the Solid Place of the *Icoſaedrum*. And ſo 20 *Spherical Baſes*, (covered over with 20 *Triangular Plain Baſes*,) compleat the whole *Spherick*. Fig. XLIII.

F E O C is One of the 20 *Pyramids* in the *Icoſaedrum*: The *Plain Triangle* B, is One of the *Hedræ* or *Baſes*: C is the *Center* of the *Body*, or *Sphere*, that circumſcribes it.

4.77 Icoſaedres
4.24 Dodecaedres
9.244 Octaedres
8.000 Cubes

} Fill a *Solid Place*, as will appear out of Precedent Practice by *Triangles*, and *Spherical Polygons*.

That is to ſay, None of the Five *Regular Bodies* fill a *Solid Place*, the *Cube* only excepted.

> Contrary to what *Potamon*, and from him *Ramus*, and all that have followed *Ramus*; to wit, *Snellius*, and others, have delivered.

JANUA MATHEMATICA.

SECTION III.

Spherical Trigonometry, *Geometrically Performed.*

Fig. XLIV. AS the *Sides* of *Plain Triangles* are Three *Right Lines*, interſecting each other upon a *Plain:* And a *Right Line* is, The *Shorteſt Extenſion* between any Two *Points* upon a *Plain Superficies:* So the *Sides* of a *Spherical Triangle* are Three *Arches*, of Three *Great Circles* of the *Sphere*, interſecting each other upon the *Globe*. And an *Arch* of a *Great Circle*, paſſing through any Two *Points* upon a *Spherical Superficies*, is the neareſt *Diſtance* between thoſe Two *Points*.

In Purſuance of the Work in this *Chapter* intended, I ſuppoſe the Reader to be acquainted with the *Circles* of the *Sphere*; that is, to know their *Names* and *Situations* upon the *Globe*, to what *Uſe* each of them ſerveth; and alſo, how to project any of them upon a *Plain*, anſwerable to any *Poſition* of the *Globe:* For to ſuch as do not, this Section will be but of little Uſe; and therefore I would adviſe my *Reader*, before he enter upon this *Geometrical* Way of reſolving *Spherical Triangles*, to peruſe the Beginnings of the Second and Third *Sections* of the *Third Part* hereof; which treat of the *Circles* of the *Sphere*, and their ſeveral Poſitions and Affections; in which he may receive very much Satisfaction concerning thoſe Particulars. So that it ſhall ſuffice, in this Place, that I declare,

1. What a *Great Circle* is.
2. How to *Project* ſuch a *Circle* of the *Sphere* upon a *Plain*, ſuitable to any *Day* and *Time* of the *Day*, at any time of the *Year*, and in any *Latitude*.
3. To diſcover the *Triangle*, which is made by the Interſection of thoſe *Great Circles* ſo projected.
4. To find the *Poles* of thoſe *Great Circles*. And,
5. To *Meaſure* the *Sides* and *Angles* of the *Triangle* ſo laid down: Which to do, is uſually called, *The Doctrine of the Dimenſion of Triangles*.

I. *What*

Fig. XLIV.

Fig.
XLII

I. *What a* Great Circle *is.*

Fig. XLIV.

A *Great Circle* of the *Sphere* is ſuch a *Circle*, as divideth the whole *Sphere* into *Two Equal Parts* or *Hemiſpheres*: Of which, there are generally accounted Six, *viz.*

The { Meridian / Horizon / Æquinoctial } The { Æquinoctial / Solſtitial } Colure. / Ecliptick,

Beſides theſe Six Principal *Great Circles* of the *Sphere* there are other *Circles*, which are *Great Circles* alſo. As are all *Azimuth* or *Vertical Circles.* Alſo all intermediate *Meridians* or *Home Circles.* And theſe are ſuch as we have moſt occaſion to make uſe of in this Place: And here note, That all *Great Circles* projected upon a *Plain*, *Steriographically* (or *Circular*,) are either, (1.) *Perfect Circles* (as the outward or *Primitive Circle* is. Or, (2.) *Semi-Great Circles*, projected within the *Primitive Circle*. Or, (3.) They are *Streight Lines*, paſſing through the Centre of the Primitive Circle, as afterwards will appear.

II. *How to* Project *the* Circles *of the* Sphere *upon a* Plain, *and ſwerable to a Prefixed* Time *and* Place.

PROBLEM.

LET it be required to *Project* ſuch *Circles* of the *Sphere in Plano*, upon the *Plain* of the *Meridian*, in the *Latitude North*, 40 Deg. Upon the 10th of *June*, at the time of the *Sun's Riſing* or *Setting*; and alſo at 10 in the *Morning*, or 2 in the *Afternoon*, the ſame Day: The Sun then having 23 Deg. 30 Min. of *North Declination.*

Firſt, With 60 Deg. of a *Scale* of *Chords*, upon the Point A, deſcribe the Primitive Circle Z H N O, repreſenting the *Meridian* of the Place.

Secondly, Draw the Right Line H A O for the Horizon of the Place; and at Right Angles thereto the Line Z A N, for the *Æquinoctial Colure*, Z being the *Zenith*, and N the *Nadir* Points.

Thirdly, Take 40 Deg. (the *Latitude* given) out of your Scale of *Chords*, and ſet them from O to P, from Z to Æ, from H to S, and from N to *æ*; and draw the Line P A S for the *Axis* of the World, and *Hour-Circle* of Six; and the Line Æ A *æ*, for the *Æquinoctial*,

Fourthly, Becauſe the *Sun's Declination* at the time given is 23 Deg. 30 Min. take 23 Deg. 30 Min. and ſet them from Æ to ♋, and from *æ* to ♋: And if you lay a Ruler from Æ or *æ*

Fig. XLIV. to ♋, it will croſs the *Axis* of the World P A S, in the Point F; ſo have you Three Points, ♋ F ♋, through which you may draw the Circle ♋ F ♋; which will cut the *Horizon* H A O, in the Point ⊙, the Place where the *Sun* will riſe that Day.

Fifthly, This Point ⊙ being found, you have Three other Points given, *viz.* P, ⊙ and S; through which you may deſcribe the Circle P ⊙ S which is the *Hour* at which the *Sun Riſeth.*

And thus have you *Projected*, ſo far of the *Problem*, as concerns the Time of the *Sun's Riſing.* Now for his Place of being at Ten in the Morning, or Two in the Afternoon.

Sixthly, Ten in the Morning, or Two in the Afternoon, are (either of them) Two Hours, or 30 Deg. diſtant from the *Meridian:* Wherefore, take 30 Deg. from your Scale of *Chords*, and ſet them upon the *Meridian*, from Æ to G, then a Ruler laid from P to G, will croſs the *Æquinoctial* Circle Æ A *æ*, in the Point B; ſo have you Three Points, S, B and P, by which you may *Project* the Circle S B P, for the *Hour Circle* of Ten in the Morning, or Two in the Afternoon: And the Circle S B P, will cut the *Tropick* of *Cancer* ♋ A ♋ (the Line which the Sun traces that Day,) in the Point C; in which Point the Sun will be at Ten in the Morning, or at Two in the Afternoon.

Seventhly, And now have you Three other Points, N, C and Z, through which you may draw the *Azimuth Circle* Z C N, for the *Azimuth*, that the Sun will be upon at Ten and Two a Clock.

III. *Concerning the* Spherical Triangles *that are made by the Interſections of theſe* Great Circles *thus projected.*

Many are the *Triangles*, that are made by the Interſections of theſe few *Circles* upon this *Projection*, but I ſhall exemplifie only in *Two* of them; namely, the *Triangle* P ⊙ O, Right-angled at O, which is compoſed of Three *Arches* of *Great Circles* of the *Sphere*, *viz.* (1.) Of ⊙ O, an *Arch* of the *Horizon*, comprehended between O, the *North* Point of the *Horizon*, and ⊙, the Point in the *Horizon*, whereon the Sun *Riſeth* that Day, or his *Amplitude* from the *North.* (2.) Of P ⊙, an *Arch* of a *Meridian* or *Hour-Circle*, paſſing through P, the *North Pole* of the World; and ⊙, the Place of the Sun's *Riſing* that Day, equal to 66 Deg. 30 Min. the Complement of the Sun's *Declination* that Day. And, (3.) Of P O, an *Arch* of the *Meridian* of the *Place*, comprehended between O, the *North* Point of the *Horizon*, and P, the *North Pole*, equal to the Latitude given, 40 Deg. Fig. XLIV.

The other *Triangle* ſhall be the *Oblique-angled* Triangle Z C P, conſtituted by the Interſection of Three other *Great Circles* of the *Sphere*; namely, Of (1.) Z P, an *Arch* of the *Meridian* of the *Place*, comprehended between Z the *Zenith*, and P, the *North Pole*, equal to 50 Deg. the Complement of the *Latitude* of the Place. (2.) Of P C, an *Arch* of a *Meridian*, or *Hour-Circle* of Ten or Two a Clock, equal to *66* Deg. 30 Min. the Complement of the Sun's *Declination*; or, the *Sun's Diſtance from the elevated Pole* P. And (3.) Of Z C, an *Arch* of an *Azimuth* or *Vertical Circle*, paſſing through the *Hour-Circle* S C P, in the Point P, where the Sun is at Ten or Two a Clock that Day, and is the Complement of the *Sun's Altitude* at that time.

IV. *To find the* Poles *of theſe* Great Circles, *they being thus Projected.*

The *Pole* of any *Great Circle* of the *Sphere*, is, always, 90 Deg. diſtant in all Places, from the ſame Circle, upon a Right Line, or Circle, which cuts it at *Right Angles*: So,

1. The *Pole* of the *General Meridian*, or *Primitive Circle* Z H N O, is A, the *Center* thereof, in all Places diſtant 90 Deg.
2. The *Poles* of the *Horizon* H O, are Z and N, the *Zenith* and *Nadir* Points.
3. The *Poles* of the *Æquinoctial* Æ *æ*, are P and S, the *Poles* of the World.
4. The *Poles* of the *Æquinoctial Colure*, or *Axis* of the World P S, are Æ and *æ*.
5. The *Poles* of the *Prime Vertical* Circle, or *Azimuth* of *Eaſt* and *Weſt*, Z A N, are H and O, the *North* and *South* Points of the *Horizon*.

And thus you ſee, that all *Great Circles* of the *Sphere*, which, when *Projected*, become *Strait Lines*, their *Poles* fall all in the *Periferie* of the *Primitive Circle*: But for all other *Great Circles*, which conſiſt of *Circular Arches*, as P ☉ S, P B S, and Z D N, the *Poles* of them fall within the *Primitive Circle* Z H N O, upon thoſe *Great Circles*, which cut them at *Right Angles*: So, Fig. XLIV.

R	The *Pole* of the Great Circle.	P ⊙ S	will be upon the *Line.*	Æ A	at the Point	R.
T		P B S		A *a*		T.
V		Z D N		A O		V.

Which *Points* or *Poles* may be thus found.

1. For the *Pole* of the *Hour-Circle* P ⊙ Q S, lay a Ruler from P to Q, it will cut the *Primitive Circle* in *a*, ſet 90 Deg. of your *Chords*, from *a* to *b*; a Ruler laid from P to *b*, will cut the *Æquinoctial Circle* in R, ſo is R the *Pole* of the *Circle* P ⊙ Q S.
2. For the *Pole* of the *Hour-Circle* P B S, lay a Ruler from P to B, it will cut the *Primitive Circle* in G, ſet 90 Deg. from G to *c*, a Ruler laid from P to *c*, will cut the *Æquinoctial* in T, ſo is T the *Pole* of the *Hour-Circle* of 10 and 2, *viz.* P B S.
3. For the *Pole* of the *Azimuth*, or *Vertical Circle* Z D N, lay a Ruler from Z to D, it will cut the *Primitive Circle* in *d*, ſet 90 Deg. from *d* to *e*, a Ruler laid from Z to *e*, will cut the *Horizon* H A O in V, ſo is V the *Pole* of the *Azimuth Circle* Z D N.

And in this manner may the *Pole* of any *Great Circle* be found. For the *Centers* of them, they always fall in the ſame Right Line, in which their *Poles* are, they being extended, if Need be: But no more of this in this Place: For I intend not here a *Treatiſe* of *Projection*, but of the *Meaſuring* of *Triangles*, to which I now proceed.

V. *How to* Meaſure *the* Sides *and* Angles *of* Spherical Triangles, *they being thus* Projected.

You muſt underſtand. That the Quantities of the *Sides* and *Angles* of *Spherical Triangles* are meaſured upon the *Periferie* of the *Primitive Circle.*

Then,

1. In the *Right angled Spherical Triangle* P ⊙ O, *Right-angled* at O, there is given, (1.) The *Perpendicular* P O 40 Deg. equal to the *Latitude* given by the *Propoſition.* (2.) The *Hypotenuſe* P ⊙, the Complement of the *Sun's Declination* 66 Deg. 30 Min. And let it be required to to find the *Baſe* ⊙ O, whoſe Meaſure is ⊙ O, the *Sun's Amplitude.*

Lay a Ruler to Z, the *Pole* of the *Horizon*, and to the *An-* *Fig.* XLIV.
lar Point ⊙, it will cut the *Primitive Circle* in the Point *f*,
the Diſtance from *f* to O, meaſured upon the Scale of *Chords*,
ill be found to contain 58 Deg. 38 Min. and that is the
iantity of the *Side* ⊙ O; and is the *Amplitude* of the Sun's
ſing or *Setting* from the *North* Part of the *Horizon* at O;
d its Complement, 31 Deg. 22 Min. is the Quantity of the
ch A ⊙, the *Sun's Amplitude* from A, the *Eaſt* and *Weſt* Points
the *Horizon*.

The Canon for Calculation is,

As *c s* P O : Radius : : *c s* P ⊙ : *c s* ⊙ O.
50 d. : 90 d. : : 23 d. 30 m. : 31 d. 22 m.

2. By the ſame Things given to find the *Angle* at the *Perpendicular* ⊙ P O, whoſe *Meaſure* is the *Arch* of the *Æquinoctial* T *æ*, or the *Hour* from *Midnight*.

Lay a Ruler to P, the *Pole* of the World, and T, it will cut
Primitive Circle in *a*, and the Diſtance *a æ* meaſured upon the
ile of *Chords*, will be found to be 68 Deg. 36 Min. And
t is the Quantity of the *Angle* ⊙ P O: Which 68 Deg. 36
n. converted into *Time*, (by allowing 15 Deg. of the *Æqui-*
ial for One *Hour*, and 4 Deg. for *One Minute* of *Time*) makes
Iours, and 34 Min. Which is the Hour from Midnight, or
Time of the *Sun's Riſing* in the Morning: Whoſe Comple-
nt to 12 Hours, is 7 Hours, 26 Min. the Time of the *Sun's*
ting.

The Canon for Calculation is,

As Radius : *t*. P O : : *ct*. P ⊙ : *c s*. ⊙ P O.
90 d. 40 d. 23 d. 30 m. 21 d. 24 m.

. By the ſame Things Given, to find the *Angle* at the *Baſe* P ⊙ O; whoſe Meaſure is an *Arch* of a *Great Circle* intercepted between the *Hour-Circle* P ⊙ S, and the *Horizon* H ⊙ O.

oraſmuch as the *Sides* ⊙ P, and ⊙ O, of the *Triangle*
O, are leſs than *Quadrants*, we muſt meaſure the *Angle*
O, by the *Angle* H ⊙ S, which is equal to it. To perform
ch,

ay a Ruler to R, (the *Pole* of the *Hour-Circle* P Q S) and ⊙,
Angular Point, and it will cut the *Primitive Circle* in *b*;
ſet 90 Deg. from *b* to *a*: A Ruler laid from the *Pole* R, to

a,

Fig. LXIV. *a*, will cut the *Hour-Circle* P Q S in *k*: Then lay a Ruler from ⊙ to *k*, it will cut the *Primitive Circle* in *l*, and the Diſtance from *l* to H, meaſured upon the Scale of *Chords*, will be found to contain 44 Deg. 30 Min. for the Quantity of the *Angle* H ⊙ S, which is equal to the *Angle* P ⊙ O required: And this is the *Angle* of the *Sun's Poſition*, (in reſpect of the *Pole* P, and *North* Part of the *Horizon* O,) at the Time of his *Riſing* or *Setting*.

The Canon for Calculation is

As *s.* P ⊙ : Radius :: *s.* P O : *s* P ⊙ O.
66 d. 30 m. : 90 d. :: 40 d. : 44 d. 30 m.

And thus are all the *Sides* and *Angles* of the *Right-angled Spherical Triangle* P ⊙ O meaſured: And the like may be done in any other *Right-angled Triangle*; of which, in this *Scheme* there are many.

VI. *And now for the* Oblique-angled Spherical Triangle Z C P.

In this *Triangle* there is given, (1.) The Side Z P, 50 Deg. equal to the Complement of the *Latitude* given. (2.) The *Side* C P, 66 Deg. 33 Min. equal to the Complement of the *Sun's Declination* given. And, (3.) The *Angle* Z P C, 30 Deg. the *Horary Diſtance* of the Sun from the *South* Part of the Meridian; namely, at Ten or Two of the Clock. And from theſe Things given, let it be required to find the third *Side* Z C, the Complement of the *Sun's Altitude* at the Time of the Queſtion, *viz.* at Ten in the Morning, or at Two in the Afternoon. Which, to perform,

Lay a Ruler to V, the *Pole* of the *Azimuth Circle* Z D N, and to the Angular Point C, it will cut the *Primitive Circle* in *m*, and the Diſtance from Z to *m*, by the *Scale* of *Chords*, will be found to 30 Deg. 7 Min. whoſe Complement D C, 59 Deg. 53 Min. is the *Sun's Altitude* at Ten or Two of the Clock, that Day.

The Canons for Calculation is

(1.) As *ct.* P Z : Radius :: *cs.* P : *t* P A.
40 d. : 90 d. :: 60 d. : 45 d. 56 m.

Which ſubſtracted from P C, 66 d. 30 m. there remains 20 Deg. 34 Min.

(2.) As *cs.* P A : *sc.* Z P :: *cs.* Rem. : *cs.* Z C.
44 d. 4 m. : 40 d. :: 69 d. 26 m. : 59 d. 53 m.

To

To find the Angle Z C P.

Fig. XLIV.

This is the *Angle* of the Sun's *Poſition* at the time of the *Queſtion*, with Reference to the *Pole* and *Zenith*; and may be thus found.

In regard the Two Sides C Z and C P, comprehending the enquired *Angle* Z C P, are both of them leſs than *Quadrants*, we muſt make uſe of the alternate and oppoſite *Angle* S C N, which is equal to Z C P. And, whoſe *Meaſure* is the *Arch* of a *Great Circle* intercepted between the *Meridian* or *Hour-Circle* S C P, and the *Azimuth-Circle* Z C N; at 90 Deg. Diſtance from the *Angular Point* C: And to find the Quantity hereof, you muſt,

Lay a Ruler to V, the *Pole* of the Circle Z C D N, and the Point C, it will cut the *Primitive Circle* in *m*, ſet 90 Deg. from *m*, and it will reach to *n*: A Ruler laid from V to *n*, will croſs the Circle Z D N in the Point X. —Again, Lay a Ruler from T, the *Pole* of the Circle P B S, to C, the Angular Point, and it will cut the *Primitive Circle* in *o*, ſet 90 Deg. from *o* to *p*: Then a Ruler laid from T to *p*, will croſs the Circle P B S, in the Point Y. —*Laſtly*, A Ruler laid from C, to X and Y, will cut the *Primitive Circle* in *r* and *s*, ſo the Diſtance between *r* and *s*, meaſured upon the Scale of *Chords*, will give 49 Deg. 46 Min. for the Quantity of the *Angle* Y C X, which is equal to the *Angle* Z C P enquired. And is the *Angle* of the *Sun's Poſition* at the time of the *Queſtion*.

The Canon for Calculation.

As *s*. Z C : *s*. Z P C :: *s*. Z P : *s*. Z C P.
30 d. 7 m. : 30 d. :: 50 d. : 49 d. 46 m.

To find the Vertical Angle C Z P.

This *Angle* is the *Sun's Azimuth* from the *North* Part of the *Meridian* Z O N, whoſe Meaſure is the Arch of the *Horizon* D A O; and to find the Quantity of it, lay a Ruler from Z, the *Pole* of the *Horizon*, to D, it will cut the *Primitive Circle* in *d*; ſo the Quantity of the Arch *d* N O, meaſured upon the Scale of *Chords*, will be found to be 113 Deg. 57 Min. And ſuch is the Sun's *Azimuth* from the *North* Part of the *Meridian*: The Arch *d* H, 66 Deg. 3 Min. is the Sun's *Azimuth* from the *South*; and the Arch *d* N, 23 Deg. 57 Min. is the Sun's *Azimuth* from the *Eaſt* and *Weſt*.

The Canon for Calculation.

Fig. XLIV.

As *s.* Z C : *s.* Z P C :: *s.* C P : *s.* C Z P.
30 d. 7 m. : 30 d. :: *66* d. 30 m. *66* d. 3 m.
(Or, 113 d. 57 m.

And thus are all the *Sides* and *Angles* of this *Oblique-angled Triangle* alſo Meaſured: And ſo may any other, being thus *Projected.*

And, to conclude, This *Projective* Way will give great *Light* to *Calculation*; for by the true delineating of your *Triangle* upon the *Projection*, you ſhall thereby diſcover whether your *Side* or *Angle* be *More* or *Leſs* than a *Quadrant*, and ſo be poſitive in your *Reſolution*; which otherwiſe you muſt have *Given*, or render a double *Solution:* As in this laſt *Angle* C Z P, whether it were *66* Deg. 3 Min. Or 113 Deg. 57 Min. The ſame *Sine* in the *Canon* anſwering to both.

Fig.
XLIV.

JANUA MATHEMATICA.

SECTION IV.

Of Spherical Trigonometry, *Inſtrumentally Explained and Performed.*

THE *Explanatory Inſtrument* here deſcribed, is for the better Information of the Fancy, by Speculation; and it is deduced from the *Catholick*, or *Univerſal Propoſition*, before treated of in *Part* II. *Sect.* II. *Chap.* III. of this Book. Notwithſtanding, for the Convenience of the Reader, I ſhall here, again, inſert it.

Propoſition Univerſal.

The Sine *of the* Middle Part, *and the* Radius, *are* Reciprocally Proportional, *with the* Tangents *of the* Extreams Parts Conjunct, *and with the* Co-ſines *of the* Extreams Disjunct.

In every *Right-angled Spherical Triangle*, there are *Five Parts*, beſides the *Right Angle*, and they are called CIRCULAR PARTS: Of which, thoſe *Three* which lye moſt remote from the *Right Angle*, (as the *Hypotenuſe*, the *Angle* at the *Perpendicular*, and the *Angle* at the *Baſe*) are noted by their *COMPLEMENTS*.

Of theſe *Five Circular Parts*, any *Two* of them (beſides the *Right Angle*) being given, a *Third* may be found. And,

Of *Three Parts*, (Two given, and One required) *One* muſt (neceſſarily) be in the *Middle*, and muſt be called the *MIDDLE PART*.

Of the other *Two Extream Parts*, they muſt either *Join* to the *Middle Part*, or be *Separate from it*.

If they be { *Joined* to it, / *Separate* from it, } then are they called *Extreams* { *Conjunct.* / *Disjunct.*

If the *Extreams* be *Conjunct*, the *Proportion* muſt be performed by *Sines* and *Tangents* jointly. —But,

If the *Extreams* be *Disjunct*, the *Proportion* may be performed by *Sines* only.

—— And in both *Caſes*, ——

If the *Part Sought* (whether *Side* or *Angle*) fall out to be the *Middle Part*, then the *Radius* muſt be the *Firſt* (or leading) *Term* in the *Proportion.* —But,

If one of the *Extreams* (whether *Conjunct* or *Disjunct*) poſſeſs the *Middle Part*, then the other *Extream* muſt be the *Firſt Term* in the *Proportion.*

Of the Inſtrument.

THis *Inſtrument* was the Coutrivance (many Years ſince) of the Right Worſhipful Sir *Charles Scarborow*, M. D. and divers of them have been made in *Silver* and *Braſs* about the Bigneſs of a *Crown Piece*, with *Verſes* in *Latin* about the Rimb, for the better bringing the *Rules* to be obſerved, in the Uſe of it, to Memory, which were to the ſame Effect with thoſe which I have even now laid down in Engliſh.

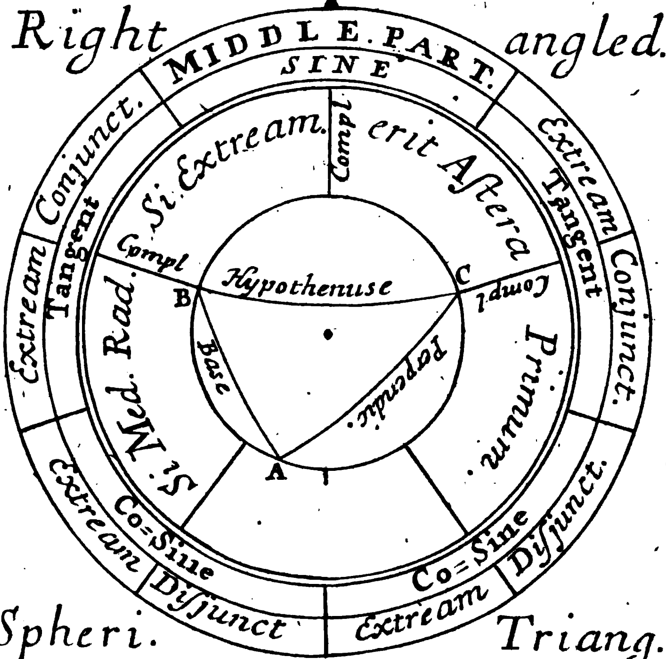
The *Inſtruments* conſiſts of Two *Parts*, the undermoſt is a round Piece, divided into *Five* Equal Parts, which repreſent the *Five Circular Parts* before ſpoken of, it hath Two Circular Margins, near the outer Edge thereof; in One of the Five Parts is engraven *Middle Part*, and under it the Word *Sine.* —On either Side of *Middle Part*, there is written or engraven *Extream Conjunct*; and under them, the Word *Tangent.* —In the Two other of the *Five* Diviſions, which are oppoſite to *Middle Part*, are engraven *Extream Disjunct*, and under them, *Co-ſine.* All which Words anſwer directly to the Words of the *Univerſal Propoſition:* And this is all that is upon the Under Plate of the *Inſtrument.*

The Upper Plate of the *Inſtrument* is a Circle alſo, divided into Five Equal Parts as the former, and within that another Concentrick Circle, between which are Five Lines drawn from the Centre thereof. —Within the innermoſt of theſe Circles, there is deſcribed a *Right-angled Spherical Triangle*, the Five Circular Parts whereof do lye directly againſt the Five Lines, drawn from the Centre between the Two Circles: And upon thoſe Three Lines which lye againſt the *Hypotenuſe*, the *Angle* at the *Perpendicular*, and the *Angle* at the *Baſe*, is written,

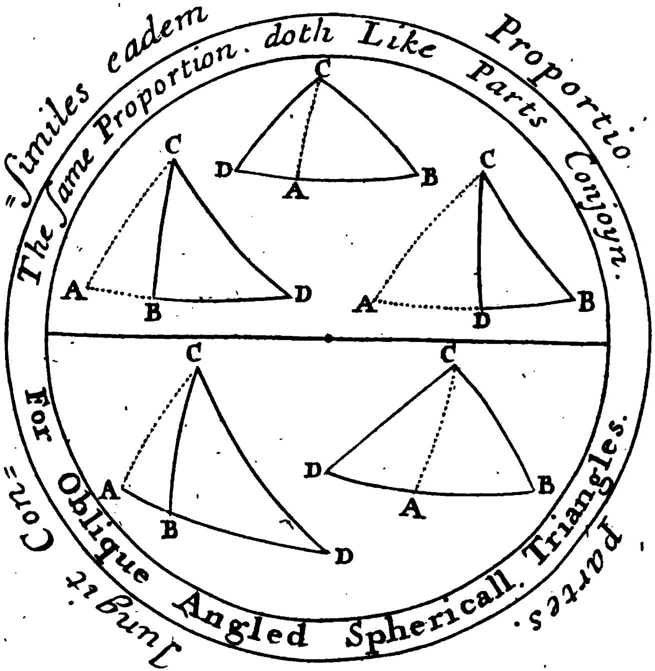
ten,

A
SYNOPSIS.
OF
Sphericall Trigonometry.
FOR

Right angled



Spheri. Triang.


=Similes eadem Proportio Conjoyn.
The same Proportion. doth Like Parts
For Oblique Angled Sphericall Triangles.
Jungit Con= partes.
A
B
C
D
A
B
C
D
A
B
C
D
A
B
C
D

And
As the Tangent of half the Base,
Is to the Tang. of half the Sum of the Sides,
So is the Tang. of half the Difference of ye Sides
To the Tang. of half the Difference of the
Segments of the Base.

Plate XII.

ten, or engraven, *Complement*; but upon the other Two Lines, which iſſue from the *Perpendicular*, and the Baſe, there is nothing written. —This Upper Plate is to move upon the Under Plate by a Rivet (or ſuch like) through the Centers of both Plates: And ſo is your *Inſtrument* finiſhed.

The Uſe of the Inſtrument.

THE Uſe of the Inſtrument is principally to give you by Inſpection the *Order* of the *Terms* of the *Proportion* in any *Caſe*.

Example. In the *Triangle* A B C, ſuppoſe there were given the Perpendicular C A, and the *Angle* at the Baſe B, to find the *Angle* at the Perpendicular C. *Fig.* XLV.

Here it is evident, that the *Angle* at the Baſe B, is the Middle Part: Turn the Rundle about till you bring the *Angle* at the Baſe againſt Middle Part: Then ſhall you find that the Perpendicular C A, and the *Angle* at the Perpendicular C, will ſtand againſt *Extreams Disjunct*; which tells you, that C A and C, are *Extreams Disjunct*: And now (the Rundle thus reſting) you ſee that One of the *Extreams*, as C, is ſought; therefore, the other *Extream* C A muſt be the firſt *Term* in the Proportion, againſt which you find *Co-ſine*. Wherefore ſay,

As the Co-ſine of the Perpendicular A C,
Is to the Radius:
So is the Middle Part the Angle at the Baſe B, (againſt which ſtands *Sine* and *Complement*, that is *Co-ſine* B.)
To the *Angle* at the Perpendicular C, againſt which ſtands *Co-ſine* and *Complement*; that is *Sine*; for, *Co-ſine Complement* is the *Sine* it ſelf.

And ſo the Proportion in ſhort is this:

As *c s*. C A : to Radius : : So *c s*. B : to *s*. C.

Another Example.

In this *Triangle* A B C let there be given the *Hypotenuſe* C B, and the *Angle* at the Perpendicular C, to find the Baſe B A. *Fig.* XLVI.

It is evident that B A is the *Middle Part*. Bring B A againſt the *Middle Part*, then will C B and C ſtand againſt *Extreams Disjunct*: And (becauſe the *Middle Part*) B A is ſought, the *Radius* muſt be the *Firſt Term* in the Proportion, and the Two *Extreams* the *Second* and *Third*, againſt both which ſtands *Comp. Coſine*, (which is *Sine*); and againſt B A the Middle

Part ſought, there ſtands *Sine* alſo: So that your Proportion will run thus:

As Radius,
To the Sine of the *Angle* the Perpendicular C:
So is the Sine of the *Hypotenuſe* C B,
To the Sine of the Baſe B A.

A Third Example.

Fig. XLVII. In this *Triangle* A B C, let there be given the Perpendicular C A, and the *Angle* at the Baſe B, to find the Baſe B A.

It is here evident that A B is the *Middle Part*: Turn the *Triangle* about till the Baſe A B lye againſt the *Middle Part*; then will the Perpendicular C A, and the *Angle* at the Baſe B, lye againſt the *Extreams Conjunct*; and ſeeing the *Middle Part* is ſought, the Radius therefore muſt be the *Firſt Term* in the Proportion: And becauſe the *Extreams* are *Disjunct*, the Proportion will be in *Sines* and *Tangents* jointly, as by the *Inſtrument* appears: And the Proportion will be

As the Radius,
Is to the *Tangent* of the Perpendicular C A:
So is the *Tangent* Complement of B,
To the Sine of the Baſe B A.

But if the ſame Things were given, and the *Angle* at C, the Perpendicular, had been required, then B had been the *Middle Part*, and C A and B, the *Extreams Disjunct*, and the Proportion in *Sines* only. As thus:

As the Co-ſine of C A,
To Radius:
So is the Co-ſine of B,
To Co-ſine Comp. (that is, to *Sine*) of C.

And thus, by this *Inſtrument*, may the Proportions for the Solution of any *Right-angled Spherical Triangle* be readily ſet down, and the *Triangle* reſolved in any of the XVI. *Caſes*.

On the Back-ſide of this *Inſtrument*, there is another Contrivance of the forementioned Gentleman's, with *Latin Verſes* for bringing the *Rules* for the Solution of *Oblique-angled Spherical Triangles* to Memory.

I ſhall not here give you any Account of it more than the *Figure*, and the *Latin Verſes* (or *Rules*) in Engliſh: Referring you, for the farther underſtanding of it, to what is delivered in the IVth Chapter hereof, Page 68.

Of the Solution of Spherical Triangles, *by* PLANISPHERE.

A *Planiſphere*, is a *Projection* of the *Sphere* (or *Globe*) in *Plano:* Of which, there are principally Two; One called *Stereographical*, projected by *Circles*; The other *Orthographical*, projected by *Ellipſes*, and generally known by the Name of *Analemma*. Fig. XLVIII.

To either of which *Planiſpheres*, there belongs proper *Indexes*, to move upon the Centres of the *Planiſpheres.* As to the *Stereographical*, an *Index* having Two *Legs* as a *Sector*; both which are to be divided, according to the *Tangents* of *half Arks*, as the Semidiametre of the *Planiſphere* it ſelf is divided, and muſt be numbred by 10, 20, 30, *&c.* to 90 Deg. both Ways, and on both *Legs.*

To the *Analemma*, or *Orthographical* Projection, there muſt be an *Index* of the whole Length of the Diametre thereof, to move about upon the Centre of the Projection, which muſt be divided as the Diametre of the *Planiſphere* is; namely, as a *Scale* of *Natural Sines*, and muſt be numbred both Ways, from the *Centre*, by 10, 20, 30, *&c.* to 90 Deg. —Upon this *Index*, (by help of a *Groove* made through the former *Index*) another *Index* is to be made to move upon, and with, the former, and always keeping at *Right Angles* with it: And this *Index* is to be divided as a *Scale of Sines*, as one half of the other; and ſo numbred, by 10, 20, 30, *&c.* to 90, both Ways: And this ſecond *Index* (in the Uſe of this *Planiſphere*) I ſhall call the *Curſor.* And thus much for the Deſcriptions.

Concerning their Uſe, I ſhall only lay down ſuch *General Rules* as are neceſſary for the counting of the Quantities of the *Sides* and *Angles* of *Spherical Triangles* upon the ſeveral *Planiſpheres* and their *Indexes*; in all *Caſes*, both of *Right and Oblique Triangles.* Not inſiſting upon particular *Examples*, for that throughout all this Book there are ſuch Variety. All which (by theſe few *General Rules* here delivered) may be wrought upon either of theſe Two Projections.

Fig. LXVIII.

The Solution of Spherical Triangles, *by the* Stereographical Projection.

I. *Of* Right-angled Spherical Triangles.

1. TO retain the Method before obſerved in the 16 *Caſes* of *Right-angled Spherical Triangles*; I will here alſo follow the ſame Order, wherein I ſhall note the *Triangle* to be reſolved by the Letters A B C, ſetting A at the *Right Angle*, and B and C at the other Two *Acute Angles*. So ſhall the *Baſe* be A B; The *Cathetus*, (or *Perpendicular*) C A; The *Hypotenuſe* B C. —The *Angle* at the *Baſe* B; and the *Angle* at the *Cathetus* C.

2. There are therefore, beſides the *Right Angle*, Five *Circular Parts*; namely, Three *Sides*, and Two *Angles*: Of which, Four come into the *Account* at once; Two of them are given, and the other Two found out.

3. *If the Four Parts, which at once come into the Account, be the Three Sides, and One Angle.*—Let that *Acute Angle* be evermore noted with the Letter B; and the *Triangle* may be reſolved thus:

Set the *Angle* B at the Centre, reckoning it upon the *Limb* from the *Diameter*; and the *Baſe* B A upon the *Diameter* from the *Centre*; and the *Cathetus* C A upon the *Great Circles* from the *Diameter*, by help of the *Parallels*; and the *Hypotenuſe* B C upon the *Index* from the *Centre*.—*Note*, That if any of thoſe *Accounts*, fall not juſt upon ſome Line in the *Inſtrument*, either of the *Great Circles* or *Parallels*, the Exceſs is to be eſtimated in *Minutes* or *Parts* of a Degree.

Example. The Perpendicular C A, *and the* Angle *at the* Baſe B, *given*; *To find*, (1.) *The* Baſe B A. (2.) *The* Hypotenuſe C B. (3.) *The* Angle *at the* Cathetus C.

1. Reckon on the *Limb*, from the *Diameter*, the Quantity of the given *Angle* B; and to the End thereof, ſet either of the *Legs* of the *Index*.

2. Upon the *Limb*, from the *Diameter*, reckon the *Cathetus* C A; and from the end of that Arch, eſtimate reaſonably, a *Parallel Circle*, till it meet with the *Index*: For that Point of Interſection ſhall ſhew both the *Hypotenuſe* B C, upon the *Index*: And the *Baſe* B A, upon the *Great Circle*, meeting alſo in that Point.—Then, to find the *Angle* at C: Take C A for the

the *Baſe*, and C A for the *Cathetus*; and to the end of that Cathetus apply the *Index*; ſo ſhall you have the *Angle* ſought for C, upon the *Limb*. *Fig.* XLVIII.

3. Otherwiſe, you may ſet the *Angle* B at the *Pole*, reckoned upon the *Diameter* from the *Limb*; and B A upon the *Limb* from the *Pole*; and C A upon the *Index* from the *Limb*; and B C upon a *Great Circle* from the *Pole*.

4. But if the *Four Parts* which at once come into the Account, be the Two *Oblique Angles*, C and B, and the Two *Right Sides* C A and B A, then the *Triangle* may be reſolved thus.

1. Reckon the *Leſſer Angle* on the *Index* from the *Centre*, and the *Greater Angle* on a *Great Circle* from the *Pole*: So both of theſe, with the *Axis*, ſhall include a *Quadrantal Triangle*: And the *Greater Right Side* ſhall be on the *Limb* from the *Pole*: And the *Leſſer Right Side* ſhall be upon the *Diameter* from the *Centre*.

Laſtly, Two of the Four Parts being had, there will be no Difficulty in finding out of the *Hypotenuſe*.

II. *Of* Oblique-angled Spherical Triangles.

1. In an *Oblique-angled Spherical Triangle* B C D, Five of the *Circular Parts* come into the Account at once; namely, *One Side* B D; and the *Two Oblique Angles* B and D, and the Two other *Sides* D C and B C: The third *Angle* C, oppoſite to B D, is not here enquired, but only the *Place* of the *Angular Point* C, both for D C and B C.

In the firſt of the Three *Schemes*, the inner *Angle* D is *Obtuſe*, and B *Acute*: In the ſecond the inner *Angle* D is *Acute*, and B *Obtuſe*: In the third, both the inner *Angles* D and B are *Acute*. *Fig.* XLIX.

2. The *Side* B D, is evermore underſtood to be *Given*: And in every *Operation*, the firſt thing to be done, is to open the *Legs* of *Index* to the Wideneſs of B D, and thereto ſcrew them faſt: Alſo I call the Two *Legs* of the *Index*, the *Leg* or *Index* B, and the *Leg* or *Index* D, as they are noted with thoſe Letters.

The *Angles* D and B, of every *Triangle* propoſed, are reckoned upon the *Diameter* from the *Limb*, by the *Great Circles*; namely, the *Angle* B, and the *Conjunct* of the *Angle* D, from ♋; but the *Angle* D, and the *Conjunct* of the *Angle* B, from ♑: And the *Sides* D C and B C, are to be reckoned by

Fig. XLIX. the *Parallels*; namely, the *Sides* themſelves, from the *Pole*; or their Complements and Exceſſes, from the *Diameter*.

Note, That of a *Triangle*, the *Conjunct Angle*, or the *Outer Angle*, is all one.

4. *Two* Sides *of an* Oblique-angled Spherical Triangle, *whereof one is* B D, *with the* Angle *intercepted, being given: To find out, at one Work, the* Third Side; *and the other* Angle *at* B D.

There are theſe Two Varieties

Given,	D C and D	to find out	B C and B.
	B C and B		D C and D.

5. The *Rule*. If the *Angles* D and B, be reckoned in the *Upper Semicircle* of the *Planiſphere*, at the *Pole* N, and the *Angle* D be *Obtuſe*, or B *Acute*; the Point C ſhall be taken in the former *Semicircle* towards ♋: But, if the *Angle* D be *Acute*, and B *Obtuſe*, the *Point* C ſhall be taken in the *latter Semicircle* towards ♑; and contrariwiſe, for the lower *Pole* S. —Then, upon the *Diameter*, count the *Angle* given, D or B, either by it ſelf, or by its Conjunct, according as was taught in *Sect.* 3. And from the end of that reckoning, imagine an Arch of a *Great Circle* aptly traced, until it meet with the *Parallel* of the Complement of the reſpective *Side* given D C or B C, for that Point of *Concurrence* ſhall be the Point C, for the given *Side*, either D C or B C. To this Point C, thus found, apply the *Index* noted with the proper *Letter* of the *Angle* given, D or B; and mark the Diſtance of it from the *Centre*; for this Diſtance being exactly taken on the other *Index*, ſhall in the *Planiſphere* give the true Place of the Point ſought: Which being aptly eſtimated, will, upon the *Diameter*, ſhew the *Angle* ſought; either B, or the Conjunct D, from ♋; or elſe, the Conjunct of B, or the *Angle* D, from ♑. And, it will alſo ſhew upon the *Limb*, the *Complement* of the *Side* ſought.

By what hath been before ſhewed, (that the Point C, both of the *Sides* D C and B C, are evermore equally diſtant from the *Centre*,) there is opened a Way, whereby *Any Two of Five Circular Parts; together with the* Side D B, *being given, the other Two may be found at one Operation.*

Herein

Fig. XLXI.

Herein are Four Varieties.

Given,	D C and B C	to find out	D and B.
	D C and B		D and B C.
	D and B C		D C and B.
	D and B		D C and B C.

The RULE.

In the *Upper Semicircle* of the *Planiſphere*, if the *Angle* D be *Obtuſe*: Or if the *Side* D C be much *Leſſer* than B C, the Point C ſhall be taken in the *former Quadrant* towards ♋: But, if D be *Acute*, or D C much *Greater* than B C; the Point C ſhall be taken in the *latter Quadrant* towards ♑. And, contrariwiſe, in the *Under Semicircle*. —And in both, if the *Angles* D and B be *Acute*, or if the *Sides* D C and B C, differ but little in length, the true Point ſought, ſhall fall near the *Axis*, towards ♑, within a Space, equal to the *Index* opened at the Wideneſs of B D.

Set, therefore, the *Index* thus opened; ſo that the Two Points C, may fall within it. And among the Great Circles and Parallels, or both (according as Reaſon ſhall direct) reckon the Two Circular Parts given, tracing them proportionally, with a Pin in each Hand, until they ſhall both exactly meet with their proper *Legs* of the *Index*, at an equal Diſtance from the Centre, and as near to it as poſſibly may be: So have you upon the *Planiſphere*, the true Places of both the Points C; and thereby, the Two Circular Parts ſought, reſpectively, to each Point C; both of D C, and B C.

And note farther, That if any Point C, falleth, either among the Parallels, near the *Diameter*; or among the *Great Circles*, near the *Limb*; where they be almoſt equidiſtant, there may be ſome Uncertainty in finding the true Points exactly: The remedying whereof requireth the more Diligence in the *Computer*; but this may be remedied, by reducing the *Oblique Triangle* into Two *Right-angled*.

Other Inconveniencies may ariſe in the Uſe of this, as in all other *Inſtrumental Operations*; for the remedying whereof, *Uſus optimus Magiſter*.

The

Fig. XLIX.

The Solution of Spherical Triangles, *by the* Orthographical Projection, *or* Analemma.

I. *Of* Right-angled Spherical Triangles.

1. THE *Hypotenuſe* is, always, repreſented upon the *Index.*

2. The *Right Angle*, at the Section of any *Meridian*, and the *Æquinoctial.*

3. One *Leg* in the *Meridian*, and the other in the *Æquinoctial.*

4. One *Angle* only entering the Queſtion, is repreſented at the Centre, between the *Æquinoctial* and the *Index*, and is numbered in the *Limb.*

5. But the Two *Angles* entering the Queſtion, you muſt turn the *Caſe* into an *Angle*, and its oppoſite *Leg.* For in the *Triangle* A B C, whoſe *Right Angle* is A; the other Two *Angles* A B C, and A C B, both entering the Queſtion, you muſt lengthen the *Hypotenuſe* to a full *Quadrant* to E; as alſo, the *Legs* adjacent to that Extremity of the *Hypotenuſe*, from which it was lengthned. Thus in the *Scheme*, the *Leg* A C, adjacent to C, from which Extremity the *Hypotenuſe* was lengthened: And ſo, in the *Triangle* C D E, *Right-angled* at E: The *Angle* D C E is equal the *Angle* B C A; and the *Leg* D E, equal to he *Complement* of the other *Angle* C B A.

Fig. L.

II. *Of* Oblique-angled Spherical Triangles.

1. *Three* Sides *being given; To find an* Angle *oppoſite to any of them.*

Reckon the *Greateſt Leg* from the Pole upon the *Limb*, and where it endeth, apply the *Index*; upon the *Index* reckon, from the *Limb*, the *Baſe*; (that is, the *Side* oppoſite to the *Angle* ſought:) And to the Point of the *Index*, where this *Baſe* endeth; apply (or bring to) the *Curſor*: Then look where the *Curſor* cutteth the Parallel of the *Leſſer Leg*, to be reckoned from the Pole, and obſerve what *Meridian* paſſeth by this Section of the *Curſor* and Parallel; for, where this *Meridian* cutteth the *Æquinoctial*, there you have the *Meaſure* of the *Angle* ſought; to be accounted from thence to the *Limb.*

2. *Three* Angles *given, to find a* Side.

Turn the *Angles* into *Sides*, and deal with them, as with the *Sides.*

3. *The*

3. *The* Parts Given *and* Sought, *being altogether* Oppoſite. Fig. I.

Reckon the *Greater* of the Two firſt *Terms* upon the *Index*, and where this Number endeth upon the *Index*; apply that Point to the leſſer of the ſaid Two firſt *Terms*; which Parallel is to be reckoned from the *Æquinoctial*; then order the *Terms*, reckoning the *Firſt* and *Third* both upon the *Index*; or both upon the *Parallel*; and ſo likewiſe do with the *Second* and *Fourth* Terms.

4. *Two* Sides, *with an* Angle *between them Given: To find the Third* Side.

Reckon the *Greater Side* given, from the *Pole* upon the *Limb*; and to the end of it, apply the *Index*. Reckon the other *Side Given* upon the *Parallels*, from the *Pole*: And the *Angle* given, upon the *Æquinoctial*, from the *Limb*; and the *Meridian* it comes to, purſue till it comes to the *Parallel* of the *Leſſer Leg*: And to this *Section* of the ſaid *Meridian* and *Parallel*, apply the *Curſor*, ſo it may juſtly lye on the *Index*; and mark what Point of the *Index* it pointeth out; For, this Point of the *Index*, counted from the *Limb*; is the *Third Side* required. And after the *Third Side* is found, you may find either of the Two unknown *Angles*, by the *Rate* for *Oppoſite Parts*.

5. *Two* Sides, *and an* Angle *oppoſite to the* Leſſer *of them*, Given: *To find the* Third Side.

Reckon the *Difference* of the given *Sides*, and the *Sum* of them (one and the ſame Way) from the *Pole*; and mark the Points where both of them do end. Count alſo, the *Angle* given, upon the *Æquinoctial* from the *Limb*, and mark what *Meridian* it cometh to. Then extending a ſtrait Line (or applying a ſtrait Ruler) between the Two Points marked in the *Limb*; it will cut the ſaid *Meridian* in Two Places: So you are to obſerve the *Parallels*, where the ſtrait Line (or Ruler) cutteth the ſaid *Meridian*; for, theſe being reckoned from the *Pole*, will give you the *Quantity* of the *Third Side* required: For, this ſame *Third Side*, may be of a Twofold *Quantity*: The *Leſſer* it is, when the *Angle* oppoſite to the *Greater* given *Side* is *Obtuſe*, and the *Greater* it is, when the ſaid *Angle* (oppoſite to the *Greater* given *Side*) is *Acute*.

6. Two

Fig. L. 6. *Two* Sides, *and an* Angle *oppoſite to the* Greater *of them Given ; To find the third* Side.

Reckon the *Difference* of the given *Sides* from the *Pole*, one way ; and the *Sum* of the ſame, the other way. Count alſo, the given *Angle* upon the *Æquinoctial :* And extend the ſtrait Line (or lay a Ruler) cutting the *Meridian*, as in the laſt ; for now it will cut but once ; and ſo the *Third Side* will admit only of one ſingle Anſwer.

The End of the Second Part.

POSTSCRIPT.

MAny are the Ways by which *Plain* and *Spherical Triangles* may be both *Geometrically* and *Inſtrumentally* performed : As by *Scales* of *Natural Sines*, *Tangents*, *Secants*, and *Equal Parts*, by *Protraction :* Alſo, by *Scales* of *Artificial Numbers*, *Sines*, *Tangents*, and *Verſed Sines*, as Mr. *Gunter* long time ſince contrived them, to be uſed with Compaſſes : And I have now lately contrived an *Inſtrument*, which I call *TRISSOTETRAS*, which printed on a large Sheet of Paper, and paſted upon a Board, *all Triangles*, both *Plain* and *Spherical*, may be *Reſolved* by Inſpection, without *Pen*, *Compaſſes*, *opening Joints*, or other Moveable ; ſave only the Extenſion of a *Thread*, or thin *Streight Ruler*, upon the Inſtrument ; the Deſcription and Uſe whereof I may hereafter publiſh by it ſelf. But, the beſt and moſt abſolute Way of *Reſolving Triangles* ; and to what Uſes ſoever they be applied, is by the *Canons* of *Artificial Sines*, *Tangents* and *Logarithms* ; both Decimal and Sexaginary : And ſuch a Canon was intended to be joined to this Book, at this time ; but muſt be referred till farther Opportunity, which may be ſhortly.

ANCILLA

ANCILLA MATHEMATICA.

VEL,

Trigonometria Practica.

PART III.

WHEREIN
The Doctrine of PLAIN and SPHERICAL TRIANGLES is applied to Practice: In

Geometry. And therein of
- Longimetria,
- Planometria,
- Geodæcia,
- Steriometria,

Or Meaſuring of
- Heights,
- Diſtances, &c.
- Plains,
- Land,
- Solids.

Coſmography.
- Geographical
- Aſtronomical

As to the
- Diſtances of Places on Land or Sea.
- Motion of the Sun, and Fixed Stars.

Sciographia: or Dialling, By Calculation: After a new Method.

Navigation: Or Sailing by
- The Plain Chart.
- Mercators Chart.
- Middle Latitude.

By *William Leybourn*, Philomathemat.

LONDON:
Printed *Anno Domini*, M.DCC.IV.

ANCILLA MATHEMATICA.

VEL,

Trigonometria Practica.

SECTION I.

OF

GEOMETRY.

IN this *Section* of *GEOMETRY*, I ſhall treat only of ſuch *Practical Parts* thereof, as the *Doctrine* of *Plain* (or *Right-lined*) *Triangles*, (both for their *Illuſtration* and *Demonſtration*) becomes ſubſervient: As,

I. In *ALTIMETRIA:* By which the *Height* of any *Object* (acceſſible or inacceſſible) may be obtained; As of *Towers*, *Steeples*, *Trees*, &c.

II. In *LONGIMETRIA:* By which the *Diſtance* of one *Object* from any Place, or of many *Objects* one from another, (whether approachable, or in-approachable) may be known, their true Poſitions laid down, and a Map made of them.

III. In *PLANOMETRIA:* By which all Kinds of *Superficies*, (Regular or Irregular) as *Plains*, *Land*, &c. may be *Meaſured*.

CHAP. I.

Of ALTIMETRIA.

I. *Of an* Altitude *that is Acceſſible.*

Fig. I. SUppoſe A C to be a *Tower*, *Steeple*, or other upright *Object*, and you ſtanding at B, were required to tell the Height thereof. —Firſt, Meaſure the Diſtance from B, to the Foot of the *Object* A, which ſuppoſe to be 432.5 Feet. —Secondly, At B, (by a *Quadrant*, or other *Graduated Inſtrument*) look to the top of the *Object* at C, where we will ſuppoſe the Degrees found by the *Inſtrument* to be 32.25 deg. —By this *Obſervation*, the *Diſtance Meaſured*, and the *Object*, you have a *Right-angled Triangle* conſtituted; in which, there is *Given*, (1.) A B the *Diſtance Meaſured* 432.5 Foot. (2.) The *Angle* at B, obſerved by your Inſtrument, 32.25 deg. And, (3.) The *Right Angle* at A: To find the Leg. C A, which will be the *Height* of the *Object*: (By *CASE* II. of *R*, *A*, *P*, *T*,) thus:

As Radius, Tangent 45 deg.
  Is to the *Diſtance Meaſured* A B, 432.5 Feet;
So is the Tan. of the *Angle* obſerved by Inſtrument at B, 32.25 d.
  To the Height of the *Object* C A, 272.89 Feet.

And thus having found the Height of the *Object* to be 272.89 F. you may find the Length of the *Viſual Line* (which is the *Hypotenuſe*) C B, (by *CASE* V. of *R*, *S*, *P*, *T*.) For,

As the Sine of the *Angle Obſerved*, B, 32.25 deg.
  Is to the *Altitude* of the *Object*, 272.89 Foot;
So is the *Radius*, Sine 90 deg.
  To the *Length* of the *Viſual Line* C B, 511.39 Foot.

II. *Of an* Altitude *Un-acceſſible.*

Fig. II. Suppoſe D E to be an *Object*, as *Steeple*, *Tower*, or the like: And that you ſtanding at G, were required to know the Height thereof; but (by reaſon of ſome broad Moat, or other Impediment, you cannot come to meaſure from G to E. In this Caſe, —Firſt meaſure from G, towards E, as far as conveniently you can, ſuppoſe to F, 95.25 F. —Then making Obſervation at G,

you find the Degrees of your *Quadrant* to be 52.50 d. and *Obſerving* at F, you find the Degrees to be 63.25. Now from this *Diſtance Meaſured*, and the Two *Obſervations* by the *Inſtrument* made; the *Altitude* D E may be attained unto by *Trigonometrical Calculation*, thus: *Fig.* II.

Firſt, Upon a Sheet of Paper, draw a Line at Pleaſure, as H K, upon which aſſume a Point for the Place of your firſt ſtanding, as at G, and upon G, protract the *Angle* obſerved 52.50 d. drawing a Line through thoſe Degrees at Liberty.

Secondly, By help of a Scale, ſet your *Meaſured Diſtance* 95.25 F. from G to F; and upon F, by a *Scale* of *Chards*, protract an *Angle* of 63.25 d. as you obſerved them to be; and through them draw another Line at Pleaſure, which will cut the former Line drawn, in the Point D, which will repreſent the top of the *Object* to be meaſured; and a *Perpendicular* let fall from D, upon the Ground-line H K, will fall in the Point E, and ſo will the Line D E repreſent the *Object* it ſelf.

Thirdly, By theſe Lines thus drawn, you will have conſtituted Two *Triangles*; one D E F, *Right-angled* at E; and the other D F G, *Obtuſe-angled* at F; by the reſolving of which, the height of the *Inacceſſible Object* D E, will be found. For,

1. In the *Oblique-angled Triangle* D F G, you have given, the *Side* F G (which was the *Meaſured Diſtance*) 95.25 F. and the obſerved *Angle* D G F, 52.50 d.—And the *Angle* obſerved at F being 63.25 d. the Complement thereof to 180 d. *viz.* 116.75 d. is the Quantity of the *Obtuſe Angle* D F G, ſo have you in the *Oblique-angled Triangle* D F G, Two *Angles* given; the Sum of which, *viz.* 169.25 d. taken from 180 d. there remains 10.75 d. for the *Angle* F D G: And now in the *Triangle* D F G you have given Two *Angles* F D G, and D G F, with the *Side* F G, oppoſite to F D G; whereby, you may find the *Side* D F, (by *Ax.* II.) For,

As the *Sine* of F D G, 10.75 d.
  Is to the *Side* F G, 95.25 Foot.
So is the *Sine* of F G D, 52.50 d.
  To the *Side* D F, 405.14.

2. In the *Right-angled Triangle* D E F, (having found the *Hypotenuſe* as before) you have given, (1.) The *Obſerved Angle* at F, 63.25 d. (2.) The *Hypotenuſe*, laſt found, D F 405.14 Foot; by which you may find D E, the *Height* of the *Object*, (by *Caſe* IV. of R, A, P, T,) thus:

As

Fig. II. As the Radius, *Sine* 90 d.
Is to the *Hypotenuſe* D F, 405.14 F.
So is the *Sine* of D F E, (the *Angle obſerved* at F) 63.25 d.
To the Leg. D E, 367.77 Foot: Which is the Height of the Object.

And now (if you pleaſe) you may find the *Viſual* Line G D, (by *Caſe* V. of *R, A, P, T,*) thus:

As the *Sine* of the *Angle* at G, 52.50 d.
Is to the height of the *Object* D E, 361.77 F.
So is Radius, *Sine* 90 d.
To the *Length* of the *Viſual Line* D G 456.00 F.

And alſo, the Diſtance E F, (by *Caſe* II. of *R, A, P, T,*) thus:

As the *Sine* of the *Angle* obſerved at F. 63.25 d.
Is to the *Height* of the *Object*, D E, 361.77 F.
So is the *Co-ſine* of the *Angle* at F, 63.75 d.
To the *Diſtance* E F, 182.35 Foot.

Unto which, if you add the *Diſtance* F G, 95.25 Foot, their *Sum* will be 277.6 Foot. And that is the whole *Diſtance* from G to E, the Foot of the *Object.*

III. *Of the* Altitude *of an* Object *ſtanding upon a* Hill, *Un-acceſſible.*

Fig. III. Suppoſe M O to be ſuch an *Object*; and you ſtanding at L, were required to tell the *Height* thereof.

Firſt, Upon Paper, or the like, draw a Right-line at Pleaſure, as Q R; and therein, aſſume any Point, at Pleaſure, for the Place of your ſtanding, as L; where, with your Inſtrument directed to the top of the *Object*, you find the Degrees cut, to be 40.52 d. and directed to the bottom of the *Object* at O, the Degrees cut, to be 22.25 d. Wherefore upon L, protract an *Angle* of 40.52 d. and draw a Line L, at Pleaſure: And alſo *b*, an *Angle* of 22.25 d. and draw the Line L *c* at Pleaſure.

Secondly, Go forwards, in a Right-line towards the *Object*, ſome conſiderable Diſtance, as to N, 212.5 Foot; and there, by your Inſtrument directed to the top of the *Object* at M, you find the Degrees cut, to be 61.82 d. through which draw a Line at Pleaſure, as N *a*, croſſing the Line L *b* in the Point M, which is the top of the *Object*: From whence, a *Perpendicular* let fall upon the Ground.

Ground-line Q R, as M P, that Line ſhall be equal to the *Altitude*, of the *Object*, and the Hill together. Fig. III.

Now, by the Interſections of theſe Four *Viſual Lines*, L M, L O, N M, and N O, there are conſtituted Four *Right-lined Triangles*, viz. L M P, and N O P, both *Right-angled* at P: And L M N, and N M O, *Oblique-angled*: By the reſolving of which, from the *Diſtance Meaſured*, L N, and the ſeveral *Angles obſerved*, at L and N, the required *Altitude* may be obtained. For,

1. In the *Oblique-angled Triangle* L M N, there is given, (1.) The *Angle* M L N, 40.52 d. (2.) The *Angle* L N M, 118.18 d. (it being the Complement of the *Angle* O N P 61.82 d. to 180 d.) (3.) The *Side Meaſured* L N, 212 5 Foot: And having the *Angles* at L and N, the Sum of them 158.70 d. taken from 180 d. there will remain 21.30 d. for the *Angle* L M N. From which Things given, the Two other *Sides*, L M and M N, may be found, by *Axiom* II. thus:

As the *Sine* of L M N, 21.30 d.
Is to the *Side* L N, 212.5 Foot;
So is the *Sine* of M L N, 40.52 d.
To the *Side* M N, 380.08 Foot:

And ſo is the *Sine* of 61.82 d. (the Complement of M N L, to 180 d.)
To the *Side* L M. 515.66

In the *Right-angled Triangle* N M P, there is given, (1.) The *Hypotenuſe* M N, 380.08 Foot. (2.) The *Angle* M N P, 61.82 d. whereby you may find N P, (by *Caſe* IV. of *R, A, P, T,*) thus:

As the Radius, Sine 90 d.
Is to the *Hypotenuſe* M N, 380.08 Foot;
So is the *Sine* of the *Angle* M N P, 61.82 d.
To the *Side* M P, 335.03 Foot:

Which is the *Height* of the *Object* and the *Hill* together.

And ſo is the *Co-ſine* of M N P, *viz.* N M P, 28.18 d.
To to *Leg.* or *Side* N P, 179.49 Foot.
To which, if you add the meaſured Diſtance L N, 212.05, their Sum will be 391.54, for the whole length L P. Then,

3. In the *Triangle* L O P, (*Right-angled* at P) you have given, (1.) The *Side* L P, 391.54 Foot. (2.) The *Angle* O L P, 22.25 d. whereby

Fig. III. whereby you may find O P, (By *Caſe* II. of *R, A, P, T,*) thus:

As the Radius, Tang. 45 d.
Is to the *Side* L P, 391.54 Foot;
So is the *Tangent* of the *Angle* O L P, 22.25 d.
To the Height of the *Hill* O P, 160.18 Foot.

Which ſubſtracted from M P (the whole Height) 335.93, there remains 175.75 Foot, for the *Altitude* of the *Object* M O.

IV. *How the* Altitude *of the* Sun *may be taken, by the* Shadow *of a* Staff, *or other* Object, *of a known Length.*

Fig. IV. Upon A B, being a level Plain, let there be erected a ſtreight *Staff*, or the like, of any Length, ſuppoſe 60.00 Inches, or 5 Foot) as C D; and the *Sun* ſhining, ſuppoſe it caſts the *Shadow* thereof upon the plain Ground to E; which meaſured, ſuppoſe to contain 108 Inches, or 9 Foot.

Draw a Line at Pleaſure, as A B; upon any Point thereof, as D, erect a *Perpendicular*, upon which ſet 60 Inches, the length of your Staff, from D to C; and the length of the Shadow thereof 108 Inches, from D to E; drawing the Line of *Umbrago* C E, and thus have you conſtituted a *Right-angled Plain Triangle* C D E, in which there is given, (1.) The Leg. C D, 60 Inches. (2) The Leg. D E 108 Inches, by which you may find the *Angle* C E D, (by *Caſe* I. of *R, A, S, T,*) thus:

As the Length of the *Shadow* C D 108.00 Inches,
Is to the Radius, *Tang.* 45 d.
So is the Length of the *Staff* C D, 60.00 Inches,
To the *Tangent* of the C E D, 29.05 d.
And ſuch is the *Sun's Altitude* at that time.

V. *How the* Height *of an* Acceſſible Object *may be obtained, by the* Length *of the* Shadow *of it.*

Fig. V. Suppoſe that the Sun ſhould caſt the Shadow of ſome upright *Object*, as F G, 84.5 Foot, from G to L, and at the ſame time, your Staff of 5 Foot, does caſt its Shadow from L to K, 6.32 Foot: And from hence I would compute the *Altitude* of the *Object* F G.

The Two Lines of the *Object* F G, and *Staff* H L, together with the Two Lines of *Shadow*, G L and L K, being laid down, do conſtitute Two *Right-angled Plain Triangles*, viz. F G L, and H L K, *Equiangled*, and their *Sides Parallel*, and therefore *Proportional* (by *Theorem* VIII. Lib. I.) And therefore, Fig. V.

As the Length of the *Shadow* of the *Staff* L K, *6 32* Foot,
Is to the Height of the *Staff* H L, 5.00 Foot;
So is the Length of the *Shadow* of the *Object* G L 84.5 Foot,
To the *Altitude* of the *Object* F G, *66.93* Foot.

CHAP. II.

Of LONGIMETRIA.

I. *How (ſtanding upon an* Object *of a known Height) to find the* Diſtance *from thence, to ſome other remote* Object.

SUppoſe C A to be the Side of a *Fort* or *Bulwark* 22.5 Foot high, and being upon the *Platform* at C, you ſee a *Tree*, or other *Object* at B, whoſe *Diſtance* you would know, from the Foot of the Wall at A. Fig. VI.

The Lines A B and A C being drawn, and the Height of the Wall 22.5 Foot, ſet from A to C, where by your Inſtrument directed to B, you find the Degrees cut to be 71.25, which *Angle* lay down, ſo have you the *Right-angled Triangle* C A B, in which there is given, (1.) C A, the Height of the Wall 22.5 Foot. (2.) The *Angle* obſerved at C, 71.25 Deg. by which you may find the *Diſtance* A B, (by *Caſe* I. of *R, A, P, T.*) thus:

As Radius, Tangent 45 Deg.
To C A, the Height of the Wall 22.5 Foot;
So is the Tangent of A C B, the *Angle* obſerved, 71.25 Deg.
To the Diſtance A B, *66.28* Foot.

And if you would find the Length of the *Viſual Line* C B, you may (by *Caſe* V. of *R, A, P, T,*) thus:

As the Sine of the *Angle* obſerved at C, 71.25 Deg.
Is to the Diſtance B A, *66.28* Foot;
So is the Radius, Sine *90* Deg.
To the *Viſual Line* C B, *69.98* Foot.

II. *To take a* Diſtance, *(Acceſſible or In-acceſſible) at Two Stations.*

Fig. VII. Being in a *Field* at E, there is a *Windmill* (or other *Object*) in another *Field* at F, (ſeparated by a *River* or other Impediment) whoſe Diſtance is required.

In any other Part of the *Field*, remote from E, cauſe a *Mark* to be ſet up, as at D; and meaſure the *Diſtance* between E and D, which let be 115.00 Foot. —Then by your Inſtrument at E, find the Quantity of the *Angle* F E D, which ſuppoſe to be 106.50 Deg. And going from E to D, make Obſervation of the *Angle* F D E, which ſuppoſe to be 57.10 Deg.

By theſe Two *Angles*, and the *Meaſured Diſtance*, you have conſtituted an *Oblique Triangle* D E F, in which there is given, (1.) The *Meaſured Diſtance* E D, 115 00 Foot. (2.) The Two *Angles* at D and E, 106.50 Deg. and 57.10 Deg. And, (3.) The *Angle* at F, (the Complements of the other Two to 180 Deg.) 16.40 Deg. Whereby you may find the *Sides* E F and D F, (by *Axiom*. II.) thus:

As the Sine of the *Angle* at F, 16.40 Deg.
Is to the *Meaſured Diſtance* D E, 115.00 Foot;
So is the Sine of the *Angle* at F, 57.10 Deg.
To the *Diſtance* E F, 341.98 Foot.
And ſo is the *Angle* at E, 106.50 Deg. (or 73.50.)
To the Diſtance D F, 390.54 Deg.

III. *If there were* Three *(or more)* Ships *on the* Sea, *and you being upon the* Land, *deſire to know how far thoſe* Ships *are from you; and alſo, how far they are diſtant one from the other.*

Fig. VIII. LET the *Three Ships* be A, B, C; and you being upon the *Shoar* at M, are required to tell how far thoſe *Ships* are from you, and alſo, how far from each other.

Firſt, Being at M, make choice of ſome other Place upon the Shoar, at ſome conſiderable Diſtance, as at O 130.00 Fathom.

Secondly, Being at M, obſerve the Quantity of the *Angle* A M O, which we will ſuppoſe to contain 104.50 Deg. and then removing to O, and obſerving the *Angle* M O A, you find it to be 37.82 Deg. through which Degrees, Lines being drawn from M and O, will croſs each other in A, which is the Place of the firſt *Ship*; which, with the Line of Diſtance M O, will form the *Oblique-*

Oblique-angled Triangle A M O: In which there is given, (1.) The Side M O 130.00 Fathom. (2.) The *Angle* A M O, 104.50 Deg. And, (3.) The *Angle* A O M, 37.82 Deg. And having Two *Angles* given, you have the third alſo given, *viz.* M A O, 37.68 Deg. whereby you may find the other Two Sides M A and O A (by *Axiom* II.) thus:

As the Sine of M A O, 37.68 Deg.
Is to the Side M O, 130.00 Fathom:
So is the Sine of A O M, 37.82 Deg.
To the Side M A, 130,41 Fathom, the Diſtance of the *Ship* at A, from M:
And ſo is the Sine of A M O, 104.50 (or 75.50 Deg.)
To the Side O A, 205.91 Fathom, the Diſtance of the *Ship* at A, from O.

Again,

Fig. VIII.

Obſerving from M and O, to B, you found the *Angle* B M O to contain 65.50 Deg. and M O B 68.00 Deg. wherefore, if upon M and O, you lay down thoſe *Angles*, you ſhall have another *Oblique-angled Triangle* M B O: In which you will have given as in the former, (1.) The *Angle* B M O, 65.50 Deg. (2.) B O M 68.00 Deg. And conſequently M B O, 46.50 Deg. together with the meaſured Diſtance M O 130 Fathom, by which you may find the other Two Sides M B and O B, by *Axiom* II. as in the former. For,

As the Sine of M B O, 46.50 Degrees,
Is to M O, 130.00 Fathom:
So is to the Sine of B M O, 65.50 Degrees,
To the Side B O, 163.08 Fathom:
And ſo is the Sine of the *Angle* B O M 68.00 Degrees,
To the Side B M, 166.18 Fathom.

Laſtly,

Obſerving again from M and O, to C, you find the *Angle* C M O to be 11.75 Deg. and M O C 132.75 Deg. the which *Angles* laid down, you have a third *Oblique-angled Triangle* C M O, wherein there is given, as before, all the *Three Angles*, and *One Side*, M O, whereby you may find the other Two, M C and O C, (by *Axiom* II.) as in the former.

So will M C be 164.39 Fathom, and O C 45.62 Fathom.

Fig. VIII. And thus have you the *Diſtances* of all the *Ships*, from the Two places, M and O, on the *Shoar*.

Now for their *Diſtances* one from another:

And *Firſt*, For the *Diſtance* A B.

In the *Oblique Triangle* A B M, there is given, the *Sides* A M 13041 Fathom, and B M 168.18 Fathom, and the *Angle* contained by them, A M B 39.00 Deg. whereby the third *Side* AB may be found, (By *Caſe* II. of *O, A, P, T,*) thus:

As the Sum of the Sides, A M and B M, 296.59 Fathom,
Is to the Difference of thoſe Sides, 35.77 Fathom:
So is the Tangent of half the *Angles* at A and B, 70.50 Deg.
To the Tangent of half the Difference of thoſe *Angles*, 18.81 Deg.

Which added to 70.52 Deg. gives 89.31 Deg. for the greater *Angle* B A M; and ſubſtracted therefrom, leaves 51.69 Deg. for the leſſer *Angle* A B M.

Then ſay, (By *Axiom* II.)

As the Sine of the *Angle* A B M, 51.69 Degrees,
Is to the Side M A, 130.41 Fathom:
So is the Sine of the *Angle* A M B, 39.00 Degrees,
To the Side A B, 104.59 F.

And in the ſame manner may the *Diſtance* from B to C, and from C to A, be found.

According to this Method may the *Diſtances* of many *Places* upon the *Land*, one from another be obtained, by making of *Obſervation*, by a *Theodolite*, *Semi Circle*, (or other *Graduated Inſtrument*) from Two Places, from whence all the other may be ſeen: An *Example* whereof I ſhall give, with the manner of making the *Obſervations*; and protracting of them, whereby the *Triangles* to be reſolved for the Performance will be conſpicuous: And for the *reſolving* of them, (it being altogether the ſame with that foregoing) I ſhall leave to the Ingenuity of the Practitioner. Wherefore,

Let A B C D E be ſeveral *Places*, as *Churches* in a *Town* or *City*, or ſuch like *Objects*.

Fig. IX. 1. Make Choice of Two ſuch *Places*, from either of which, you may ſee all the *Places* whoſe *Diſtances* you require; which *Places* let be F and G, diſtant from each other 1000 Foot, more or leſs.

2. Set up your *Inſtrument* at F, and direct the *Sights* on the *Diameter* thereof, to the other Place at G, and there fix it: Which done, direct the *Sights* to the ſeveral *Places*, A, B, C, D and E, noting what *Degrees* of the *Inſtrument* are cut by the *Index* at every *Obſervation*. Fig. IX.

3. Then removing your *Inſtrument* to the ſecond *Place* G, direct the *Sights* which are upon the *Diameter* thereof, back towards the *Firſt Place* at F, where fix it: Then, turning the *Index* about, direct it to the ſeveral *Places*, A, B, C, D and F, as before; noting the *Degrees* cut by the *Index*. Which we will ſuppoſe to be ſuch as are noted in this *Table*.

From F to	A	the Degrees cut were	70.00	From G to	A	the Degr. cut were	55.00
	B		98.50		B		64.50
	C		140.00		C		96.00
	D		200.40		D		185.00
	E		290.60		E		282.00

And the *Diſtance* from F to G 1000.

From theſe *Obſervations*, to make a *Plot* or *Map* of the Situation of the ſeveral *Places*.

1. Upon a Sheet of Paper, draw a *Right Line*, as F G, to contain 1000 Foot, of any *Scale*, which repreſents the Line of the *Diſtance* of the *Two Places*, where you *Obſerved* at F and G.

2. Place the *Center* of a *Protracter* upon F, and the *Diameter* thereof upon the *Line* F G; and there holding it faſt, make *Marks* againſt the ſeveral *Degrees* that were cut by the *Index*, when the *Sights* were directed to the ſeveral *Objects* at A, B, C, *&c.* when you *Obſerved* at F, and through thoſe *Points* draw *Right Lines*, at Pleaſure; as F A, F B, F C, F D, F E.

3. Lay the *Center* of the *Protractor* to the Point G, and the *Diameter* thereof upon the Line F G, and there holding it faſt, make *Marks* againſt thoſe Degrees of the *Protractor*, as the *Index* did cut upon the *Inſtrument*, when you made *Obſervation* at G; and through thoſe *Points*, and the *Point* G, draw *Lines*; as G A, G B, G C, G D, and G E, croſſing the former *Lines* (drawn from F,) in the reſpective *Points*, A, B, C, D and E. Which *Points* will lye upon your *Paper*, in the ſame *Poſition* as the *Places* you took notice of, were ſituate on the Ground on which they ſtood: And being thus laid down, if you take with your Com-

paſſes

Fig. IX. paſſes the *Diſtance* between any *Two* of them, and meaſure it upon the ſame *Scale* you laid down the *Line* F G by, it will give you the *Diſtance* between thoſe Two *Places*.

But, their *Diſtances* may be more exact and accurately attained unto by *Trigonometrical Calculation*.

For, in every *Triangle*, as in F G A, F G B, F G C, F G D, and F G E, there is given, (1.) The *Angle* A F G, obſerved at F. (2.) The *Angle* A G F, obſerved at G; and, (3.) The *Side* F G, (the *Stationary Diſtance*) included between them, to find the other *Sides*, A F and A G. The *Practice* whereof I commit to the Ingenuity of the *Practitioner*.

CHAP. III.

Of PLANOMETRIA.

OR,

Of the Menſuration *of* Plain Superficial Figures.

I. *Of the* Geometrical Square A B C D, *whoſe* Side A B *is* 27.32 *Foot*. By Logarithms.

Fig. X. As 1 F : to 27.32 F : : 27.32 F : to 746. 38 F.

Logar. of 27.32	1.436481
Logar. of 27.32	1.436481
Logar. of 746.38	2.872962

The *Superficial Content* of the *Square* in *Feet*.

II. *Of the* Parallelogram (*or* Long Square) E F G H, *whoſe* Length E F *is* 27.25 *Yards*, *and* Breadth E G, 6.29 *Yards*.

Fig. XI. As 1 : 27.25 : : 6.29 : 17.03

Logar. of 27.25	1.4353665
Logar. of 6.25	0.7958800
Logar. of 17.03	2.2312565

g. XII.

XIII.

XIV.

p51

Fig. IX.

Fig. X

Fig. X

III. *Of the* Triangle K L M, *whose* Longeſt Side L M *is* 267.26 Pole *or* Perches; *and its* Perpendicular L N 160.25 Perches.

~~As 1 P~~ : $\frac{1}{2}$ L M 133.63 P : : L N 160.25 P : 21414 P. *Fig.* XII.

Logar. 160.25 2.204798
Logar. 133.63 2.125904
Logar. of 21414 4.330702

IV. *Of the* ~~Trapezia~~ (*or Figure of Four un-equal* Sides) O P Q R *The Diggonal, whereof* O Q *is* 27.32 Perches, *the* Perpendicular P S 9.53 P. *and the* Perpendicular R T, 21.06 P.

As 1 : $\frac{1}{2}$ O P 13.66 : : P S + R T 30.59 : 417.85 *Fig.* XIII.

Logar. of half O Q 13.66 1.135451
Logar. of P S and K T, 30.59, 1.485579
Logar. of 417.85 *Perches* 2.621030
Which is the *Area*, or *Content* in *Perches*.

IV. *Of an* Irregular Plot, *conſiſting of many unequal* Sides *and* Angles: *As the Figure* A B C D E F G.

Before ſuch *Irregular Figures* can be meaſured, they muſt be reduced in *Triangles* or *Trapezia's*, by drawing of *Lines* from *Angle* to *Angle* at the beſt Advantage, as in this, by the *Lines* F C and F C, which Two *Lines* divides the whole *Plot* into the Two *Trapezia's* A B C F; F C D E, and One *Triangle* F E G. And then the *Baſes* and *Perpendiculars* being ſuch as are expreſſed in the *Figure*, they may be *meaſured*, as is ſhewed in the Two foregoing *Sections* hereof; and according to the following *Operations*. *Fig.* XIV.

I. *For the* Trapezia A B C F.

Logarithm of half A C, 12.16 1.084933
Logarithm of B H and H F, 8.13 0.910089
Logarithm of the *Area* of A B C F, 98.86 1.995023

II. *For the* Trapezia F C D E.

Logarithm of half F D, 12.96 1.112605
Logarithm of C L and E M, 23.11 1.263800
Logarithm of the *Area* of F C D E 299.51 2.476405

III. For

III. *For the* Triangle F E G.

Logarithm of	half E F, 9.50	0.977723
	G N, 5.82	0.764923
	the *Area* of F E G, 55.29	1.742646

The *Area* of	A B C F	98.86
	F C D E	299.51
	F E G	55.29
The *Area* of A B C D E F G		453.66

V. *Of a* Circle A B C D, *whoſe* Diameter B C *let be* 14.00.

Fig. XV. The Proportion of the *Diameter* of any *Circle*, to the *Circumference* thereof, is (in the leaſt Terms) as 7 to 22: But in greater Numbers, as 113 to 355; and of theſe I ſhall make uſe.

I. *By the* Diameter, *to find the* Circumference.

As 113 : 355 : : 14 Diam. : 43.98 *Cir.*

Logar. of 113	2.053078
Logar. of 355	2.550228
Logar. of 14.00 the Diameter	1.146128
	3.696356
The Logar. of 43.98 the *Circumference*	1.643278

II. *By the* Circumference, *to find the* Diameter.

As 355 : 113 : : 43.98 *Circum.* : 14.00 *Diam.*

Logar. of 355	2.550228
Logar. of 113	2.053078
Logar. of 43.98 *Circ.*	1.643278
	3.696356
Logar. of 14.00, the *Diam.*	1.146128

III. By

III. *By the* Diameter, *to find the* Area.

Fig. XVI.

As 113 in 4 (*viz.* 452.)	2.655138
Is to 355	2.550228
So is the *Diam.* 14 in 14 (*viz.* 196)	2.292256
	4.842484
To the Logar. of 159.94, the *Area*	2.187346

IV. *By the* Circumference, *to find the* Area.

As 355 ✠ 4 (*viz.* 113)	3.152288
Is to 113	2.053078
So is 43.98 ✠ 43.98, *Area* Squa.	1.643255
The ſame again	1.643255
	5.339588
To the Logar. of the *Area* 153.49	2.187300

V. *By the* Diameter, *to find the* Area.

As 10 : 8.862 : : 14 Di. : 12.41 : : 12.41 : 154.00

Log. 8.862	0.947532
Log. 14	1.146128
Logar. 12.41	1.093660
The double of it	2.187320 Log. of 154 the *Area.*

CHAP. IV.

Of GEODÆCIA: *Or,* Land Meaſuring.

I. *How* (*by any* Graduated Inſtrument) *to take the true* Plot *of any large* Piece *of* Ground, *as* Common Field, Park, Wood, *&c.*

IN going round about a *Field* to Survey it, there are Two ways; for in going round about it, you muſt either go on the *inſide* or on the *outſide*; and ſometimes you may be conſtrained to go ſometimes *within*, and ſometimes *without*.

Let A B C D E F, be ſuch a Field to be *ſurveyed* and *plotted.*

Fig. XVII.

1. Begin at any *Angle* thereof, as at A, and there ſetting up your *Inſtrument*, lay the *Index* and *Sights* upon the *Diameter* thereof; and turning it about, direct the *Sights* to B, and there

Fig. XVII.

ſcrew it faſt; and turn the *Index* about, till through the *Si* you ſee the *Angle* at F; and there note what Degrees of the *ſtrument* were cut by the *Index*, which we will ſuppoſe to 300 Deg. for the Quantity of the *Exterior Angle* F A B, out the Field: Or, 60 Deg. for the Quantity of the *Interior* *gle* within the Field: Then meaſure the Side A F, which poſe to be 41.00 *Pole* or *Perches*, and the Side A B 30.00 *Per* Theſe *Diſtances*, with the Quantity of the *Angle* before obſe 300, or 60 Deg. ſet down in a *Book* or *Paper*, as you ſee in following *Table*.

2. Remove your *Inſtrument* to B, and laying the *Index* the *Diameter*, turn the *Inſtrument* about, till through the *S* you ſee the Place where the *Inſtrument* laſt ſtood at A, and ſcrew it faſt; and turn the *Index* about, till, through the *Sights* ſee the *Angle* C, and note the Degrees which the *Index* cut which are here 145 Deg. for the *Exterior*, or 215 Deg. f *Interior Angle* A B C; and the *Diſtance* B C meaſured is *Perches*: Both which, note down as before.

3. Remove your *Inſtrument* to C, and placing the *Index* *Diameter*, look back to B, and there ſcrew it faſt; and turn th about, till you ſee the *Angle* at D; where the *Index* 270.25 Deg. for the *Exterior*, or 89.75 Deg. for the *Interi* *gle* B C D, and the meaſured Diſtance C D is 23.40 Deg.

4. Remove the *Inſtrument* to D, the *Index* on the *Diamet* look back to C, and then ſcrew it faſt; then turn the *Inde* till by the *Sights* you ſee the *Angle* at E; the *Index* cutti Deg. for the *Exterior*, or 97 Deg. for the *Interior Angle* the *Meaſured Diſtance* D E being 28.00 *Perches*: Whi down as the other.

5. Place your *Inſtrument* at E, the *Index* on the D look back to D, and then ſcrew it faſt, and turn the *Ind* till through the *Sights* you ſee the *Angle* at F, the *Ind* cutting 220 Deg. for the *Exterior*, or 140 Deg. for the *Angle* D E F; the *Meaſured Diſtance* E F being 42.80

6. Place the *Inſtrument* at F, and the *Index* lying on *meter*, look back to E, and then ſcrew it; and turn the bout, till through the *Sights* you ſee the *Angle* at A, (or where you firſt placed your *Inſtrument*) where the *In* 210 Deg. for the *Exterior*, or 150 Deg. for the *Interi* E F A, the *Meaſured Diſtance* F A being 41.00 *Perches*, All which being noted down according to theſe *Obſerv* *Meaſured*, will ſtand as in this *Table*.

Fig. XVII.

The Angle at	*Exterior.*	*Inferior.*	The Diſtance from	*Perches.*
A	300.00	60.00	A to B	30.00
B	145.00	215.00	B to C	28.40
C	270.25	89.75	C to D	23.40
D	263.00	97.00	D to E	28.00
E	220.00	140.00	E to F	42.80
F	210.00	150.00	F to A	41.00

Theſe *Obſervations* of *Sides* and *Angles*, as they were taken in the *Field*, being noted down in a *Book* or *Paper*, as in this *Table*; a *Plot* of the *Field* may be drawn upon *Paper* or *Parchment*, by the following *Directions*.

To Protract *the former* Work.

1. Upon a Sheet of Paper or Parchment draw a Line A B, to contain 30.00 Perches, of any *Scale*: And upon the end thereof A, place the Centre of a *Protracter*, laying the Diameter of it upon the Line A B: —Then, the *Exterior Angle* at A, being 300.00 Deg. make a Mark againſt 300.00 Deg. of your *Protractor*; and through that Point, and the Point A, draw a Right Line, downwards, as A F, to contain 41.00 Perches, of the ſame *Scale*.

2. Apply the Centre of the *Protractor* to the Point B, and the Diameter upon the Line A B; then againſt 145.00 Deg. (the *Angle* at B) make the Mark, and through that Mark, and B, draw a Right Line B C, to contain 28.40 Perches.

3. Lay the Centre of the *Protractor* upon C, and the Diameter upon B C, and againſt 270.25 Deg. make a Mark; through which, and the Point C, draw the Line C D, to contain 23.40 *Perches*.

4. Apply the Centre of the *Protractor* to the Point D, and its Diameter to the Line C D; making a Mark againſt 263.00 Deg. through which, and the Point D, draw the Line D E, to contain 28.00 *Perches*.

5. Lay the Center of the *Protractor* upon the Point E, and its Diameter upon D E, and againſt 220.00 Deg. make a Mark; through which, and the Point E, draw a Right Line; which (if you have committed no former Error) will cut the Line, firſt drawn, in the Point F: And, by this Means, you will have upon your *Paper*, the exact *Figure* of your *Piece* of *Ground*: Which you may caſt up, and find the Quantity thereof in *Perches*, (by the Directions of the Third Chapter hereof) which being divided by 160, will give the Content in *Acres* and odd *Perches*.

Fig. XVII. In this *Example*, we have wrought by the *Exterior*, (or *Outward*) *Angles*: But if you would *Protract* the ſame by the *Interior* (or *Inward*) *Angles*; it is done, by ſubſtracting the *Exterior Angle* from 360 Deg. and the Remainder will be the *Interior Angle*: So, the *Angle* at A, being 300 Deg. that ſubſtracted from 360 Deg. leaves 60 Deg. for the *Interior Angle* at A. And ſo of all the reſt, as they are ſet down in the *Table*.

CHAP. V.

Of STEREOMETRIA:

OR,

Menſuration of Solids.

Fig. XVIII.

I. *Of the* Cube ABCDEF, *whoſe* Side *is* 5.20 *Foot*.

As 1 : 5.20 : : 5.20 27.04 : : 27.04 : 140.61 *Foot*.

Logar. 5.20	0.716003
Logar. 5.20	0.716003
Logar. 27.04	1.432006
Logar. 5.20	0.716003
Log. of 140.61	2.148009

Equal to the *Solidity* of the *Cube* in *Feet*.

Fig. XIX. This might have been done more eaſily, for the *Logar*. of the *Side* 5.20, which is 0.716003, multiplied by 3, would have produced 2.148009, the *Logar*. of 140.61, the *Solidity*.

II. *Of a* Long Cube *or* Parallelipipedon, GHIKL, *whoſe* Side *of the* Square *at the* End, KI, *is* 1.27 *Foot*, *and of its* Length GH, 5.32 *Foot*.

As 1 : 1.27 : : 1.27 : 1.61 : : 5.32 : 8.58 Foot.

Log. 1.27	0.103804
Log. 1.27	0.103804
Log. 5.32	0.725911
Log. 8.58	0.933519

The *Solid Content*.

III. *Of an* Oblong Parallelipipedon M N O P Q R, *whoſe* Breadth *at the* End N O *is* 3.40 *Foot, and* Depth M N 6.50 *Foot, and the* Length *thereof* M P, 16.25 *Foot.* *Fig.* XX.

As 1 : 3.40 : : 6.50 : 22.10 : : 16.25 : 159.12 Foot.

Logar. 3.40	0.531479	
Logar. 6.50	0.812913	
Logar. 16.25	1.210853	
Logar. 359.12	2.555245	The Solidity.

IV. *Of a* Pyramis S T V X N, *whoſe* Side *of its* Baſe S T, *&c. is* 1.12 *Foot, and* Altitude N X 12.90 *Foot.* *Fig.* XXI.

As 1 : 1.12 : : 1.12 : 1.25 : $\frac{1}{3}$ N X : 4.30 : 5.39 the Sol. Con.

Logar. 1.12	0.049218	
Logar. 1.12	0.049218	
Logar. 4.30	0.633468	
Logar. 5.39	0.731904	the *Solid Content.*

V. *Of a* Cone A B C D, *whoſe* Diameter B C *is* 6.12, *and* Altitude A D 12.90 *Foot.* *Fig.* XXII.

As 7 in 4 (28) : 22

So 6.12 in 6.12 (37.45) : 29.30

The *Area* of the *Circle* of the *Baſe.*

As 1 : 29.30 : : So 4.30 : 126.00

The *Solidity.*

Logar. 28	1.447058	
Logar. 22	1.340422	
Logar. 37.45	1.573452	
	2.913874	
Logar. 29.30	1.466816	the *Area.*
Logar. 4.30	0.633468	
Logar. 126.00	2.100284	the *Solidity.*

VI. *Of the* Fruſtum *of a* Pyramis *or* Cone. *Fig.* XXIII. XXIV.

Let A B C H G F be the *Fruſtum* of a *Square Pyramid*, the *Side* of the *Square* at the *Leſſer* end A B, is 1.60 Foot; and therefore the *Area* of that *Square* A B C D is 2.56 Foot: —The *Side* of the *Square* at the *Greater* End F G, is 2.30 Foot; and therefore the *Area* thereof is 5.29 Foot, and the *Altitude* D E 39.3 Foot.

Then,

The Logarithm of A B C D 2.56 is		0.408240
The Logarithm of E F G H 5.29 is		0.723455
Their Sum		1.131695
Their half Sum		0.565847
Which is the Logarithm of	3.68	
The Sum of the Three Products	11.53	
The Logar. of the Sum	11.53	1.061829
The Log. of $\frac{1}{3}$ of the Altitude	11.31	1.117271
The Log. of	151.04	2.179100

Which is the *Solidity* of the *Fruſtum* A B C D E F G H.

In the ſame manner the *Fruſtum* of a *Cone* of the ſame *Altitude*, and the *Area's* of the *Circles*, at both *Ends*, the ſame with the *Area's* of the *Squares*; the *Solidity* of ſuch a *Fruſtum Cone*, will be found to be the ſame.

Fig. XXV.

VII. *Of a* Globe *or* Bullet, *whoſe* Diameter *is* 1.75 *Foot.*

The Logar. of the Diameter 1.75	0.243038
Multiplied by 3	0.729114
The Logar. of 355	2.550228
Their Sum	3.279342
Logar. of [678 always] Subſtract	2.831229
Reſts the Logar. of 2.806 Foot	0.448113

Which is the *Solidity* of the Bullet.

Fig.
XXV.

ANCILLA MATHEMATICA.

VEL,

Trigonometria Practica.

SECTION II.

OF

COSMOGRAPHY.

COSMOGRAPHY, in Greek Κοσμογραφία, is derived à Κοσμος; i. e. *Mundus*; & γραφὴ; i. e. *Scriptum*, the Deſcription: So that COSMOGRAPHY is a Deſcription of the World; conſider'd under the Names of the Heavens, as the *Cæleſtial*; and of the Earth and Water, as the *Terreſtrial* Parts thereof. A true and perfect Repreſention of both which, may be (and uſually is) deſcribed upon Two Round Bodies, called GLOBES: And therefore we ſhall firſt treat of them.

CHAP. I.

Of a material Sphere *or* Globe; *and of ſuch* Circles, Lines *and* Points, *as are deſcribed upon it, and that are appendant to it. And how to rectifie the* Globe, *fitting it for the reſolving of ſuch Problems as are to be performed by it.*

I. *Of the* Circles *of the* Sphere.

A *Globe* or *Sphere* is an Artificial Repreſentation of the *Starry Heaven*, or of the *Earth* and *Water*, under the Form or Figure Fig. XXV.

Fig. XXV. gure of *Roundneſs*, which they are ſuppoſed to have: Shewing in a juſt *Proportion* every particular *Conſtellation* in the *Heavens*: And ſuch a *Globe* is called [A *Cæleſtial Globe*] and every particular *Region* or *Country* on the *Earth*, and *Iſland* in the *Sea*: And ſuch a *Globe* is called [A *Terreſtrial Globe*.]

Upon the *Convex Superficies* of either of theſe *Globes* (beſides the *Conſtellations* of the *Stars*, upon the *Cæleſtial*; and the *Countries*, *Kingdoms*, *Sea-Coaſts*, *Iſlands*, &c. upon the *Terreſtrial*) there are deſcribed ſeveral *Circles*; ſome *Great Circles*, which divide the whole Body of the *Globe* into Two *Equal Parts*; and ſome *Leſſer Circles*, which divide the Body of the *Globe* into Two *Unequal Parts*.

The *Great Circles* deſcribed upon the Body of either *Globe* are,

The *Æquinoctial* } { The *Æquinoctial* } *Colure.*
The *Ecliptick* } { The *Solſtitial* }

The *Leſſer Circles* are,

The Two *Tropicks*; and, The Two *Polar Circles*.

I. *Of the* ÆQUINOCTIAL.

The *Æquinoctial* is a *Great Circle*, dividing the Body of the *Globe* into Two *Equal Parts*, paſſing through Æ and æ; which *Points* are, each of them, equidiſtant from P and S, the Two *Poles* of the *World*; and ſo the *Line* or *Axis* P A S paſſing through the Body of the *Globe*, cutteth this Circle always at *Right Angles*.

II. *Of the* ECLIPTICK *and* ZODIACK.

The *Zodiack* is a *Great Circle*, croſſing both the *Horizon* and *Æquinoctial* in A, the *Eaſt* and *Weſt* Points of the *Horizon* at *Oblique Angles*, viz. the *Horizon* at an *Angle* equal to the *Sun's* greateſt *Meridian Altitude* in any *Latitude*, but the *Æquinoctial*, always, at an *Angle*, equal to (23.5 Deg.) the *Sun's greateſt Declination*. This *Circle* is ſuppoſed to have Breadth; for, through the middle of it, there paſſeth a *Line*, noted in the Figure with the Twelve *Signs* of the *Zodiack*, ♈ ♉ ♊ ♋ ♌ ♍ on the upper Part, and with ♎ ♏ ♐ ♑ ♒ ♓ on the lower Part; which *Line* is called the *Ecliptick*, or *Via Solis*, for that the *Sun* always keepeth in this *Line*.

III. *Of*

III. *Of the Two* COLURES.

Fig. XXV.

Theſe are Two *Great Circles*, which paſs through the *Poles* of the *World*, where they cut each other at *Right Angles:* One of theſe cutteth the *Horizon* in A, and the *Æquinoctial* alſo in the Point ♈ ♎, and is therefore called the *Æquinoctial Colure*. The other *Colure* cutteth the *Horizon* in H and O, the *North* and *South* Points thereof, and alſo the *Ecliptick* in the *Points* ♋ and ♑, and is therefore called the *Solſticial Colure*. In the Figure the *Æquinoctial Colure* is noted with P A S (and is the ſame with the *Sixth Hour-Circle*.) the *Solſticial Colure*, with H P O S, and is the ſame with the *Meridian*.

IV. *Of the* TROPICK *of* CANCER.

This is a *Leſſer Circle* of the *Sphere*, deſcribed upon the Superficies of the *Globe*, parallel to the *Æquinoctial*, on the *North Side* thereof, at 23.5 Deg. diſtant therefrom, equal to the *Sun's greateſt Declination*. It is called the *Northern Tropick*, for that it lyeth towards the *North Pole*, and is repreſented in the *Figure* by the *Small Circle*, ♋ *6* ♋.

V. *Of the* TROPICK *of* CAPRICORN.

This is a *Leſſer Circle* alſo, deſcribed upon the *Globe*, parallel to the *Æquinoctial* on the *South* Side thereof, at the Diſtance of 23.5 Deg. towards the *South Pole*, and is called the *Southern Tropick:* It is noted in the *Figure* with ♑ *6* ♑.

VI. *Of the Two* POLAR CIRCLES.

Theſe are *Two Leſſer Circles* deſcribed about the *Poles* of the *World*, parallel to the *Æquinoctial* and *Tropicks*, and ſo far diſtant from either *Pole*, (*viz.* 23.5 Deg.) as the *Tropicks* were from the *Æquinoctial*: That deſcribed about the *North Pole*, noted with B *6* C, is called the *Artick Circle*; and the other, noted with G *6* E, deſcribed about the *South Pole*, is called the *Antartick Circle*.

Beſides theſe *Circles*, which are deſcribed upon the Convex Superficies of either *Globe*, there are others, which are framed without the Body of the *Globe*; and thoſe are, principally, Two, Fig. XXV. of the *Meridian*, and the *Horizon*; the one of *Braſs*, the other of *Wood*. —*Circles*, indeed they are not ſo properly called; for, in the rigorous Senſe, no *Line* is ſuppoſed to have any *Breadth*, but both theſe have *Breadth* allowed them; that ſuch Things might be written, or engraven upon them, as might render them more uſeful in all Poſitions of the *Globe*: And therefore, (they being of a *Circular Form*) notwithſtanding the Impropriety of Speech, we will have it ſo; and we muſt call them, The *Meridian* and *Horizontal Circles*. The *Meridian* in the Figure, is noted with the Letters Z Æ N æ, and the *Horizon* with H A O.

Unto this *Brazen Meridian*, there belong Two other Appendants, *viz.* An *Hour-Circle*, with its *Index*; and a *Quadrant* of *Altitude*.

Through the Body of either *Globe*, there runs a ſtrong *Wyre*; the Ends whereof are ſo fixed in the *Brazen Meridian*, that the *Body* of the *Globe* may turn about together with the *Wyre*. —This *Wyre* is called the *Axis* of the *World*, and the Ends thereof, the *Poles* of the *World*, one the *Artick* or *North Pole*, noted with P; the other, the *Antartick*, or *South Pole*, noted with S.

Unto this *Braſs Meridian* alſo, there belongs another Appendant, called *A Quadrant of Altitude*; which is a thin Plate, divided into 90 equal Parts or Degrees, and fitted with a *Nut* and a *Screw*, to move to any Degree upon the *Meridian*; but generally in the Vertical Point Z, the Zenith of any Place.

II. *Of the ſeveral* Poſitions, *that a* Globe *or* Sphere, *may be poſited in its* Horizon.

THere are but *Three Poſitions*, in which a *Globe* may be ſeated in its *Horizon*; viz. (1.) *Direct.* (2.) *Parallel.* (3.) *Oblique.* Of which, the Two firſt are *Particular*, the third more *General*.

I. *Of* Direct Poſition.

The *Globe* may be ſo placed in the *Frame*, that both the *Poles* thereof may reſt upon (or lye directly in) the *North* and *South* Points of the *Horizon*, neither *Pole* having any Elevation: The *Zenith* Point being in the *Æquinoctial Circle*, and the *Axis* of the World in the Plain of the *Horizon*. And this is called *Direct Poſition*.

II. *Of* Parallel Poſition.

Fig. XXV.

The *Globe* may be ſo placed in the Frame, that one of the *Poles* ſhall be in the *Zenith*, and the other in the *Nadir Points*; the *Poles* having 90 Deg. of *Elevation* above the *Horizon:* And in this Situation, the *Æquinoctial Circle* will be in the *Horizon.* This *Poſition* is called *Parallel Poſition.*

III. *Of* Oblique Poſition.

The third *Poſition* of the *Globe* is more *General*; for it hath relation to all *People* living between the *Æquinoctial* and either *Pole:* And according as the *Poles* are *Elevated* or *Depreſſed*, accordingly are the *People* ſaid to be *ſituate:* That is, if the *Globe* be placed in the *Horizontal Frame*; ſo that the *Pole* be elevated above the *Horizon* 10 Deg. then is the *Globe* elevated, or fixed, to reſolve ſuch *Queſtions Geographical* or *Aſtronomical*, as relate to ſuch *Places* and *People*, who have the *Pole* elevated (or who live in the *Latitude* of) 10 Deg. And this *Poſition* of the *Globe* is called *Oblique*, becauſe the *Axis* of the *World*, the *Æquinoctial*, and all the *Parallels* of the *Sun's Declination*, are cut by the *Horizon* at *Oblique Angles*; whereas in the Two former *Poſitions*, they cut one the other *Right Angles.*

III. *How to Rectifie the* Globes, *fitting them for* Uſe *in any* Latitude *or* Part *of the* World.

Being provided of a *Pair of Globes*, the *Meridian*, *Horizon*, and *Hour-Circle*, truly turned and divided; alſo the *Balls* truly hung or poized upon the *Axis*, and the *Meridian* and *Horizon* (in all Poſitions) cutting each other at *Right Angles*; the *Papers* truly joined in their paſting upon the *Bodies*, &c. All which are to be performed by the *Workman*, (yet the *Buyer* ought alſo to have Inſpection thereunto) you may proceed to *Rectifie* them for *Uſe* in this manner.

1. Put the *Braſs Meridian* into the Two *Notches* that are cut in the *North* and *South* Parts of the *Horizon*; the *Graduated*, or *Divided*, Part thereof, towards the *Eaſt Part* of the *Horizon*; and the plain, or undivided Side thereof, towards the *Weſt*, and let the *Meridian* alſo reſt in the *Notch* which is in the Foot, or bottom, of the *Frame* of the *Horizon.*

Fig. XXV. 2. Place the *Braſs Hour-Circle*, or *Wheel*, about the Pole; ſo that the *Hour-Lines* of 12 and 12 do lye directly over the *Eaſt*, or *Graduated*, Side of the *Meridian*; and that the Point of the *Axis* of the *Globe* do paſs directly through the *Centre* of the *Hour-Wheel*; then ſhall the Two *Twelves* repreſent the Two Hours of 12 and 12: That towards the South Part of the *Meridian* 12 at Noon, and the other towards the North Part, 12 at Midnight: And the Two Sixes ſhall repreſent the Two Hours of Six a Clock: That towards the *Eaſt*, 6 in the *Morning*, and the other 6 at *Night*. Then put the *Index* (or *Pointer*) upon the end of the *Axis*; ſo that as the *Globe* being turned, which way ſoever, the Pointer may move with it; and ſo is your *Hour-Circle* rectified.

3. Elevate the Pole of your *Globe* (whether *North* or *South*) according to the *Latitude* of the Place of that what part of the World you are in: As ſuppoſe *London*, which hath 51 Deg. 30 Min. of *North Latitude*: The *Meridian* being in the Notches of the *Horizon*, and alſo in the Notch at the bottom of the Frame, as is before directed. Move the *Meridian* upwards or downwards in the Notches, till you find 51 Deg. 30 Min. of the *Meridian*, juſtly to touch the upper part of the *Horizon*, on the *North* part thereof: And ſo is your *Globe* rectified to the *Latitude* of 51 Deg. 30 Min.

4. The next thing to be rectified is the *Quadrant* of *Altitude*; which muſt be done, by having reſpect to the *Latitude* alſo: Wherefore, the *Latitude* being 51 Deg. 30 Min. Count 51 Deg. 30 Min. *North*, upon the *South* part of the *Meridian*, from the *Equinoctial Circle*, towards the *North* (or elevated) *Pole*, and there put on the *Nut*, which is at the end of the *Quadrant*; ſo that the edge of the *Diviſions* of the *Quadrant*, may be directly under the Degrees of the *Latitude*, viz. 51 Deg. 30 Min. and there ſcrew the *Nut* faſt. And thus is your *Globe* Rectified for the Solution of all ſuch Queſtions Coſmographical, as are to be wrought thereby in that *Latitude* of 51 Deg. 30 Min.

CHAP. II.

Coſmographical ELEMENTS,

Neceſſary to be known.

I. THere are Two kind of *Motions* in the Heavens; the firſt is called the *Common Motion* of the *fixed Stars* and *Planets* together; by which they go all about in 24 Hours from *Eaſt* to *Weſt*. The ſecond Motion is called the proper *Motion*; by which they go about, every one in his own *Time* or *Period*, from *Weſt* to *Eaſt*. Fig. XXV.

II. Theſe Two *Motions* are the Original of Two *Circles*, the *Æquinoctial* and the *Ecliptick*; for the *Diurnal Motion* is done about the Pole of the *Æquinoctial* either in the *Æquinoctial* it ſelf, or in a *Leſſer Circle*, parallel unto it: But the proper *Motion*, is about the *Poles* of the *Ecliptick*, either in the *Ecliptick* it ſelf, or in a *Leſſer Circle*, parallel unto it.

III. The Sun's *Center* keepeth always upon the *Ecliptick Line*, but the other *Planets* do go from the *Ecliptick* on both Sides 8 Deg. Hence the broad *Circle*, whoſe Middle is the *Ecliptick*, doth ariſe, and is called the *Zodiack*.

IV. The *Æquinoctial* is in the Heavens about that Streak, which the Sun doth make by his *Diurnal Motion* on the Days of the Two *Æquinoxes*, viz. the 10th of *March*, and the 12th of *September*.

V. The *Zodiack* is known by the Twelve *Aſteriſms* of *fixed Stars*, called, The *Twelve Signs*.

VI. The Two *Luminaries* are the *Sun* ☉, and the *Moon* ☾, the *Moon* cometh round by her proper *Motion*, in a *Month*, the *Sun* in a *Year*.

VII. The other *Planets* are either the *Superior*, as *Saturn* ♄, coming about in his proper Motion, once in 30 Years. *Jupiter*

Fig. XXV. *ter* ♃ in 12, *Mars* ♂ in 2 Years. The *Inferior Planets* are *Venus* ♀ and *Mercury* ☿; *Venus* is 9 Months *Morning Star*, and other 9 Months *Evening Star*. Theſe Two Planets keep always near to the *Sun*, ſo that *Mercury* ☿ is for the moſt part covered with its Beams.

VIII. The *fixed Stars* move alſo from *Weſt* to *Eaſt*, either in the *Ecliptick*, or in a Parallel to the *Ecliptick*, but very ſlowly, *viz.* One Degree in 70 Years. Hence the *Signs* are diſtinguiſhed in *Starred* and *Un-ſtarred*. The *Starred Signs* are the Twelve *Aſteriſms* of the *Zodiack*; but the *Un-ſtarred* are every one a Twelfth part of the *Ecliptick*. Now the *Starred Signs* left their former Places, and are preceded in ſome 1800 Years almoſt One whole *Sign*; ſo the *Starred Aries* ♈, ſtands now in the Place of the *Un-ſtarred Taurus* ♉; and the *Starred Taurus* ♉, in the Place of the *Un-ſtarred Gemini* ♊, *&c.*

IX. The *Æquinoctial* and *Ecliptick* are immutable, for there is never but One *Æquinoctial*, and One *Ecliptick*: But the *Horizon* and *Meridian* are mutable: For every Body walking upon the Superficies of the *Earth*, doth carry along with him his *Horizon*; So this *Circle* is as manifold as there are divers Points upon the Surface of the *Earth*. The *Horizon* is determined by the Eye of the Man turning about in an even open Field, where the Heaven ſeemeth to join with the *Earth*; and its Office is to ſhew the *Riſing* and *Setting* of all *Heavenly Bodies*.

X. The *Meridian* is not alter'd by going on ſtreight towards *South* or *North*, but only when you walk never ſo little towards the *Eaſt* or *Weſt*, you have preſently another *Meridian*. It is obſervable in the Heaven, by letting fall a Plummet or Perpendicular from the *Vertex*, by the *Sun* (or any *Star*) being at its higheſt.

XI. Every one of theſe Four *Circles* hath its *Poles*, which the *Circle* is juſt between, and every way equally diſtant from it, exactly dividing the *Sphere* into Two equal *Hemiſpheres*, and they divide each other into Two equal *Semicircles*: And by the *Poles* of each, there are deſcribed *Secondary Circles* (the *Meridian* only excepted) which *Secondary Circles* do cut their *Principal Circle* into Two Equal Parts, and at *Right Angles*.

Fig. XXV.

XII. The *Poles* of the *Æquinoctial* are the ſame with the *Poles* of the *World*; the one of which is called the *Artick Pole*, becauſe it is near to the Two *Arktos* or *Bears*: The other oppoſite to it, is called *Antartick*: And the ſtreight Line, which paſſeth between, through the Centre of the *Sphere* (from one *Pole* to the other) is called the *Axis* of the *World*. The *Æquinoctial* divideth the *Ecliptick* into Six *North*, and Six *South*, *Signs*: The *Secondary Circles* of the *Æquinoctial*, are called in the Heavenly Sphere *Circles* of *Declination*. Amongſt theſe is one of chiefeſt Note, the *Meridian*; and beſides it, Eleven *Hour-Circles*, paſſing by every 15th Degree of the *Æquinoctial*, to be reckoned from the *Meridian*, and ſo they divide the whole *Æquinoctial* into 24 equal *Hours*. There are alſo Two chief *Secondary Circles* of the *Æquinoctial*, which are called *Colures*; the one paſſing by the *Vernal* and *Autumnal Section*, is called the *Colure* of the *Æquinoxes*; the other paſſing by the Two Solſticial Points, *viz.* the beginning of *Cancer* ♋ and *Capricorn* ♑, is called the *Colure* of the *Solſtice*. This latter, divideth the *Ecliptick* in *Aſcending* and *Deſcending Signs*; becauſe in the firſt the Sun doth aſcend to our *Zenith* in *Capricorn*, *viz.* in ♑ ♒ ♓ ♈ ♉ ♊, which are alſo called the *Signs* of *Short Aſcenſion*, becauſe they riſe in a ſhort time equal to the *Shorteſt Day* of the Year: But in the *Deſcending Signs*, the Sun doth deſcend every Day more and more from our *Zenith*, and thoſe are ♋ ♌ ♍ ♎ ♏ ♐: Theſe are called alſo *Signs of Long Aſcenſion*, becauſe they Riſe in a time, equal to the *Longeſt Day* in the Year. Both *Colures* together divide the *Ecliptick* into Four *Quadrants*; the *Vernal* containing *Aries* ♈, *Taurus* ♉, *Gemini* ♊; The *Summer* Quadrant, *Cancer* ♋, *Leo* ♌, *Virgo* ♍; The *Autumnal*, *Libra* ♎, *Scorpio* ♏, *Sagittarius* ♐; the *Winter* Quadrant, *Capricorn* ♑, *Aquaries* ♒, *Piſces* ♓.

XIII. The Secondaries of the Terreſtrial *Æquinoctial* are called *Meridians*, and they are 18, paſſing by every 10th Degree of the *Æquinoctial*, and in ſome *Globes*, through every 15th Degree of the *Æquinoctial*, which is equal to One Hour, in Time. The firſt of theſe doth paſs by the Iſlands of *Azores*, in ſome *Globes*. But by the neweſt, and beſt made *Engliſh*, *Globes*, at St. *Michael's* Iſland in the *Azores*.

Fig. XXV. XIV. The *Parallels* to the *Æquinoctial*, are *Leſſer Circles*, dividing the *Sphere* into Two *unequal* Parts: They are Two *Tropicks*, which the Sun deſcribeth, the one in the *Longeſt Day*, the other in the *Shorteſt Day*. Their Name *Tropicks*, ſignifieth a returning, *viz.* of the *Sun*. And Two other *Leſſer Circles* are deſcribed by the *Diurnal Revolution* of the *Poles* of the *Eclipiick*; the one called *Artick*, the other *Antartick*.

XV. The Two *Tropicks*, together with the Two *Polar Circles*, diſtinguiſheth the whole Surface of the *Earth* into Five *Zones*; the *Hot* or *Torrid Zone* is between the Two *Tropicks*, but from the *Tropick* of *Cancer* ♋ to the *Artick Circle* is the North *Temperate Zone*, and from the *Tropick* of *Capricorn* ♑ to the *Antartick*, is the South *Temperate Zone*. What lyeth within the *Artick Circle*, is called the *North Cold or Frigid Zone*; and that which lyeth within the *Antartick*, the *South Frigid Zone*.

XVI. The *Inhabitants* of the *Torrid Zone* are called *Amphiſcii*, becauſe their *Mid-days Shadow* falleth now towards the *South*, and then towards the *North*: But the *Inhabitants* of the Two *Temperate Zones* are called *Heteroſcii*, becauſe their *Mid-days Shadow* falleth only towards the *North* in our *Hemiſphere*, and only towards the *South* in the other *Hemiſphere*: And laſtly, the *Inhabitants* of the Two *Frigid Zones* are called *Periſcii*, becauſe their Shadow goeth round about them in 24 Hours.

XVII. There are *Parallels* alſo which diſtinguiſh the *Climates*; which *Climates* are, as it were, *Little Zones*: They are greateſt near the *Æquinoctial*, and from thence they grow ſmaller and ſmaller towards both the *Poles*. Their Diſtance and Largeneſs is determined by the length of the *Longeſt Day*; for as often as the *Longeſt Day* gaineth *half an Hour* above 12 Hours, there is produced a new *Climate*: So the firſt natural *Climate* ought to have its midſt where the *Longeſt Day* is 12 Hours and a half long: But the Ancients (perhaps knowing nothing of thoſe Parts of the World ſo near to the *Æquinoctial*) left out this firſt natural *Climate*, and made their firſt *Climate*, where indeed the Second ſhould have been, *viz.* where the *Longeſt Day* was 13 Hours; and becauſe the midſt of this did paſs by the Iſland which the *Nilus* maketh, and is called *Meroe*; they named their firſt *Climate Dia Meries*, the ſecond *Dia Syenes*, *Syene* being a Town lying under the Tropick of *Cancer*; the third they called *Dia Alexandria*, this being a Town ſituated upon the Month *Nilus*; the fourth, *Dia Rhodia*; the fifth, *Dia Romes*; the ſixth, *Dia Ponta*; the ſeventh, *Dia Boriſthenis*, where the Longeſt Day was 16 Hours; and here the Ancients left it: But the Modern have continued their *Climates* to the *Artick Circle*, where the Longeſt Day is 24 Hours, becauſe the Sun doth not ſet there in the beginning of *Cancer*, but only toucheth the *Horizon* with his *Circle*. And now, there are as many *Climates* on the other Side of the *Æquinoctial*. The *Circle* of *Perpetual Apparition* is deſcribed by a Point touching the *North Cardinal*, being carried about by the daily Motion; and all the Stars that are within this *Circle* of *Perpetual Apparition* are always ſeen above our *Horizon*. Another *Circle* of *Perpetual Occultation* there is deſcribed by a Point touching the *South Cardinal*, and being carried about by the daily Motion; ſo all the *Stars* that are within this, do never *Riſe* with us.

Fig. XXV.

XVIII. The *Secondaries* of the *Ecliptick*, are called *Circles* of *Latitude*; there are Six of them upon the *Cæleſtial Globe*, dividing the Signs of the *Ecliptick*, and alſo the whole *Sphere* into *Twelve* equal Parts; by which Diviſion all *Stars* are referred to that *Sign* which is between the Two next *Circles* of *Latitude*. Now Six Signs of the *Ecliptick* are always above the *Horizon*, and Six are beneath.

XIX. The *Poles* of the *Horizon*, are the *Zenith* and the *Nadir*; the *Secondary Circles* are called *Vertical* or *Aximuth*, or *Circles* of *Altitude*, amongſt which, the chiefeſt are, (1.) The *Meridian*. (2.) The *Circle of the 90th Degree of the Ecliptick*, paſſing by the *Poles* of the *Ecliptick*, the *Zenith*, and the 90th Degree of the *Ecliptick*, being counted from the *Horizon*, either from the *Eaſt* or *Weſt*. (3.) The *Vertical*, paſſing by the *Eaſt* and *Weſt Cardinals*; where the Interſection is of the *Æquinoctial* and the *Horizon*; which Sections alſo, be Poles of the *Meridian*: And the *Poles* of this *Vertical* paſſing by *Eaſt* and *Weſt*, are in the *South* and *North Cardinals*, where the *Meridian* doth divide the *Horizon* in the *Ortive* and *Occidental Semicircles*. Now the *Secondaries* of this ſaid *Vertical*, which

Fig. XXV. paſſeth by the true *Eaſt* and *Weſt*, are the *Circles* of *Poſition*, paſſing by every 30th Degree of the *Æquinoctial*, reckoning from the *Meridian* or *Horizon*, and dividing the whole *Sphere* into *Twelve Houſes*. The firſt *Houſe* is called *Horoſcope*, and is that which is next under the *Ortive Horizon*; from thence the other *Houſes* do ſucceed under the *Earth* after the Succeſſion of the *Signs*. Where every *Arch* of *Poſition* cutteth the *Ecliptick*, there is the *Cuſpides* of the *Houſes*. There are beſides theſe *Circles* of *Poſition*, an infinite Number paſſing by every Point of the *Sphere*.

XX. The *Parallels* of the *Horizon* are called *Almacantarath*, and are deſcribed upon the *Aſtrolabe*, to ſhew the *Altitude* of the *Sun*, or of the *Stars* above the *Horizon*.

XXI. The *Meridian* is the *Original* of *Winds*; there are Four *Cardinals*, and Four *Mean*: The *North* is known by the *Flower-de-Luce*, and the *Eaſt* by the *Croſs*: The *Mean* do compound their *Names* from the next adjacent *Cardinals*; being *North-Eaſt*, *North-Weſt*, *South-Eaſt*, *South-Weſt*: Now every one of the *Cardinals* and *Means* hath Two *Laterals*, bearing the ſame Name with their *Principals*; ſo they are called *North by Eaſt*; *North by Weſt*; *North Eaſt by Eaſt*; *North Eaſt by North*, &c. Now theſe *Laterals* being 16, make, together with the 8 *Principals*, 24; and juſt in the midſt, between every Two *Principals*, there are the Eight *Reſidual Winds*, bearing the ſame Name with the *Means* they are next unto; but taking a fore Name from the next *Cardinal*, ſo they are called *North-North Eaſt*, *Eaſt North-Eaſt*, &c.

XXII. Theſe 32 *Winds* being continued upon the Surface of the *Earth*, do make as many *Rumbs*; that *Rumb* which paſſeth by the *South* or *North*, is always a *Meridian*; and that which paſſeth by *Eaſt* and *Weſt*, is always either the *Æquinoctial*, or a *Parallel* unto it. The other *Rumbs* are *crooked Lines*, neither *Circular* nor *Elliptical*, and are *Seven* in every *Quadrant*, to be numbred both Ways from the *Meridian*. The general Propriety of all *Rumbs* is to cut all the *Meridians* they paſs by into equal *Angles*. There may alſo, beſides the ſaid 32 *Rumbs*, paſs one by every *Point* of the *Horizon*.

XXIII. The Effects of all the foreſaid *Circles*, both *Principal* and *Secondary*, are *Angles* and *Arks*. The *Angles* which they make are *Right*, when a *Secondary* meeteth with its own *Principal*, and the other *Angles* are altogether *Oblique*: So the *Angle* which the *Æquinoctial* maketh with the *Ecliptick* is always 23 Deg. 30 Min. as much as the greateſt *Declination* of the *Sun*. Fig. XXV.

XXIV. The general Way of *Meaſuring Angles* upon the *Sphere*, is to ſet one Foot of a Pair of *Callipers* in the Point where the *Angle* is made, and extend the other 90 Deg. from thence, and ſo deſcribe an *Ark* between the Two *Legs*, (or *Circles* which make the *Angle*) for as many Degrees as that *Ark* containeth, ſo many Degrees alſo is the *Angle*. So the *Meaſure* of the *Angle* made by the *Æquinoctial* and *Ecliptick* is taken in the *Solſticial Colure*, between the beginning of ♋, and the 90th Degree of the *Æquinoctial*. Likewiſe the *Angle* which the *Æquinoctial* makes with the *Horizon*, is meaſured 90 Deg. from thence in the *Meridian*, between the *Horizon* and *Æquinoctial*; and that which is farther in the *Meridian*, from the *Æquinoctial* to the *Zenith*, is the *Latitude* of one *Place*; into which is, always, equal to the *Poles Height*, to be reckoned from the *North Cardinal* to the *Pole*. Sailing ſtreight towards *South* or *North* One Mile, the *Æquinoctial* is raiſed or fallen One Minute; going 60 Miles, it riſeth or falleth One Degree: So at length, coming to the middle Line of the World, the *Æquinoctial* will be raiſed to the very *Zenith*, both *Poles* lying in the *Horizon*, and all the *Parallels* to the *Æquinoctial* cut the *Horizon* at *Right Angles*; whence this *Poſition* of the *Sphere* is called *Sphæra Recta*; where all the *Stars* do riſe, and abide as long above the *Horizon* as beneath; ſo there is a perpetual *Æquinoctial* all the Year: But as ſoon as one of the *Poles* doth *riſe* above the *Horizon*, and the other cometh to be under the *Æquinoctial*, and all its *Parallels* make *Oblique Angles* with the *Horizon*: And for this Reaſon, ſuch a *Poſition* of the *Sphere* is called *Sphæra Obliqua*; where, not all the *Stars* do riſe, but ſome are always *above*, and ſome always *below*, the *Horizon*. When the *Pole* cometh to unite with the *Zenith*, then the *Æquinoctial* falleth wholly in the *Horizon*, and the *Parallels* of it, are alſo parallel to the *Horizon*; and this *Poſition* of the *Sphere*

Fig. XXV. is called *Sphæra Parallela.* The Sun moving in that part of the *Ecliptick*, which is above the *Horizon*, that is on one ſide of the *Æquinoctial*, doth never ſet, but is turned continually round about, and maketh a Day of Six Months: So likewiſe, running through the other Six *Signs* that are under the *Horizon*, it doth never *riſe*, but maketh a Night of Six Months alſo.

XXV. The *Angle* which any Degree of the *Ecliptick* maketh with the *Right Horizon*, (that is, in *Sphæra Recta*) is equal to that *Angle* which the ſame Degree of the *Ecliptick*, maketh with the *Meridian:* But whether the *Ecliptick* make ſuch an *Angle* with the *Right* or *Oblique Horizon*, the ſame *Angle* is always called the *Angle Orient*; that is, of the riſing Degree of the *Ecliptick*; and its *Meaſure* is in the *Circle* of the 90th Degree, between the ſaid 90th Degree and the *Horizon*.

XXVI. The *Angle* which the *Meridian* maketh in the *Pole* of the *World* with any *Circle* of *Declination*, taketh its Meaſure in the *Æquinoctial*, between the *Meridian* and the ſaid *Circle* of *Declinatial*, and this they call, The *Diſtance of the Star from the Meridian.* So likewiſe the *Angle* which the *Meridian* maketh with the *Vertical Circle* at the *Zenith*, taketh its *Meaſure* in the *Horizon*, between the *Meridian* and the ſaid *Vertical*; and this they call, The *Azimuth of the Sun* or *Star*.

XXVII. The *Arks* to be meaſured in every *Principal Great Circle*, or its *Secondaries*, have alſo their proper *Appellations:* So the *Ark* of the *Æquinoctial*, which is comprehended between the beginning of ♈, and the *Circle* of *Declination* paſſing by any *Star*, is called the *Right Aſcenſion* of that *Star:* And the *Ark* which in the ſaid Circle of *Declination*, is between the *Æquinoctial* and the *Star*, is called, The *Declination* of that *Star.* But the *Oblique Aſcenſion* of a *Star*, is an *Ark* of the *Æquinoctial*, reckoned from the beginning of ♈, to the Point of the *Æquinoctial Riſing* with that *Star:* So likewiſe, the *Oblique Deſcenſion* of a *Star* is, the *Ark* of the *Æquinoctial* reckoned from the beginning of ♈, to that Point of the *Æquinoctial*, which is a ſetting together with the *Star*. Now the Difference that is between the *Right* and *Oblique Aſcenſion* of the *Sun*, or any *Star*, is called, *The Aſcenſional Difference.* Moreover, the *Ark* of the *Ecliptick*, which is between the beginning of ♈,

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Fig. XXV.

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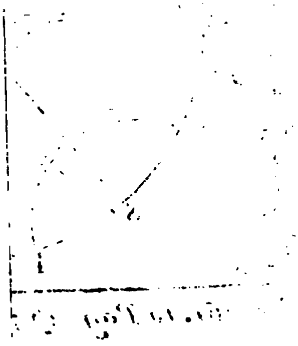

Fig. XXV.

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and *Circle* of *Latitude* paſſing by any *Star*, is called, The *Longitude* of that *Star*: And the *Ark*, which in the ſaid *Circle* of *Latitude*, is between the *Ecliptick* and the *Star*, is the *Stars Latitude*: And all this is to be underſtood of the *Cæleſtial Globe*. But upon the *Terreſtrial Globe*, the *Longitude* and *Latitude* of any *Place* are referred to the *Æquinoctial* and *Meridian*: So the *Longitude* of an *Earthly Place* is an Ark of the *Æquinoctial*, intercepted between the *Firſt Meridian*, and the *Meridian* paſſing by the ſame *Place*. And the *Latitude* of the ſame *Place* is, an Ark of the *Meridian*, to be reckoned from the *Æquinoctial* to the Place upon the *Globe*. Fig. XXV.

XVIII. In the *Horizon* we reckon the *Amplitude* of the *Sun* or any *Star*, between the true *Eaſt* or *Weſt* Points, and that Point where the *Sun* or *Star* doth Riſe or Set: And the ſaid *Amplitude* is either *North* or *South*, according to the Beaming of the *Sun* or *Star*, in reſpect of the true *Eaſt* or *Weſt Points*. The *Altitude* of the *Sun* or a *Star*, is taken in the *Vertical Circle*, paſſing by the ſame, between the *Horizon* and the ſaid *Star*: So the *Depreſſion* of the *Star* is, An *Arch* of the *Vertical Circle*, between the *Horizon* and the ſaid *Star*.

ANCILLA MATHEMATICA.

VEL,

Trigonometria Practica.

SECTION III.

OF

GEOGRAPHY.

THE following *Geographical Problems* being firſt to be performed upon the *Terreſtrial Globe*; upon which the *Spherical Triangle*, that reſolves any *Queſtion* is diſcovered, in order to the *Trigonometrical Calculation*: I conceive it neceſſary, in the firſt place, to inſert this *General*

PROBLEM.

How to Meaſure *the* Sides *and* Angles, *of all* Spherical Triangles, *upon the Convex Superficies of the* Globe.

Fig. XXVI. THE *Sides* of all *Spherical Triangles* upon the *Globe*, are *Meaſured* by the *Degrees* of thoſe *Great Circles*, that make (or conſtitute) the *Triangle*, contained between the Two *Angular Points*.

1. If the *Side*, or *Sides*, of the *Triangle* to be meaſured, do conſiſt of ſuch *Great Circles* as are actually divided into *Degrees* upon the *Globe*, or its Appendants; as the *Æquinoctial*, the *Colures*, the *Ecliptick*, the general *Meridian* or *Horizon*: Then, the number of Degrees contained in that *Great Circle*, contained between the Two *Angular Points*; is the Quantity of that *Side* of that *Triangle* in *Degrees*. But,

2. If the *Side* or *Sides* of the *Triangle* be compoſed of *Arches* of ſuch *Great Circles* as are not actually divided (as all *Circles* of *Longitude*, and other *Oblique Great Circles*) then, take the Length of ſuch Side in a Pair of *Calliper Compaſſes*, and apply it to any of the forementioned *Great Circles* (as the *Æquinoctial*, &c.) it ſhall thereupon ſhew you the Quantity of that *Side* in *Degrees*. —Or, the *Quadrant of Altitude* (but rather, a thin *Plate* of *Braſs* longer than the *Quadrant of Altitude*, divided into *Degrees*, as the *Quadrant* is) applied to the *Side* to be *Meaſured*, between the Two *Angular Points*, ſhall give you the Quantity of the *Degrees* of that *Side* of the *Triangle*. Fig. XXVI.

II. *For the* Angles.

The *Angles* of *Spherical Triangles* are *Meaſured* upon the Superficies of the *Globe*; by counting (or ſetting off) 90 Deg. from the *Angular Point*, of the *Angle* to be *Meaſured*, upon both the *Sides* which contains the *Angle* to be *Meaſured:* And at the Terminations of thoſe 90 Deg. on both the *Sides*, make Two ſmall *Marks* upon the *Globe*. Unto theſe Two *Marks*, apply the *Quadrant of Altitude*, or thin *Plate* of *Braſs*; ſo the Number of the *Degrees* thereof, contained between the Two *Marks*, is the Quantity of that *Angle*.

Geographical Problems.

PROB. I.

To find the Longitude *of any* Place, *deſcribed upon the* Terreſtrial Globe.

L*Ongitude* is the Diſtance of a Place from the firſt *Meridian* reckoned in the Degrees of the *Equator*, beginning, as was ſaid, in the *New Terreſtrial Globe*, (made by Mr. *Morden*) as St. *Michael*'s Iſland in the *Azores*.

Practice.] Bring the Place, (that is, the Mark of the Place) ſuppoſe *London*, to the *Brazen Meridian*; then count how many Degrees of the *Equator* are contained between the firſt *Meridian*, and that of *London* cut by the *Brazen Meridian*, which you will find to be 28 Deg. and that is the *Longitude* required. And in this manner you find

London

Fig. XXVI.

		D.	M.
London	to be diſtant from the firſt *Meridian* by theſe *New Globes.*	28	0
Jeruſalem		66	30
Jedo in *Japan*		167	0
Rio de la plata		32	0
Mexico		75	0
Charleton Iſle		51	30

PROB. II.

To find the Latitude *of any Place.*

THE *Latitude* of a Place, is the Diſtance of the *Equator* from the Parallel of that Place, reckoned in the Degrees of the *Braſs Meridian*; and is either *North* or *South*, according as it lyes between the *North* or *South Poles* of the *Equator*.

To find the *Latitude*, bring the Mark of the Place, ſuppoſe *London*, to be the *Brazen Meridian*; then count the Number of Degrees upon the *Meridian*, contained between the *Equator* and the Place ☉. Thus you ſhall find the *Latitude*, by this new *Globe*, of *London*, to be 51 Deg. 30 Min. and of

	D.	M.	By other *Globes* and *Maps.*	D.	M.
Labor in the *Mogul's* Country to be	31	30		23	30
The *South* Part of the *Caſpian* Sea to be	37	0		41	0
Aſtracan on the *North* Part of the *Caſpian* Sea to be	46	0		49	0
The *North* Part of *China* to be	42	0		52	0
Delli in *India* to be	28	0		21	0

PROB. III.

Two Places, which differ only in Latitude, *to find their* Diſtance.

IN this there are Two Varieties of Poſition.

1. If both the *Places* lye under the ſame *Meridian*, and on one and the ſame Side of the *Æquinoctial: Subſtract the Leſſer Latitude from the Greater, the Difference* (or Remainder) *reduced into Miles*, (by allowing 60 Deg. to One Mile) *ſhall give you the Diſtance.*

Example. *London* and *Ribadio* lye both under the ſame *Meridian*, but differ in *Latitude:* For *London* hath 51 Deg. 30 Min. of *Latitude*, at L, and *Ribadio* hath 34 Deg. of *Latitude*, at R, both *North*; the *Difference* of *Latitude* is 17 Deg. 30 Min. equal to the *Arch* L R: And that Reduced into *Miles*, makes 1050 for their *Diſtance.* Fig. XXVI.

2. If the Two *Places* lye under the ſame *Meridian*, but in different *Hemiſpheres*, *i. e.* one on the *North*, and the other on the *South* Side of the *Æquinoctial:* Then, *Add both the Latitudes together, and the Sum of them is their Diſtance.*

Example. *London*, and the Iſland *Triſtan Dacunhu*, lye both under One *Meridian*, but *London* hath 51 Deg. 30 Min. of *North Latitude*, at L, and the Iſland hath 34 Deg. of *South Latitude*, at D; the Sum of theſe Two *Latitudes* is 85 Deg. 30 Min. equal to the *Arch* of the *Meridian* L Æ D; the which reduced into *Miles*, (by multiplying the Degrees by 60, and allowing for every Minute One Mile) makes 5130 Miles, for their Diſtance.

PROB. IV.

Two P*laces*, *which differ in* Longitude *only*; *To find their Diſtance.*

IN this there are Two Varieties of Poſition.

1. If the Two Places lye both under the *Æquinoctial*, and have no *Latitude*; in this Caſe, *Their Difference of Longitude* (if it be leſs than 180 Deg.) *is their Diſtance:* But, if the Difference exceed 180 Deg. Subtract it from 360 Deg. and the Remainder is their *Diſtance*, in Degrees.

Example. The Iſland *Samatra*, and Iſland St. *Thomas*, lye both under the *Æquinoctial*: St. *Thomas* having 22 Deg. 10 Min. of *Longitude* at T, and the Iſland *Samatra* 82 Deg. 10 Min. at S. Now, the Leſſer Longitude 22 Deg. 10 Min. ſubſtracted from the Greater 82 Deg. 10 Min. leaves 60 Deg. equal to the Arch S T, for their Difference in Degrees: Which converted into Miles, makes 3600, and ſo many Miles are the Two Iſlands diſtant from each other.

2. But if the Two *Places* differ only in *Longitude*, and lye not in the *Æquinoctial*, but under ſome other intermediate *Parallel* of *Latitude:* As *Hieruſalem* at H, and *Baldo* at B; both in the *Parallel* of 31 Deg. 40 Min. of North *Latitude*, but differing in *Longitude* 60 Deg. 15 Min. equal to the *Angle* H P B, to find the Diſtance of theſe Two Places.

Fig. XXVI.

I. *By the* Globe.

Apply the *Quadrant* of *Altitude*, or *Braſs Plate*, to the Two Places, and the Number of Degrees thereof contained between the Two Places, is their Diſtance, which will be found to be 50 Deg. 32 Min.

II. *By* Trigonometrical Calculation.

The *Quadrant* of *Altitude* (or Braſs Plate) applied to the Two Places, is repreſented by the *Arch* H B, and the *Arches* of the Two *Meridians*, which paſs through the Two Places, are P B N, and P H M; and P B and P H, are equal to the Complement of of the *Latitude* of both the Places, *viz.* 58 Deg. 20 Min. So that now you have conſtituted upon the *Globe* an *Oblique Spherical Triangle* P B H; in which you have given, (1.) The Two Sides P B and P H, both equal to 58 Deg. 20 Min. the Complement of the *Latitude*. (2.) The *Angle* B P H 60 Deg. 15 Min. the *Difference* of the *Longitude* of the Two given *Places*. To find the *Side* B H, their *Diſtance*. For which this is

The Canon for Calculation. By *CASE* I. of *R. A. S. T.*

As the Radius, Sine 90 Deg.
Is to the Co-ſine of the *Common Latitude* (P H or P B) 58 D. 20 M.
So is the Sine of half the *Difference of Longitude*, (half B P H) 30 Deg. 07 $\frac{1}{2}$ Min.
To to Sine of half the Diſtance (half B H) 25 Deg. 16 Min.
The Double whereof, 50 Deg. 32 Min. is the *Diſtance* B H, which in Miles is 3032 Miles.

PROB. V.

Two Places, *which differ both in* Longitude *and* Latitude; *to find their* Diſtance.

IN this there are Three various *Poſitions*.

1. If one of the Places lye under the *Æquinoctial*, and ſo have no *Latitude*; and the other under ſome *Parallel* of *Latitude* between the *Æquinoctial*, and one of the *Poles*: As *London*, in 51 Deg. 30 Min. of North *Latitude* at L; and St. *Thomas* Iſland under the *Æquinoctial* at T, but differ in *Longitude* 18 Deg. For finding the *Diſtance* of theſe Two *Places*.

I. *Upon the* Terreſtrial Globe.

Fig. XXVI.

Bring *London* to the *Braſs Meridian*, and over it, fix the *Quadrant of Altitude:* The *Globe* being in this Poſition, bring the *Quadrant of Altitude* to lye juſt over St. *Thomas* Iſland, and you will find it cut the *Quadrant of Altitude*, in 54 Deg. 45 Min. for the *Diſtance* of the Two Places.

II. *By* Trigonometrical Calculation.

The *Globe* reſting in the former Poſition, you will find conſtituted upon it a *Right-angled Spherical Triangle* L Æ T, compoſed of, (1.) L Æ, an *Arch* of the *Braſs Meridian.* (2.) Æ T, an *Arch* of the *Æquinoctial.* And, (3.) L T, an *Arch* of a *Great Circle* (made by the *Quadrant of Altitude*) paſſing through both the Places: And in this *Triangle*, you have given, (beſides the *Right Angle* at Æ) (1.) The *Perpendicular* L Æ, the *Latitude* of *London*, 51 Deg. 30 Min. (2.) The *Angle* at L, the *Difference* of *Longitude* 18 Deg. 10 Min. To find the Hypotenuſe L T, the *Diſtance.*

The Canon for Calculation. By CASE XIV. of *R. A. S. T.*

As Tang. of the Latitude L Æ, 51 Deg. 30 Min.
Is to the Radius;
So is the Co-ſine of Æ L T, 18 Deg. 10 Min.
To the Co-tangent of L T, 52 Deg. 55 Min.
Which reduced into Miles, makes 3175 Miles, for *Diſtance* between *London*, and St. *Thomas* Iſland.

2. If both the *Places* propoſed ſhall be without the *Æquinoctial*, but both of them, either on the *North* or *South* Side thereof: As *London* in 31 Deg. 30 Min. at L, and *Hieruſalem* in 31 Deg. 40 Min. at H, both on the *North* Side of the *Æquinoctial*; and their Difference of *Longitude* 46 Deg: To find their *Diſtance*

Upon the Terreſtrial Globe.

Bring one of the Places, as *London*, to the *Braſs Meridian*, and over it ſcrew the *Quadrant of Altitude*, and keep the *Globe* there fixed, then move the *Quadrant of Altitude*, till it lye over *Hieruſalem*, and you ſhall find it to lye under 39 Deg. 32 Min. of the *Quadrant:* And that is the *Diſtance* of the Two Places.

By Trigonometrical Calculation.

Fig. XXVI. The *Globe* being in the former Poſition, you will find upon it an *Oblique-angled Spherical Triangle*, compoſed of P L, an Arch of the *Braſs Meridian*: Of P H, an Arch of the *Meridian* that paſſeth over *Hieruſalem*; and of L H, an Arch of the *Quadrant of Altitude*. In which *Triangle* you have given, (1.) P L, the Complement of the *Latitude* of *London*, 38 Deg. 30 Min. (2.) The Side P H (the Complement of the *Latitude* of *Hieruſalem* 59 Deg. 20 Min. (3.) The Angle L P H, the *Difference of Longitude* of the Two Places, 46 Deg. To find the Two Angles of *Poſition* at L and H: And, (2.) The Side L H, the *Diſtance* of the Two Places.

The Canons for Calculation.

By *CASE* III. and *CASE* IX. of *A. O. S. T.*

		D.	M.
The Side	P L is	38	30
	P H	59	20
Their Sum is		96	50
Their Difference is		19	50
The Half Sum is		48	25
The Half Difference is		9	55.

Being thus prepared, I ſay,

(1.) As the Sine of half the Sum of the Sides P L and P H, 48 Deg. 25 Min.
Is to the Sine of half their *Difference*, 9 Deg. 55 Min.
So is the Co-tangent of half the *Difference* of *Longitude* (*i. e.*) half the Angle L P H, 23 Deg.
To the Tangent of 28 Deg. 29 Min:
Which is the half *Difference* of the *Angles* P L H and P H L.

(2.) As the Co-ſine of half the Sum of the Sides P L and P H, 41 Deg. 35 Min.
Is to the Co-ſine of half their *Difference*, 80 Deg. 2 Min.
So is the Co-tangent of half the *Difference* of *Longitude*, *i. e.* half the Angle L P H 23 Deg.
To the Tangent of 74 Deg. 2 Min.
Which is the Tangent of half the Sum of the Two Angles P L H and P H L.

The

Fig. XXVI.

	D.	M.	
The Half Sum is	74	02	
The Half Difference is	28	29	
Their Sum	102	31	equal to ∠ P L H.
Their Difference	45	32	equal to ∠ P H L.

(3.) For the Side L H, which is the *Diſtance*,
As the Sine of P H L, 45 Deg. 33 Min.
Is to the Sine of P L, 38 Deg. 30 Min.
So is the Sine of L P H, the Differ. of *Longitude*, 46 Deg.
To the Sine of L H, 30 Deg. 51 Min.
Which 38 Deg. 31 Min. reduced in Miles, makes 2331 Miles for the *Diſtance* of the Two Places.

3. If the Two Places propoſed ſhould be ſo ſituate, that one have *North*, and the other *South*, *Latitude*, and under different *Meridians*. As ſuppoſe *Conſtantinople*, lying in *North Latitude* 47 D. at C. and the *Cape* of *Good Hope*, lying in 35 Deg. of *South Latitude*, at V; and differing in *Longitude* 59 Deg. To find the *Diſtance* of theſe Two Places

Upon the Terreſtrial Globe.

Bring one of the Places (as *Conſtantinople*) to the *Braſs Meridian*, and there keep the *Globe*; then apply the *Quadrant of Altitude* (or rather a thin Plate of Braſs divided as that is) to the Two *Places*, and you ſhall find 97 Deg. 42 Min. of the *Quadrant* (or *Braſs Plate*) to be contained between them, and that is their *Diſtance*.

By Trigonometrical Calculation.

The *Globe* reſting in its former Poſition, you will diſcover upon it an *Oblique-angled Spherical Triangle*, compoſed of P C, an Arch of the *Braſs Meridian*: P V, an Arch of a *Meridian*, paſſing through the Place in *South Latitude*: And of C V, the *Quadrant* (or *Braſs Plate*) repreſenting the Arch of a *Great Circle* paſſing through both the *Places*:—In which *Triangle* you have given, (1.) The Side P C, the *Complement* of the *Latitude* of *Conſtantinople*, 43 Deg. (2.) The Side P V, the *South Latitude* of the Cape of *Good Hope*, with 90 Deg. added, which make 125 Deg. And, (3.) The Angle V P C, the *Difference* of *Longitude*, 59 Deg. To find, (1.) The Angles of *Poſition* P V C and P C V: And, (2.) The third Side V C, the *Diſtance* of the Two Places.

The

Fig. XXVI. *The Canons for Calculations, as in the laſt, by* CASE III. *and* IX. *of A. O. S. T.*

(1.) As the Sine of the *Half Sum* of the *Sides* P V and P C, 84 D.
Is to the Sine of half their *Difference*, 41 Deg.
So is the Co-tangent of half the *Difference of Longitude*, *i. e.* (half the Angle V P C) 29 Deg. 30 Min.
To the Tangent of 49 Deg. 23 Min.
Which is the *Half Difference* of the Angles P V C and P C V.

(2.) As the Co-ſine of half the *Sum* of the *Sides*, P V and P C, 6 Deg.
Is to the Co-ſine of half their *Difference*, 49 Deg.
So is the Co-tangent of half the *Difference of Longitude*, 61 Deg.
To the Tangent of 85 Deg. 32 Min.
Which is half the *Sum* of the Two Angles, P C V and P V C.

	D.	M.	
The half Sum is	85	32	
The half Difference is	49	23	
Their Sum	134	55	equal to ∠ P C V.
Their Difference	36	09	equal to ∠ P V C.

The Two *Angles* of *Poſition*.

(3.) For the Side V C, which is the *Diſtance* of the Two *Places*.
As the Sine of P V C, 36 Deg. 9 Min.
Is to the Sine of P C, 43 Deg.
So is the Sine of V P C, 59 Deg.
To the Sine of 82 Deg. 18 Min.
Whoſe Complement to 180 Deg. is 97 Deg. 42 Min. for the Side V C, which is the *Diſtance* of the Two *Places*. Which in Miles is 5862.

And theſe are all the Varieties of Poſitions that any Two *Places* upon the *Globe* can be ſituate.

ANCILLA MATHEMATICA.

VEL,

Trigonometria Practica.

SECTION IV.

OF

ASTRONOMY.

ADVERTISEMENT.

WHereas, for the reſolving of the following *Aſtronomical Problems* by *Trigonometrical Calculation*, it is abſolutely neceſſary, that the true *Place* (or *Longitude*) of the *Sun* in the *Ecliptick* be firſt known; I have therefore inſerted *Aſtronomical Tables* of the *Sun's Mean Longitude* and *Anomaly* in *Years*, *Months*, *Days* and *Hours*; and alſo of *Æquation*, whereby the Sun's true Place in the *Ecliptick* may be found at any time. It is alſo to be noted, That the ſeveral *Problems* in this *Section*, being wrought firſt by the *Cœleſtial Globe*, in order to their *Trigonometrical Calculation*; I have, therefore, to inform the Fancy, and eaſe the Memory, of the Reader (in all the *Schemes* relating to them, (which conſiſt of *Great Circles* of the *Sphere*, and repreſent the *Triangle* to the *Eye*, which the *Problem* is to be reſolved by) noted the *Circles*, *Lines* and *Points*, in all the *Schemes*, with the ſame *Letters* or *Characters*; and prefixed *Them*, and their *Significations*, next before the *Problems*. *Fig.* XXVII.

TABLE

Fig. XXVII.

TABLE I.

The Sun's Mean Motion in Years.

	Longit. ☉.			Anomal. ☉				Longit. ☉			Longit. ☉		
Years.	*S.*	*D.*	*M.*	*S.*	*D.*	*M.*	*Years.*	*S.*	*D.*	*M.*	*S.*	*D.*	*M.*
B 1700	9	19	58	6	12	33	1731	9	20	27	6	12	30
1701	9	20	42	6	13	16	B 1732	9	20	12	6	12	14
1702	9	20	29	6	13	1	1733	9	20	57	6	12	58
1703	9	20	14	6	12	46	1734	9	20	43	6	12	43
B 1704	9	20	0	6	12	31	1735	9	20	29	6	12	27
1705	9	20	45	6	13	14	B 1736	9	20	14	6	12	12
1706	9	20	30	6	12	59	1737	9	20	59	6	12	56
1707	9	20	16	6	12	43	1738	9	20	45	6	12	40
B 1708	9	20	2	6	12	28	1739	9	20	30	6	12	25
1709	9	20	46	6	13	12	B 1740	9	20	16	6	12	10
1710	9	20	32	6	12	56	1741	9	21	1	6	12	53
1711	9	20	18	6	12	41	1742	9	20	46	6	12	38
B 1712	9	20	3	6	12	26	1743	9	20	32	6	12	23
1713	9	20	48	6	13	10	B 1744	9	20	18	6	12	7
1714	9	20	24	6	12	54	1745	9	21	2	6	12	51
1715	9	20	30	6	12	39	1746	9	20	48	6	12	36
B 1716	9	20	5	6	12	23	1747	9	20	34	6	12	20
1717	9	20	50	6	13	7	B 1748	9	20	20	6	12	5
1718	9	20	36	6	12	52	1749	9	21	4	6	12	49
1719	9	20	21	6	12	37	1750	9	20	50	6	12	33
B 1720	9	20	7	6	12	21	1751	9	20	36	6	12	18
1721	9	20	52	6	13	5	B 1752	9	20	21	6	12	2
1722	9	20	37	6	12	50	1753	9	21	6	6	12	46
1723	9	20	23	6	12	34	1754	9	20	52	6	12	31
B 1724	9	20	9	6	12	19	1755	9	20	38	6	12	16
1725	9	20	54	6	13	2	B 1756	9	20	23	6	12	0
1726	9	20	39	6	12	47	1757	9	21	8	6	12	44
1727	9	20	25	6	12	32	1758	9	20	54	6	12	29
B 1728	9	20	11	6	12	16	1759	9	20	39	6	12	13
1729	9	20	55	6	12	0	B 1760	9	20	25	6	11	58
1730	9	20	41	6	12	45	1761	9	21	10	6	12	42

TABLE II.

The Sun's Mean Motion in Months and Days.

Days in Years.		*JANUARY.*						*Days in Years.*		*FEBRUARY.*					
		Longit. ☉			*Anom.* ☉					*Longit.* ☉			*Anom.* ☉		
Com.	*Biſ.*	*S.*	*D.*	*M.*	*S.*	*D.*	*M.*	*Com.*	*Biſ.*	*S.*	*D.*	*M*	*S.*	*D.*	*M.*
1	0	0	0	59	0	0	59	1	0	1	1	32	1	1	32
2	1	0	1	58	0	1	58	2	1	1	2	32	1	2	31
3	2	0	2	57	0	2	57	3	2	1	3	31	1	3	31
4	3	0	3	56	0	3	56	4	3	1	4	30	1	4	30
5	4	0	4	56	0	4	56	5	4	1	5	29	1	5	29
6	5	0	5	55	0	5	55	6	5	1	6	28	1	6	28
7	6	0	6	54	0	6	54	7	6	1	7	27	1	7	27
8	7	0	7	53	0	7	53	8	7	1	8	26	1	8	26
9	8	0	8	52	0	8	52	9	8	1	9	25	1	9	25
10	9	0	9	51	0	9	51	10	9	1	10	25	1	10	24
11	10	0	10	50	0	10	50	11	10	1	11	24	1	11	24
12	11	0	11	50	0	11	50	12	11	1	12	23	1	12	23
13	12	0	12	49	0	12	49	13	12	1	13	22	1	13	22
14	13	0	13	48	0	13	47	14	13	1	14	21	1	14	21
15	14	0	14	47	0	14	47	15	14	1	15	20	1	15	20
16	15	0	15	46	0	15	46	16	15	1	16	19	1	16	19
17	16	0	16	45	0	16	45	17	16	1	17	19	1	17	18
18	17	0	17	44	0	17	44	18	17	1	18	18	1	18	18
19	18	0	18	44	0	18	43	19	18	1	19	17	1	19	17
20	19	0	19	42	0	19	43	20	19	1	20	16	1	20	16
21	20	0	20	42	0	20	42	21	20	1	21	15	1	21	15
22	21	0	21	41	0	21	41	22	21	1	22	14	1	22	14
23	22	0	22	40	0	22	40	23	22	1	23	13	1	23	13
24	23	0	23	39	0	23	39	24	23	1	24	13	1	24	12
25	24	0	24	38	0	24	38	25	24	1	25	12	1	25	12
26	25	0	25	38	0	25	37	26	25	1	26	11	1	26	11
27	26	0	26	37	0	26	37	27	26	1	27	10	1	27	10
28	27	0	27	36	0	27	35	28	27	1	28	9	1	28	9
29	28	0	28	35	0	28	35	29	28	1	29	8	1	29	8
30	29	0	29	34	0	29	34								
31	30	1	0	33	1	0	33								

TABLE II.

The Sun's Mean Motion in Months and Days.

		March.								*April.*					
		Longit. ☉			*Anom.* ☉					*Longit.* ☉			*Anom.* ☉		
Com.	*Biſ.*	*S.*	*D.*	*M.*	*S.*	*D.*	*M.*	*Com.*	*Biſ.*	*S.*	*D.*	*M.*	*S.*	*D.*	*M.*
1	0	1	29	8	1	29	8	1	0	3	29	42	2	29	41
2	1	2	0	7	2	0	7	2	1	3	0	41	3	0	40
3	2	2	1	6	2	1	6	3	2	3	1	40	3	1	40
4	3	2	2	6	2	2	6	4	3	3	2	39	3	2	39
5	4	2	3	5	2	3	5	5	4	3	3	28	3	3	38
6	5	2	4	4	2	4	4	6	5	3	4	37	3	4	37
7	6	2	5	3	2	5	3	7	6	3	5	36	3	5	36
8	7	2	6	2	2	6	2	8	7	3	6	36	3	6	35
9	8	2	7	1	2	7	1	9	8	3	7	35	3	7	34
10	9	2	8	0	2	8	0	10	9	3	8	34	3	8	34
11	10	2	9	0	2	8	59	11	10	3	9	33	3	9	33
12	11	2	9	59	2	9	58	12	11	3	10	32	3	10	32
13	12	2	10	58	2	10	58	13	12	3	11	31	3	11	31
14	13	2	11	57	2	11	57	14	13	3	12	30	3	12	30
15	14	2	12	56	2	12	56	15	14	3	13	20	3	13	29
16	15	2	13	55	2	13	55	16	15	3	14	29	3	14	28
17	16	2	14	54	2	14	54	17	16	3	15	28	3	15	27
18	17	2	15	54	2	15	53	18	17	3	16	27	3	16	27
19	18	2	16	53	2	16	53	19	18	3	17	26	3	17	26
20	19	2	17	52	2	17	52	20	19	3	18	25	3	18	25
21	20	2	18	51	2	18	51	21	20	3	19	24	3	19	24
22	21	2	19	50	2	19	50	22	21	3	20	23	3	20	23
23	22	2	20	49	2	20	49	23	22	3	21	23	3	21	22
24	23	2	21	48	2	21	48	24	23	3	22	22	3	22	21
25	24	2	22	49	2	22	47	25	24	3	23	21	3	23	21
26	25	2	23	47	2	23	46	26	25	3	24	20	3	24	20
27	26	2	24	46	2	24	46	27	26	3	25	19	3	25	19
28	27	2	25	45	2	25	45	28	27	3	26	18	3	26	18
29	28	2	26	44	2	26	44	29	28	3	27	17	3	27	17
30	29	2	27	43	2	27	43	30	29	3	28	17	3	28	16
31	30	2	28	42	2	28	42	31	30	3	29	10	3	29	15

TABLE II.

The Sun's Mean Motion in Months and Days.

		MAY.								*JUNE.*					
		Longit. ⊙			*Anom.* ⊙					*Longit.* ⊙			*Anom.* ⊙		
Com.	*Biſ.*	*S.*	*D.*	*M.*	*S.*	*D.*	*M.*	*Com.*	*Biſ.*	*S.*	*D.*	*M.*	*S.*	*D.*	*M.*
1	0	3	29	16	3	29	15	1	0	4	29	49	4	29	49
2	1	4	0	15	4	0	14	2	1	5	0	48	5	0	48
3	2	4	1	14	4	1	14	3	2	5	1	47	5	1	47
4	3	4	2	13	4	2	13	4	3	5	2	46	5	2	46
5	4	4	3	12	4	3	12	5	4	5	3	46	5	3	45
6	5	4	4	11	4	4	11	6	5	5	4	45	5	4	44
7	6	4	5	10	4	5	10	7	6	5	5	44	5	5	43
8	7	4	6	10	4	6	9	8	7	5	6	43	5	6	43
9	8	4	7	9	4	7	8	9	8	5	7	42	5	7	42
10	9	4	8	8	4	8	8	10	9	5	8	41	5	8	41
11	10	4	9	7	4	9	7	11	10	5	9	40	5	9	40
12	11	4	10	6	4	10	6	12	11	5	10	40	5	10	39
13	12	4	11	5	4	11	5	13	12	5	11	39	5	11	38
14	13	4	12	4	4	12	4	14	13	5	12	38	5	12	37
15	14	4	13	4	4	13	3	15	14	5	13	37	5	13	36
16	15	4	14	3	4	14	2	16	15	5	14	36	5	14	36
17	16	4	15	2	4	15	2	17	16	5	15	35	5	15	35
18	17	4	16	1	4	16	1	18	17	5	16	34	5	16	34
19	18	4	17	0	4	17	0	19	18	5	17	33	5	17	33
20	19	4	17	59	4	17	59	20	19	5	18	33	5	18	32
21	20	4	18	58	4	18	58	21	20	5	19	32	5	19	31
22	21	4	19	58	4	19	57	22	21	5	20	31	5	20	30
23	22	4	20	57	4	20	56	23	22	5	21	30	5	21	29
24	23	4	21	56	4	21	55	24	23	5	22	29	5	22	28
25	24	4	22	55	4	22	55	25	24	5	23	28	5	23	28
26	25	4	23	54	4	23	54	26	25	5	24	28	5	24	27
27	26	4	24	53	4	24	53	27	26	5	25	27	5	25	26
28	27	4	25	52	4	25	52	28	27	5	26	26	5	26	25
29	28	4	26	51	4	26	51	29	28	5	27	25	5	27	24
30	29	4	27	51	4	27	50	30	29	5	28	24	5	28	24
31	30	4	28	50	4	28	49	31	30	5	29	23	5	29	23

TABLE II.

The Sun's Mean Motion in Months and Days.

		JULY.					
		Longit. ⊙			*Anom.* ⊙		
Com.	*Biſ.*	*S.*	*D.*	*M.*	*S.*	*D.*	*M.*
1	0	5	29	23	5	29	23
2	1	6	0	22	6	0	22
3	2	6	1	21	6	1	21
4	3	6	2	21	5	2	20
5	4	6	3	20	6	3	19
6	5	6	4	19	6	4	18
7	6	6	5	18	5	5	17
8	7	6	6	17	6	6	17
9	8	6	7	18	5	7	16
10	9	6	8	16	5	8	15
11	10	6	9	15	6	9	14
12	11	6	10	14	6	10	13
13	12	6	11	13	6	11	12
14	13	6	12	12	6	12	11
15	14	6	13	11	6	13	11
16	15	6	14	10	6	14	10
17	16	6	15	9	6	15	9
18	17	6	16	9	6	16	8
19	18	6	17	8	6	17	7
20	19	6	18	7	6	18	6
21	20	6	19	6	6	19	5
22	21	6	20	5	6	20	5
23	22	6	21	4	5	21	4
24	23	6	22	3	6	22	3
25	24	6	23	3	6	23	2
26	25	6	24	2	5	24	1
27	26	6	25	1	5	25	0
28	27	5	26	0	6	25	59
29	28	6	26	59	6	26	59
30	29	6	27	58	6	27	58
31	30	6	28	57	6	28	57

		AUGUST.					
		Longit. ⊙			*Anom.* ⊙		
Com.	*Biſ.*	*S.*	*D.*	*M.*	*S.*	*D.*	*M.*
1	0	6	29	56	6	29	56
2	1	7	0	56	7	0	55
3	2	7	1	55	7	1	54
4	3	7	2	54	7	2	53
5	4	7	3	52	7	3	52
6	5	7	4	52	7	4	52
7	6	7	5	51	7	5	51
8	7	7	6	50	7	6	50
9	8	7	7	50	7	7	49
10	9	7	8	49	7	8	48
11	10	7	9	48	7	9	47
12	11	7	10	47	7	10	46
13	12	7	11	46	7	11	45
14	13	7	12	45	7	12	45
15	14	7	13	44	7	13	44
16	15	7	14	44	7	14	43
17	16	7	15	43	7	15	42
18	17	7	16	42	7	16	41
19	18	7	17	41	7	17	40
20	19	7	18	40	7	18	39
21	20	7	19	39	7	19	39
22	21	7	20	38	7	20	38
23	22	7	21	38	7	21	37
24	23	7	22	37	7	22	36
25	24	7	23	36	7	23	35
26	25	7	24	35	7	24	34
27	26	7	25	34	7	25	33
28	27	7	26	33	7	26	33
29	28	7	27	32	7	27	32
30	29	7	28	31	7	28	31
31	30	7	29	31	7	29	30

TABLE II.

The Sun's Mean Motion in Months and Days.

		SEPTEMBER.					
		Longit. ⊙			Anom. ⊙		
Com.	*Biſ.*	*S.*	*D.*	*M*	*S.*	*D.*	*M*
1	0	8	0	30	8	0	25
2	1	8	1	29	8	1	28
3	2	8	2	28	8	2	28
4	3	8	3	27	8	3	27
5	4	8	4	26	8	4	26
6	5	8	5	25	8	5	25
7	6	8	6	25	8	6	24
8	7	8	7	24	8	7	23
9	8	8	8	23	8	8	22
10	9	8	9	22	8	9	21
11	10	8	10	21	8	10	21
12	11	8	11	20	8	11	20
13	12	8	12	19	8	12	19
14	13	8	13	19	8	13	18
15	14	8	14	18	8	14	17
16	15	8	15	17	8	15	16
17	16	8	16	16	8	16	15
18	17	8	17	15	8	17	15
19	18	8	18	14	8	18	14
20	19	8	19	13	8	19	13
21	20	8	20	13	8	20	12
22	21	8	21	12	8	21	11
23	22	8	22	11	8	22	10
24	23	8	23	10	8	23	9
25	24	8	24	9	8	24	8
26	25	8	25	8	8	25	8
27	26	8	26	7	8	26	7
28	27	8	27	7	8	27	6
29	28	8	28	6	8	28	5
30	29	8	29	5	8	29	4
31	30	9	0	4	9	0	3

		OCTOBER.					
		Longit. ⊙			Anom. ⊙		
Com.	*Biſ.*	*S.*	*D.*	*M.*	*S.*	*D.*	*M*
1	0	9	0	4	9	0	3
2	1	9	1	3	9	1	2
3	2	9	2	2	9	2	2
4	3	9	3	1	9	3	1
5	4	9	4	1	9	4	0
6	5	9	5	0	9	4	59
7	6	9	5	59	9	5	58
8	7	9	6	58	9	6	57
9	8	9	7	57	9	7	56
10	9	9	8	56	9	8	55
11	10	9	9	55	9	9	54
12	11	9	10	54	9	10	54
13	12	9	11	54	9	11	53
14	13	9	12	53	9	12	52
15	14	9	13	52	9	13	51
16	15	9	14	51	9	14	50
17	16	9	15	50	9	15	49
18	17	9	16	49	9	16	48
19	18	9	17	48	9	17	48
20	19	9	18	48	9	18	47
21	20	9	19	47	9	19	46
22	21	9	20	46	9	20	45
23	22	9	21	45	9	21	44
24	23	9	22	44	9	22	43
25	24	9	23	43	9	23	42
26	25	9	24	42	9	24	42
27	26	9	25	42	9	25	41
28	27	9	26	41	9	26	40
29	28	9	27	40	9	27	39
20	29	9	28	39	9	28	28
31	30	9	29	38	9	29	37

TABL

TABLE II.

The Sun's Mean Motion in Months and Days.

		NOVEMBER.								DECEMBER.					
		Longit. ☉			Anom. ☉					Longit. ☉			Anom. ☉		
Co.	Bi.	S.	D.	M.	S.	D.	M.	Co.	Bi.	S.	D.	M.	S.	D.	M.
1	0	10	0	37	10	0	36	1	0	11	0	11	11	0	10
2	1	10	1	36	10	1	35	2	1	11	1	11	11	1	10
3	2	10	2	36	10	2	35	3	2	11	2	10	11	2	9
4	3	10	3	35	10	3	34	4	3	11	3	9	11	3	8
5	4	10	4	34	10	4	33	5	4	11	4	8	11	4	7
6	5	10	5	33	10	5	32	6	5	11	5	7	11	5	6
7	6	10	6	32	10	6	31	7	6	11	6	6	11	6	5
8	7	10	7	31	10	7	30	8	7	11	7	5	11	7	4
9	8	10	8	30	10	8	29	9	8	11	8	5	11	8	4
10	9	10	9	30	10	9	29	10	9	11	9	4	11	9	3
11	10	10	10	29	10	10	28	11	10	11	10	3	11	10	2
12	11	10	11	28	10	11	27	12	11	11	11	2	11	11	1
13	12	10	12	27	10	12	26	13	12	11	12	1	11	12	0
14	13	10	13	26	10	13	25	14	13	11	13	0	11	12	59
15	14	10	14	25	10	14	24	15	14	11	14	0	11	13	58
16	15	10	15	24	10	15	23	16	15	11	14	58	11	14	57
17	16	10	16	23	10	16	23	17	16	11	15	58	11	15	57
18	17	10	17	23	10	17	22	18	17	11	16	57	11	16	56
19	18	10	18	22	10	18	21	19	18	11	17	56	11	17	55
20	19	10	19	21	10	19	20	20	19	11	18	55	11	18	54
21	20	10	20	20	10	20	19	21	20	11	19	54	11	19	53
22	21	10	21	19	10	21	18	22	21	11	20	53	11	20	52
23	22	10	22	18	10	22	17	23	22	11	21	52	11	21	51
24	23	10	23	17	10	23	16	24	23	11	22	52	11	22	51
25	24	10	24	17	10	24	16	25	24	11	23	51	11	23	50
26	25	10	25	16	10	25	15	26	25	11	24	50	11	24	49
27	26	10	26	15	10	26	14	27	26	11	25	49	11	25	48
28	27	10	27	14	10	27	13	28	27	11	26	48	11	26	47
29	28	10	28	13	10	28	12	29	28	11	27	47	11	27	46
30	29	10	29	12	10	29	11	30	29	11	28	46	11	28	45
31	30	11	0	11	11	0	10	31	30	11	29	46	11	29	45

TABLE III.

The Sun's Mean Motion in Hours.

Hours.	Longitude. Mi.	Aphelion. Mi.
1	2	2
2	5	5
3	7	7
4	10	10
5	12	12
6	15	15
7	17	17
8	20	20
9	22	22
10	25	25
11	27	27
12	30	30
13	32	32
14	34	34
15	37	37
16	39	39
17	42	42
18	44	44
19	47	47
20	49	49
21	52	52
22	54	54
23	57	57
24	59	59

TABLE IV.

A Table of the Sun's Æquation.

	Sign 0.		*Sign* 1.		*Sign* 2.		*Sign* 3.		*Sign* 4.		*Sign* 5.		
	Æ. Subſ.		*Æ. Subſ.*		*Æ. Subſ.*		*Æ. Subſ.*		*Æ. Subſ.*		*Æ. Subſ.*		
	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	
0	0	0	0	58	1	41	1	57	1	43	0	0	30
1	0	2	1	0	1	42	1	57	1	42	0	58	29
2	0	4	1	1	1	43	1	57	1	41	0	56	28
3	0	6	1	3	1	44	1	57	1	40	0	55	27
4	0	8	1	5	1	45	1	57	1	39	0	53	26
5	0	10	1	6	1	46	1	57	1	38	0	51	25
6	0	12	1	8	1	47	1	57	1	36	0	49	24
7	0	14	1	10	1	47	1	57	1	35	0	47	23
8	0	16	1	11	1	48	1	57	1	34	0	45	22
9	0	18	1	13	1	49	1	56	1	33	0	43	21
10	0	20	1	15	1	50	1	56	1	32	0	41	20
11	0	22	1	16	1	50	1	56	1	30	0	39	19
12	0	24	1	18	1	51	1	56	1	29	0	37	18
13	0	26	1	19	1	52	1	55	1	27	0	35	17
14	0	28	1	21	1	52	1	55	1	26	0	33	16
15	0	30	1	22	1	53	1	54	1	25	0	31	15
16	0	32	1	23	1	54	1	54	1	23	0	29	14
17	0	34	1	25	1	54	1	53	1	22	0	27	13
18	0	36	1	26	1	54	1	52	1	20	0	25	12
19	0	38	1	28	1	55	1	52	1	19	0	23	11
20	0	39	1	29	1	55	1	51	1	17	0	21	10
21	0	41	1	30	1	56	1	51	1	15	0	19	9
22	0	43	1	32	1	56	1	59	1	14	0	17	8
23	0	45	1	33	1	56	1	49	1	12	0	15	7
24	0	47	1	34	1	57	1	48	1	10	0	13	6
25	0	49	1	35	1	57	1	48	1	9	0	10	5
26	0	51	1	36	1	57	1	47	1	7	0	8	4
27	0	52	1	38	1	57	1	46	1	5	0	6	3
28	0	54	1	39	1	57	1	45	1	3	0	4	2
29	0	56	1	40	1	57	1	44	1	2	0	2	1
30	0	58	1	41	1	57	1	42	1	0	0	0	0
	Æ. Add.		*Æ. Add.*		*Æ. Add.*		*Æ. Add.*		*Æ. Add.*		*Æ. Add*		
	11 *Sign.*		10 *Sign.*		9 *Sign.*		8 *Sign.*		7 *Sign.*		6 *Sign.*		

The

The Uſe of the Tables.

TO find the *Sun*'s true Place in the *Ecliptick* at any time, by theſe *Tables*, this is the

RULE.

From the ſeveral *Tables*, collect the *Years*, *Month*, and *Day* of the *Month*, and alſo the odd *Hours*, if any be, and ſet them down, one under another, with the reſpective *Longitudes* and *Anomalies* anſwering thereto: Then add the ſeveral *Longitudes* and *Anomalies* together, and with the *Sum* of the *Anomalies*, enter (*Table* IV.) of *Æquations*, which *Æquation*, *Add to*, or *Subſtract from*, the *Sum* of the *Longitudes*, and the *Sum* or *Remainder*, ſhall be the *Sun*'s true Place in the *Ecliptick* for the time propoſed.

How Time is to be computed.

1. Any *Day* begins upon its own *Noon*; ſo that the Firſt Day of *January*, at 12 at *Noon*, is the common *Term* of the *Old* and *New Years*.

Example.

> Let the true Place of the *Sun* in the *Ecliptick* be required, for the 16th Day of *May*, in the Year of our Lord 1703. at 5 in the *Afternoon*.

Firſt, Set down the Year 1703. then the Month and Day, *May* 16; and *laſtly*, the Hours, 5, as is here due.

	Long. ☉ *S.*	*D.*	*M.*	*Anom.* ☉ *S.*	*D.*	*M.*
Year 1703.	9	20	14	06	12	46
May, Day 19.	4	14	03	04	12	02
Hour 5			12			12
☉ Mean Lon.	1	04	29	10	25	00
Æquat. Add.		01	06			
☉ true Place.	2	05	35	♊ *Gemini.*		

Secondly, Look for 1703. (in *Table* I.) againſt which ſtands 9 S. 10 D. 14 M. for the ☉ *Longitude*; and 6 S. 12 D. 46 M. for the ☉ *Anomaly*; both which ſet down as here you ſee.

Thirdly, Look for *May* 16. (in *Table* II.) againſt which ſtands 4 S. 14 D. 3 M. for the *Long.* And 4 S. 12 D. 2 M. for the *Anomaly*; both which ſet under the former, as you ſee.

Fourthly, Look for 5 Hours (in *Table* III.) againſt which ſtands 12 Minutes, both for the *Longit.* and *Anomal.* both ſet down, as you ſee.

Fifthly, Add all the *Longitudes* together, and they make 14 Sig. 04 Deg. 29 Min. (from which abate 12 *Signs*, and there remains only Two *Signs.*) —Alſo *Add* the *Anomalies* together, and they make 10 Sig. 25 Deg. 00 Min.

Sixthly, (in *Table* IV.) Look for 10 *Signs* at the *Bottom* of the *Table*, and 25 Deg. in the laſt Column towards the *Right Hand*, ſo againſt it, over 10 *Signs*, you ſhall find 1 Deg 6 Min. to be *Added*. Set them under the *Mean Longitude*, and add them to it, ſo will the *Sum* be 2 Sig. 05 Deg. 35 Min. And that is the true *Place* of the Sun in the Ecliptick, which is 35 Deg. diſtant from the *Æquinoctial Point* ♈ *Aries*. Note, that

Signs	0	1	2	3	4	5	6	7	8	9	10	11
Is	♈	♉	♊	♋	♌	♍	♎	♏	♐	♑	♒	♓

Other Examples.

	Longit. ☉	*Anom.* ☉
	S. D. M.	*S. D. M.*
Year 1720.	9 20 07	6 12 21
Febr. Day 29	1 28 09	1 28 09
Hours 6	15	15
Mean Motion	11 18 31	8 10 45
Æquat. Add.	1 52	
☉ true Place.	11 20 23	♓ *Piſces.*

In this *Example*, it being *Leap-Year*, I take the Day of the Month out of the Column that hath *Biſſex.* at the Head of it.

	Longit. ☉	*Anom.* ☉
	S. D. M.	*S. D. M.*
Year 1713	9 20 48	6 13 10
April 23	3 21 23	3 21 22
At Noon	0 00 00	0 00 00
Mean Motion	1 12 11	10 04 32
Æqua. Add.	01 35	
☉ true Place.	1 13 46	♉ *Taurus.*

	Longit. ☉	*Anom.* ☉
	S. D. M.	*S. D. M.*
Year 1761	9 21 10	6 12 42
Jan. Day 12	11 50	11 50
Hours 16	39	39
Mean Motion	10 3 39	6 25 11
Æquat. Add.	51	
☉ true Place.	10 4 30	♒

Note alſo, That if the *Sum* of the Sun's *Anomalies* be leſs than Six *Signs*, they will be found at the *Head* of the *Æquation Table*, and the *Degrees* in the *Firſt Column* towards the *Left Hand*, and the *Æquation* muſt (always) be *Subſtracted* from the *Mean Longitude*; whereas in all theſe *Examples* it hath been *Added*.

In all the *Problems* in this *Section*, it is to be underſtood, That (in all the *Spherical Schemes*, or *Figures* following:

P	Repre-ſents the	*Pole* of the World.	
Æ Q		*Æquinoctial* Circle, or *Æquator*.	
P P		*Parallels* (or ſmaller Circles) of *Declination* of the *Sun*, or of a *Star*.	
F		*Pole* of the *Ecliptick*:	
E C		*Ecliptick*.	
F F		*Parallels* (or leſſer *Circles*) of *Altitude* of the *Sun*, or of a *Star*, or the *Latitude* of a *Place* or *Country*.	
♈		*Vernal*	Interſection of the *Exliptick* and *Æquator*.
♎		*Autumnal*	
A		*Aſcenſion*	of any Point of the *Ecliptick*.
D		*Deſcenſion*	
Z		*Zenith*.	
N		*Nadir*.	
H O		*Horizon*.	
Z N		*Prime Vertical* Circle, or *Azimuth* of *Eaſt* and *Weſt*.	
O *r*		*Eaſt*	*Pole* of the *Meridian*: Or the Place in the *Horizon*, where the *Sun*, or a *Star*, *Riſes* or *Sets*.
O *c*		*Weſt*	
R		*Right Angle*.	
☉		*Sun*.	
✱		*Star*.	
S		*Side*, *Sun* (and ſometimes) *Star*.	
b		*North*	*Latitude*, *Declination*, *Amplitude*.
m		*South*	

R. A. P. T.	Right-angled	Plain Triangles.
O. A. P. T.	Oblique-angled	
R. A. S. T.	Right-angled	Plain Triangles.
O. A. S. T.	Oblique-angled	

Aſtronomical Problems.

PROBLEM I.

The Longitude, *or* Place *of the* Sun, *in the* Ecliptick, *being given; To find,*

1. *The Sun's* Right Aſcenſion.
2. *The Sun's* Declination.
3. *The* Angle of Poſition *made by the Interſection of the* Meridian *and the* Ecliptick.

IN this *Problem* there is given the *Sun*'s or a *Star*'s *Place*, in Reſpect of the *Ecliptick*; and his Place, in reſpect of the *Æquinoctial*, is required,

I. *By the* Cœleſtial Globe.

Example. Let the Place of the Sun be in 29 Deg. of *Taurus* ♉, (that is, 59 Deg. from the beginning of *Aries* ♈, which is the neareſt *Æquinoctial Point.*)

The *Globe* being in any *Poſition*, (for in this *Problem* there is no regard to be had to the *Latitude*) count the Sun's Place in the *Ecliptick* upon the *Ecliptick Circle*, from ♈; and bring that Point to the Graduated Side of the *Braſs Meridian*. Then, (1.) Will the *Braſs Meridian* cut the *Æquinoctial Circle* in 56 d. 46 m. counted alſo from ♈; and that is the *Right Aſcenſion*. And, (2.) The Number of Degrees of the *Braſs Meridian*, comprehended between the *Æquinoctial* and *Ecliptick Circles*, will be 20 d. which is the *Sun*'s *Declination.* —And, (3.) The *Angle* made by the Interſection of the *Braſs Meridian*, and the *Ecliptick Circle* in the Point of the Sun's Place, will be 77 d. 23 m. which is the *Angle of Poſition*, in reſpect of the *Meridian* and *Ecliptick*.

II. *By* Trigonometrical Calculation.

The *Globe* being in this *Poſition*, there is repreſented upon the *Superficies* thereof, Two *Right-angled Spherical Triangles*, ſuch as are expreſſed in the *Diagrams*; and are there noted with ⊙ R ♈ and ⊙ R ♎; in which, the Sides ⊙ ♈ and ⊙ ♎ are Arches of the *Ecliptick Circle*, and is the *Sun*'s *Longitude*:

Fig. XXVII. XXVIII.

Fig. XXVII. XXVIII. The Sides ♈ R and ♎ R are Arches of the *Æquinoctial*, and is the **Sun's** ***Right Aſcenſion*****; and the Sides ☉ R are Arches** of the *Braſs Meridian*, and is the Sun's *Declination*.—Alſo, the Angle at ♈ or ♎, is the Angle of the greateſt *Obliquity* of the *Ecliptick*, (and is equal to the Sun's greateſt *Declination* 23 d. 31 m.) The Angle R is a *Right Angle*, and the Angle ☉ is the Angle at the *Sun*'s *Poſition*, in reſpect of the *Ecliptick* and *Meridian Circles*.

To reſolve this *Problem Trigonometrically*, you are to conſider the *Quadrant* of the *Ecliptick*, in which the Sun is, which in the *Figures* are ſignified by one of theſe Numbers, 1. 2. 3. 4. Of which, the firſt is of the *Spring*, from the beginning of *Aries* ♈, to the beginning of *Cancer* ♋, &c. Then in the Triangle R ♈ ☉ or R ♎ ☉.

There is given, beſides the *Right Angle* at R,

1. The Angle at ♈ or ♎, 23 d. 31 m.
2. R ♈ or R ♎: The Sun's Diſtance from the next *Æquinoctial Point*; to be numbred from ♈ or ♎, unto the Degrees of the Sun's *Longitude* or *Place* in the *Ecliptick* given.

And,

There is required,

1. R ☉, The Sun's *Declination* for that Sign which he is in; whether *North* or *South*.
2. The Arch ♈ R, or ♎ R, which if it be in Quadrant,
    1. The Degrees found by the Canon, are the Degrees of
    2. The Degrees found, muſt be ſubſtracted from 180, and the remainded are the Degrees of
    3. The Degrees found muſt be added to 180, and the Sum are the Degrees of
    4. The Degrees found muſt be ſubſtracted from 360 d. and the Remainder are the Degrees of

    Right Aſcenſion.
3. The Angle ☉, or the Angle of *Poſition*, in reſpect of the *Meridian* and *Ecliptick Circles*.

The Canons for Calculation.

The Sun being in 29 d. of *Taurus* ♉, which is 59 d. from *Aries* ♈.

1. For the *Right Aſcenſion* ♈ or ♎ R. By *Caſe* III. of *R. A. S. T.*

Fig. XXVII. XXVIII.

As the Sine of 90 d.

Is to the Co-ſine of the greateſt Obliquity of the Ecliptick 66 d. 29 m.

So is the Tangent of the Sun's Diſtance from ♈ or ♎, 59 d.

To the Tangent of 56 d. 46 m. Which is the Sun's *Right Aſcenſion*; becauſe his Place given was in the Firſt *Quadrant*:—But if the Sun had been in 1 d. of *Leo* ♌, or 29 d. of *Scorpio* ♏, or 1 d. of *Aquarius* ♒. All which Points are 59 d. diſtant from ♈ or ♎; the *Right Aſcenſion* would be found the ſame, as before, *viz.* 56 d. 46 m. But by the Rule before given in this *Problem.*

			D.	M.
If the Sun's Place given had been in	1 d. of *Leo*, in Quad. 2.	The R. *Aſcenſion* would have been	123	14
	29 d. of *Scorpio*, in Qu.3.		236	46
	1 d. of *Aquarius*, in Qu.4.		303	15

2. For the *Sun's Declination* R ⊙. By *Caſe* II. of *R. A. S. T.*

As the Sine of 90 d.

Is to the Sine of the greateſt *Obliquity* of the *Ecliptick* 23 d. 31 m.

So is the Sine of the Sun's Diſtance from ♈ or ♎ 59 d.

To the Sine of 20 d. the Sun's preſent *Declination.*

Which is *North*, becauſe he is in a *Northern Sign.*

3. For the Angle of the Sun's *Poſition* ⊙. By *Caſe* III. of *R. A. S. T.*

As the Sine of the Sun's preſent *Declination* 20 d.

Is to the Sine of greateſt *Declination* 23 d. 31 m.

So is the Sine of the *Sun's R. Aſcenſion* 56 d. 46 m.

To the Sine of 77 d. 23 m. The Sun's *Angle of Poſition*, made by the *Meridian* and *Ecliptick Circles.*

PROB. II.

The Right Aſcenſion, *or* Declination *of the Sun given*; *To find his* Longitude (*or* Place) *in the* Ecliptick.

THIS is the Converſe of the foregoing *Problem*; for in this, the Sun's *Place*, in reſpect of the *Æquinoctial Circle*, is given: And his *Place*, in reſpect of the *Ecliptick Circle*, is required: And may be reſolved as followeth.

I. *By the* Cœleſtial Globe.

Fig. XXVII. XXVIII. *Example.* Let the Sun's *Right Aſcenſion* be 303 d. 14 m. and his *Declination* 20 d. Southward: And let his *Longitude* (or *Place* in the *Ecliptick*) be required.

Count 303 d. 14 m. the *Right Aſcenſion* given, upon the *Æquinoctial* Circle, from the *Vernal Æquinoctial* Point *Aries* ♈, and bring that Point to the graduated Side of the *Braſs Meridian:* Then will the *Braſs Meridian* cut the *Ecliptick Circle* in 1 d. of *Aquarius* ♒, and in that Sign and Degree the Sun is at *Noon*, when his *Right Aſcenſion* is 303 d. 14 m.

But to find his Place by his *Declination*, Count 20 d. (the *Declination* given) upon the *Braſs Meridian*, downwards towards the South Pole (becauſe the *Declination* given was *Southerly*) and turn the *Body* of the *Globe* about till 20 d. of the *Meridian* do cut the *Ecliptick Circle*; which it will do in Two Points, *viz.* One in 29 d. of *Scorpio* ♏, in the third *Quadrant*; and the other in 1 d. of *Aquarius* ♒, in the fourth *Quadrant*; in both which Points the Sun being, he hath 20 d. of *South Declination*, and which of thoſe Points you ſeek is determined by the Degrees of *Right Aſcenſion* given, as here 303 d. 14 m. and the *Globe* being in this Poſition, will ſign out the ſame Triangles, mentioned in the foregoing *Problem.*

II. *By* Trigonometrical Calculation.

Conſider the *Quadrant* of the *Ecliptick* in which the Sun is, (which the Degrees of *Right Aſcenſion* given, will determine) and in this Example will be the fourth *Quadrant:* But the given *Declination* is indifferent in all the Four *Quadrants.*

Wherefore, in the Triangle R ♈ ☉.

There is given, beſides the *Right Angle* at R.

1. The Angle at ♈ or ♎, 23 d. 31 m. the *Sun's greateſt Declination.*
2. The Side R ☉, the Sun's preſent *Declination* 20 d. And alſo either of the other Sides R ♈ (or R ♎) an Ark of the *Equator*, to be numbred from the neareſt *Æquinoctial* Point, to the Degrees of *Right Aſcenſion given.*

And there is required { The Side ☉ ♈ (or ☉ ♎) the Sun's Diſtance in the *Ecliptick Circle*; from the neareſt *Æquinoctial Point*, from which Point the *Longitude* required is to be numbred upon the *Ecliptick Circle*. *Fig.* XXVII. XXVIII.

So the *Right Aſcenſion* 303 d. 14 m. given, it being in the *Fourth Quadrant*, will give in the *Ecliptick* 1 d. of *Aquarius* ♒.

The Canons for Calculation.

1. By the *Right Aſcenſion* given.

The *Right Aſcenſion* given being 303 d. 14 m. are found in the *Fourth Quadrant*, and therefore muſt be ſubſtracted from 360 d. and the Remainder will be 56 d. 46 m. which muſt be made uſe of in the Calculation, inſtead of 303 d. 14 m. And then in the Triangle ♎ ☉ R,

1. By the *Right Aſcenſion* given.

As the Tangent of the Side ♎ R, 56 d. 46 m. the *Right Aſcenſion*,

Is to the Sine of 90 d. or Radius:

So is the Co-ſine of the Angle at ♎ 66 d. 29 m.

To the Co-tangent of ♎ ☉, 59 d. 00 m. Sun's *Longitude*.

Now, becauſe the *Right Aſcenſion* given was in the *Fourth Quadrant*, the *Longitude* 56 d. 46 m. thus found, muſt be in the *Fourth Quadrant* alſo, which will be in 1 d. of *Aquarius* ♒.

2. By the *Declination* given.

As the Sine of the Sun's *greateſt Declination*, 23 d. 31 m. the Angle at ♎.

Is to the Sine of the Side ☉ R, 20 d. the Sun's *preſent Declination* given:

So is the Radius, Sine 90 d.

To the Sine of the Side ♎ ☉, 59 d. from *Libra* ♎.

And being in the *Fourth Quadrant*, it gives the Longitude of the Sun to be in 1 d. of *Aquarius* ♒, as before.

PROB.

PROB. III.

Fig. XXIX.

The Latitude *of the Place*, 51 *d.* 30 *m. and the Sun's Place in the* Ecliptick, 29 *d. of* Taurus, *given, to find,*

1. *The Sun's* Declination.
2. *The Sun's* Amplitude.
3. *The Hour from Midnight or Aſcenſion all* Difference.
4. *The* Time *of the Sun's* Riſing *and* Setting.
5. *The* Length *of the* Day *and* Night.
6. *The* Angle *of the Sun's* Poſition *at the time of his* Riſing *or* Setting.

I. *By the* Cœleſtial Globe.

BRing the 29 d. of *Taurus* to the *Braſs Meridian*, and you will find 29 of *Taurus* to lye under 20 d. of the *Meridian*, and that is the Sun's *Declination* North, the Sun being in a *Northern* Sign. Then, ſet the *Index* of the *Hour-Circle* to the *South* 12, and turn the Body of the Globe Eaſtward, till 29 d. of *Taurus* do juſt touch the *Horizon*, and there fix it. Then ſhall you find 29 d. of *Taurus* to cut the *Horizon* in 33 d. 18 m. counted from the *Eaſt*, which is the *Amplitude* of the Sun's *Riſing* from the true *Eaſt* Point of the *Horizon* Northward: And alſo, the *Index* of the *Hour-Wheel*, will point out 11 m. after 4 in the Morning, at which time the Sun *Riſeth*. —And if you turn the Body of the Globe about Weſtward, till 29 d. of *Taurus* doth touch the *Weſt* Side of the *Horizon*; then ſhall the *Index* point at 49 m. after 7 at Night, at which Time the Sun *Setteth*.

Again, Turn the Globe about, till 29 d. of *Taurus* touch the *Eaſt* Side of the *Horizon* (as before) and ſet the *Index* of the *Hour-Circle* to the *North* (or undermoſt) 12: And then turning it Weſtward till 29 d. of *Taurus* touch the *Horizon* on the *Weſt* Side; and then ſhall the *Index* point at 3 ho. 38 m. more than 12 Hours, from the *South* 12: Which ſhews that the *Day* is then 15 ho. and 38 m. *Long*.—And if you count the Hours between the *North* 12 and the *Index*, you ſhall find them to be 8 ho. and 22 m. and that is the *Length* of the Night.

And then, for the Angle of the *Sun's Poſition*, that is to be found, as is directed in the 24th *Element*, and in the *Problem* to this Second Part, and will be found to be 56 d. 23 m.

The Globe being in this Poſition, there is repreſented upon the Superficies of it a Spherical Triangle P ⊙ O, as in *Fig.* compounded of the Arches of ſuch *Great Circles* of the *Sphere*, as are ingredient in the *Propoſition*, *viz.* Of P O, an Arch of the *Braſs Meridian*; of ⊙ O, an Arch of the *Horizon*; and of P ⊙, an Arch of that *Meridian*, which cutteth the *Ecliptick* in that Point in which the Sun is. —And in this Triangle, the Angle P O ⊙ is a *Right Angle:* ⊙ P O, is the *Hour* from *Midnight:* And the *Angle* P ⊙ O, is the Angle of *Poſition* made by the *Meridian* and *Horizon*, at the time of the *Sun's Riſing*. Fig. XXIX.

II. *By* Trigonometrical Calculation.

In the Triangle P O ⊙, you have given, (beſides the *Right Angle* at O). (1.) The Hypotenuſe P ⊙, the Complement of the Sun's *Declination*, (70 d.) (2.) The Perpendicular P O, the *Latitude* 51 d. 30 m. —To find, (1.) The Baſe ⊙ O, the Sun's *Amplitude* at his Riſing. (2.) The Angle ⊙ P O, the *Hour* from *Midnight:* And, (3.) The Angle P ⊙ O; the Angle of the *Sun*'s *Poſition* at his *Riſing*.

The Canons for Calculation.

1. For the *Amplitude*, ⊙ O. By *Caſe* VI. of *R. A. S. T.*
As the Co-ſine of P O, (the Latitude) 38 d. 30 m.
Is to the Radius, 90 d.
So is the Co-ſine of P ⊙, (the Sun's *Declination*) 20 d.
To the Co-ſine of ⊙ O, (the *Amplitude*) 33 d. 20 m.
Which is equal to the Arch of the *Horizon*. *Or* ⊙, which is the *Amplitude* of his *Riſing* from the *Eaſt* Northward, becauſe the Sun was in a *Northern* Sign: For if he had been in a *Southern* Sign, as in 59 d. of *Scorpio*, the *Amplitude* would have been the ſame, but from the *Eaſt* Southward.

2. For the *Hour* from *Midnight* ⊙ P O. By *Caſe* V. of *R. A. S. T.*
As the Radius,
Is to the Tangent of P O (the *Latitude*) 51 d. 30 m.
So is the Co-tangent of P ⊙ (the Sun's Diſtance from the Pole) 20 d.
To the Co-ſine of ⊙ P O (the *Hour from Six*) 27 d. 14 m.
Whoſe Complement, 62 d. 46 m. is the *Hour* from *Midnight*; which turned into *Time*, (by allowing 15 d. for 1 h. and 4 d. for 1 m. of *Time*) makes 4 h. and 11 m. Which is the *Hour* from *Midnight*.

Fig. XXIX. 3. For the *Aſcenſional Difference.*

The Hours from *Midnight*, 4 h. 11 min. ſubſtracted from 6 h. there remains 1 h. 49 m. which is the *Aſcenſional Difference.* Or, The *Time* that the *Sun Riſes* before *Six* in the *Summer*, or after *Six* in the *Winter.*

4. and 5. For the *Time* of the *Sun's Riſing* and *Setting*, and (conſequently) the *Length* of the *Day* and *Night.*

The *Aſcenſional Difference*, 1 h. 49 m. being added to 6 h. gives 7 h. 49 m. for the *Semidiurnal Ark* in *Summer*; or ſubtracted from 6 h. gives 4 h. 11 m. for the *Semidiurnal Ark* in *Winter.*

The *Semidiurnal Ark*, 7 h. 49 m. ſubſtracted from 12 h. there remains 4 h. 11 m. for the time of the *Sun's Riſing:* And that ſubſtracted from 12 h. leaves 7 h. 49 m. for the time of the *Sun's Setting.* And,

The *Semidiurnal Ark*, 7 h. 49 m. doubled, gives 15 h. 38 m. for the *Length* of the *Day:* —And 15 h. 38 m. ſubſtracted from 24 h. there will remain 8 h. 22 m. for the *Length* of the *Night.*

6. For the *Angle* P ⊙ O, of the Sun's *Poſition*, at the time of his *Riſing.* By *Caſe* IV. of *R. A. S. T.*

As the Sine of P ⊙ (the *Co-Declination*) 70 d.
Is to the Radius:
So is the Sine of P O (the *Latitude*) 51 d. 30 m.
To the Sine of P ⊙ O (the *Angle* of *Poſition* at the *Sun's Riſing*) 56 d. 23 m.

PROB. IV.

To find the Sun's Meridian Altitude, *and his* Depreſſion *at* Midnight, *he being in any Point of the* Ecliptick.

LET the Place of the Sun be the ſame, as in the former *Problem*, *viz.* in 29 d. of *Taurus.* Then,

I. *By the* Cœleſtial Globe.

Turn the Globe about, till the 29 d. of *Taurus* be juſt under the *Meridian*; then ſhall you find the Number of Degrees of the *Meridian*, which are comprehended between that Point and the Horizon to be 58 d. 30 m. which is the *Meridian Altitude.* And if you bring the 29 d. of *Scorpio*, which is the oppoſite Point of the

Fig. XXIX.

the *Ecliptick* in which the *Sun* is, to the *Meridian*, the Number of the Degrees of the *Meridian* between that Point and the *Horizon* will be found to be 18 d. 30 m. which is the *Sun*'s Depreſſion at *Midnight*.

In like manner,

	D.	M.			D.	M.		D.	M.
When the *Sun* is in	*16*	00	♌	You ſhall find the *Meridian Altitude.*	54	*36*	And the *Depreſ.*	22	24
	25	00	♍		*19*	25		57	25
	29	00	♒		*26*	38		50	22
	13	00	♈		43	39		33	21

PROB. V.

To know when Twilight *Begins and Ends.*

Twilight begins, when the Sun is 18 d. below the *Horizon* before its *Riſing:* And endeth, when the Sun cometh to be 18 d. below the *Horizon*, after its *Setting*.

The Globe Rectified, and the *Sun* in 29 d. of *Taurus*, find the oppoſite Point thereunto, which is the 29 d. of *Scorpio*; and bring that Point, as alſo the *Quadrant of Altitude*, both of them on the Weſt-ſide of the *Meridian*; and then move both the Body of the *Globe*, and the *Quadrant of Altitude* alſo, till the 29 d. of *Scorpio* lye directly under 18 d. of the *Quadrant of Altitude:* Which done, keep them both together, and then ſee how many Hours the *Index* is removed from 12, which you ſhall find to be 1 h. and 8 m. So that *Twilight* begins at 8 m. after 1 in the Morning. And this being taken from 4 h. 11 m. the time of the *Sun*'s *Riſing* that Day, there will remain 3 h. 3 m. which is the Length or Continuance of the *Twilight*. Alſo if you double the time of the beginning of *Twilight* 1 h. 8 m. you ſhall have the length of dark Night, which will be but 2 h. 16 m.

In like manner, if you would know when the *Twilight* endeth after *Sun-ſetting*, you muſt bring the 29 d. of *Scorpio* (the Point oppoſite to the *Sun*) on the Eaſt-ſide of the *Meridian*, making it, and 18 d. of the *Quadrant of Altitude*, to meet, then the *Index* will ſhew 10 h. 52 m. and till that time of Night doth *Twilight* continue.

Fig. XXIX.

And ſo;

The *Sun* in	D.		Day-break will be at	H.	M.		Twilight laſt till	H.	M.	
	29	♋		0	10	aft. Midn.		11	50	at Night.
	8	♐		5	52			6	8	
	2	♎		4	6			7	54	
	0	♉		2	41			9	19	

And if you go about to find the time of the beginning and end of *Twilight*, all the Time that *Sun* is paſſing from 2 d. of *Gemini* to 30 d. of *Cancer*, which is from about the 12th of *May* to the 12th of *July*, you ſhall find that there will be no *Twilight* at all, but all that Time continual Day: For all that Space of Time the *Sun* never deſcendeth ſo much as 18 d. under the *Horizon*, in the Latitude of 51 d. 30 m.

The former of theſe Two laſt *Problems* needeth no *Trigonometrical Calculation:* But, if to the Sun's *Declination* for the Day (or *Place* of the *Sun*) given, 20 d. you add the Complement of the *Latitude* given, 38 d. 30 m. the *Sum* of them 58 d. 30 m. is the *Meridian Altitude* for that Day that the Sun is in 58 d. of *Taurus*.

But if the Sun had been in 29 d. of *Scorpio*, a Southern Sign, then the *Declination* 20 d. ſubſtracted from 38 d. 30 m. the Complement of the *Latitude*, the Remainder 18 d. 30 m. will be the *Meridian Altitude.*

For the *Depreſſion* of the *Sun*, he being upon the *Meridian* at Midnight, it is the ſame with the *Meridian Altitude*, when the Sun is in the oppoſite *Sign*.

So the *Depreſſion* at *Midnight* when the Sun is in

29 d. of *Taurus*, } is { 18 d. 30 m.
29 d. of *Scorpio*, } is { 58 d. 30 m.

PROB.

PROB. VI.

The Latitude *of* Place, *and the* Sun's Place *in the* Ecliptick *(and conſequently his* Declination) *being given; To find*

1. *What* Altitude *the Sun ſhall have at Six a Clock.*
2. *What* Azimuth *he will be upon, at Six a Clock, Morning or Evening.*
3. *The* Angle *of the* Sun's Poſition, *made by the Interſection of the* Vertical Circle, *and the* Meridian *which paſſes by the* Sun *at the time of the* Queſtion.

ALL theſe *Problems* are only in uſe, when *Sun* is in a *Northern Sign*, and ſo hath *North Declination*: For the *Sun* is never above the *Horizon* (in either of the *Temperate Zones*) when he is in a *Southern Sign*, or hath *South Declination.* Fig. XXIX.

I. *By the* Cœleſtial Globe.

1. For the *Altitude* at Six. Bring the 29 d. of *Taurus* to the *Meridian*, and ſet the *Index* of the *Hour-Circle* to 12; then turn the *Globe* Eaſtward, till the *Index* of the *Hour-Circle* come juſt to Six a Clock: Then holding the *Globe* there, lay the *Quadrant of Altitude* juſt over the 29 d. of *Taurus*, and there you ſhall find it to cut 15 d. 30 m. of the *Quadrant.* And ſuch *Altitude* ſhall the *Sun* have at Six a Clock in the Morning, and the ſame at Six at Night.

And ſo,

	D.	*M.*			D.	*M.*
When the *Sun* is in	16	00	♌	The *Sun*'s *Altitude* at Six will be found to be	12	32
	13	00	♈		4	2

2. For the *Azimuth* at Six. Bring the 29 of *Taurus* to the *Meridian*, and ſet the *Index* of the *Hour-Wheel* to 12; then move the *Globe* till the *Index* lye upon Six; and holding the *Globe* there, lay the *Quadrant of Altitude* juſt over 29 d. *Taurus*: Then ſhall you find, that there are 77 d. 14 m. of the *Horizon* contained between the Interſection of the North Part of the *Meridian*, and the *Quadrant of Altitude* which is the *Azimuth* from the North; or 12 d. 46 m. from the Eaſt, which is the *Azimuth* from the Eaſt, or 102 d. 46 m. from the South, which is its *Azimuth* therefrom.

Fig. XXIX.

In like manner,

	D.M.		D. M. *North*	D. M. *Eaſt* or *Weſt*	D. M. *South*
Sun in	16 0 ♌	The *Sun*'s *Azimuth* at 6 will be found to be from the	79 49	10 11	100 11
	13 0 ♈		86 47	3 13	93 13

II. *By* Trigonometrical Calculation.

The *Globe* being in this Poſition, you may diſcover upon the Fig. XXX. Superficies thereof a *Spherical Triangle*, ſuch as in the *Figure*, is noted with Z P ⊙; in which, Z P is an Arch of the *Meridian*, P ⊙ an Arch of the *Hour-Circle* of Six, and Z ⊙ an Arch of an *Azimuth*, or *Vertical Circle*: In which *Triangle* there is given, (beſides the *Right Angle* at P). (1.) The Perpendicular Z P, 38 d. 30 m. equal to the Complement of the *Latitude*. (2.) The Baſe ⊙ P, 70 d. equal to the Complement of the Sun's *Declination*: To find, (1.) The Hypotenuſe Z ⊙, the Complement of the Sun's *Altitude* at Six. (2.) The *Angle* ⊙ Z P, the Sun's *Azimuth* at Six a Clock: (3.) Z ⊙ P, the *Angle* of the Sun's *Poſition*.

The Canons for Calculation.

1. For the *Hypotenuſe* Z ⊙, (the Complement of the Sun's *Altitude* at Six) by *Caſe* XIV. of *R. A. S. T.*

As the Radius,
Is to the Co-ſine of Z P, 51 d. 30 m.
So is the Co-ſine of ⊙ P, 20 d.
To the Sine of 15 d. 32 m. (the Complement of Z ⊙.)

And ſuch *Altitude* the Sun will have, at Six a Clock, (Morning or Evening) when he is in 29 d. of *Taurus*.

2. For the *Angle* P Z ⊙, the Sun's *Azimuth* at Six a Clock, by *Caſe* XIII. of *R. A. S. T.*

As the Tangent of ⊙ P (the *Co-Declination*) 70 d.
Is to the Radius:
So is the Sine of Z P (the *Co-Latitude*) 38 d. 30 m.
To the Tangent of 12 d. 46 m.

Whoſe Complement 77 d. 14 m. is the *Angle* ⊙ Z P, the Sun's *Azimuth* from the North Part of the *Meridian*.

3. For the *Angle* of the Sun's *Poſition*, Z ⊙ P: By *Caſe* XI. of R. *A. S. T.* *Fig.* XXVII.

As the Sine of ⊙ P, 70 d.
Is to the Sine of ⊙ Z P, 77 d. 14 m.
So is the Sine of Z P, 31 d. 30 m.
To the Sine of Z ⊙ P, 40 d. 15 m.
Which is the *Angle* of the Sun's *Poſition.*

PROB. VII.

Having the Latitude, 51 *d.* 30 *m. the Sun's* Place, 59 *of* Taurus, (*or* Declination 20 *d.*) *Given, as in the foregoing* Problem; *To find,*

1. *At what* Hour *the* Sun *ſhall be upon the* Eaſt *or* Weſt Azimuth.
2. *What* Altitude *the Sun ſhall have when he is upon the* Eaſt *or* Weſt Azimuth.
3. *The* Angle *of the Sun's* Poſition.

I. *By the* Cœleſtial Globe.

BRing the 29 d. of *Taurus* to the *Meridian*, and the *Index* to 12 of the Clock. Alſo bring the beginning of the Degrees of the *Quadrant of Altitude* to the Eaſt Point of the *Horizon*, and turn the Globe about till the 29 d. of *Taurus* do touch the Degrees of the *Quadrant of Altitude*; then ſhall the *Index* point at 7 m. paſt Seven; at which time, in the Morning, will the Sun be exactly upon the Eaſt *Azimuth*, or Point of the Compaſs. And if you carry the *Quadrant of Altitude* to the Weſt Point of the *Horizon*, and turn the *Globe* about till 29 d. of *Taurus* touch the edge of Degrees thereof, the *Hour-Index* will point at Four of the Clock, and 53 m: at which time, in the Afternoon, will the Sun be upon the Weſt *Azimuth* or Point of the Compaſs.

In the ſame manner,

	D. M.		*D.M.*		*D.M.*
When the Sun is in	16 00 ♌	It will be due *Eaſt* at	6 57	*Weſt* at	5 03
	13 00 ♈		6 17		5 43

Then for the Sun's *Altitude* when he will be upon the *Eaſt* or *Weſt Azimuth.*

Bring 29 d. of *Taurus* to the *Meridian*, and the *Quadrant of Altitude* to the Eaſt or Weſt Points of the *Horizon:* Then turn the

Fig. XXX. *Globe* about, till the 29 d. of *Taurus* touch the *Quadrant of Altitude*, and you ſhall find it to touch at 25 d. 55 m. of the *Quadrant of Altitude*; and ſuch *Altitude* hath the Sun, when he is upon the *Eaſt* or *Weſt Azimuth.*

In like manner,

When the Sun is in	D.	M.		its *Altitude*, when *Eaſt* or *Weſt*, will be found,	D.	M.
	16	00	♌		20	19
	13	00	♈		6	36

II. *By* Trigonometrical Calculation.

The *Globe* being in any of the former *Poſitions*, you will find repreſented upon the Superficies of it a *Spherical Triangle*, Z P ⊙, (ſuch as in the *Figure*) *Right-angled* at Z, which is compoſed of Z P, an Arch of the *Braſs Meridian*, equal to the *Complement* of the *Latitude* 38 d. 30 m. Z ⊙, an Arch of the *Quadrant of Altitude:* And of P S, an Arch of that *Meridian* (or *Hour-Circle*) which cutteth the *Eclip tick* in the Place the Sun is in.—And in this *Triangle* there is given (beſides the *Right Angle* at Z, (1.) The *Perpendicular* P Z, the *Complement* of the *Latitude* 38 d. 30 m. (2.) The *Hypotenuſe* P ⊙, the *Complement* of the Sun's *Declination* 70 d. To find, (1.) The *Angle* Z P ⊙, the *Hour* at which the Sun will be upon the *Eaſt* or *Weſt Azimuth.* (2.) The *Side* Z ⊙, the *Complement* of the *Altitude* he ſhall then have: And, (3.) The *Angle* Z ⊙ P of his *Poſition.*

Fig. XXXI.

The Canons for Calculation.

1. For the *Angle* Z P ⊙, (the *Hour* from *Noon*, that the Sun will be due *Eaſt* or *Weſt*; by *Caſe* V. of *R. A. S. T.*

As the Radius,
Is to the Tangent of Z P, 31 d. 30 m.
So is the Co-tangent of P ⊙, 20 d.
To the Co-ſine of Z P ⊙, 73 d. 10 m.
Which converted into Time, is 4 h. 53 m. Whoſe Complement is 7 h. 7 m. at which Time the Sun will be due *Eaſt* or *Weſt.*

Fig. XXXI.

2. For the Baſe Z ⊙, the Complement of the Sun's *Altitude*, when due *Eaſt* or *Weſt* : By *Caſe* VI. of *R. A. S. T.*

As the Co-ſine of Z P, (the *Co-Latitude*) 51 d. 30 m.
To the Radius:
So is the Co-ſine of P ⊙ (the *Declination*) 20 d.
To the Co-ſine of Z ⊙ (the *Altitude*) 25 d. 55 m.
And that is the Sun's *Altitude*.

3. For the Angle Z ⊙ P, of Poſition: By *Caſe* IV.

As the Sine of P ⊙, 70 d.
To the Radius:
So Sine Z P, 38 d. 30 m.
To the Sine of Z ⊙ P, 41 d. 29 m.
Which is the Angle of Poſition.

PROB. VIII.

The Latitude *of the* Place, *the Sun's* Place *in the* Ecliptick *(or his* Declination) *being given, to find,*

1. *What* Altitude *the Sun ſhall have at any time of the* Day.
2. *What* Azimuth *he ſhall then have. And,*
3. *The Sun's* Angle *of* Poſition.

Let the Latitude *given be* 51 *d.* 30 *m.* North, *the Sun's* Place *in the* Ecliptick 29 *d. of* Taurus, *(and ſo his* Declination 20 *d.) And let the* Hour *be* 9 *in the* Morning, *or* 3 *in the* Afternoon.

I. *By the* Cœleſtial Globe.

BRing the 29 d. of *Taurus* to the *Meridian*, and ſet the *Hour-Index* to 12 a Clock. Then turn about the *Globe*, till the *Hour-Index* point to the given Hour, (ſuppoſe 9 in the Morning, or 3 in the Afternoon:) There keep the *Globe*; and laying the *Quadrant of Altitude* over the 29 d. of *Taurus*, you ſhall find 43 d. cut thereby; and ſuch *Altitude* ſhall the Sun have at 9 in the Morning, or 3 in the Afternoon. And at the ſame time you will find the *Quadrant of Altitude* to cut the *Horizon* in 24 d. 37 m. counted from the *Eaſt* or *Weſt* Points thereof; and ſuch *Azimuth* will the *Sun* have at 9 a Clock in the Morning, or 3 in the Afternoon, when he is in 29 d. of *Taurus*. And by this *Problem* the Sun's *Altitudes* in any Sign or Degree of the *Ecliptick* at all Hours may be found, as in theſe following *Synopſis* or *Tables*.

Fig. XXXI.

A Table, *ſhewing what* Altitude *the Sun ſhall have* Hour *of the* Day, *in the beginning of every of th* Signs *of the* Zodiack.

So the *Sun* being in the beginning of		*Cancer*		*Gemi.* or *Leo.*		*Taur.* or *Virgo.*		*Aries* or *Libra.*		*Scorp.* or *Piſces.*		*Aqu* or *Sagi*
At the H.		*D.*	*M.*	*D.*	*M.*	*D.*	*M*	*D.*	*M*	*D.*	*M.*	*D.*
XII.	His *Altitude* will be	62	00	58	42	50	00	38	30	27	1	18
XI. I.		59	43	56	34	48	12	36	58	25	40	17
X. II.		53	45	50	55	43	12	32	37	21	51	13
IX. III.		44	42	43	3	36	0	26	7	15	58	8
VIII. VI.		35	41	34	13	27	31	18	8	8	33	1
VII. V.		27	17	24	56	18	18	9	17	0	6	
VI.		18	11	15	40	9	0					
V. VII.		9	32	6	50							
IV. VIII.		1	32									

'able, *ſhewing what* Altitude *the Sun ſhall have, he being on every* Tenth Azimuth *from the South, in the beginning every of the Twelve Signs.* Fig. XXXI.

muth from the South, viz.

will be found to be

ː Sun in ıe begin- ng of	Cancer.		Leo or Gemi.		Virgo or Taur.		Libra or Aries.		Scorp. or Piſces		Srgitt. or Aqua.		Capric.	
d. *Az.*	*D.*	*M.*	*D.*	*M.*	*D.*	*M.*	*D.*	*M.*	*D.*	*M.*	*D.*	*M.*	*D.*	*M.*
0	62	0	58	42	50	0	38	30	27	0	28	18	15	0
10	61	43	58	24	49	38	38	4	26	30	17	45	14	25
20	60	51	57	28	48	33	36	46	25	10	16	5	12	41
30	59	52	55	52	46	40	34	34	22	27	13	15	9	45
40	57	20	53	29	43	51	31	21	18	48	9	14	5	34
50	54	3	50	12	40	11	27	5	13	58	3	57	0	6
60	49	56	45	53	35	23	21	41	8	0				
70	44	40	40	25	29	27	15	13	1	0				
80	28	11	33	46	21	29	7	52						
90	30	38	26	10	14	25								
100	22	27	18	2	6	45								
110	14	14	9	58										
120	6	34	2	30										

ıeſe Tables are of good uſe for the making of *Cylinders*, *drants*, and other Inſtruments, that give the *Hour* and *Azi-* by the height of the Sun, and alſo to inſert the Tropicks other Signs of the *Zodiacks* and *Azimuths* into Sun-Di- *&c.*

II. *By* Trigonometrical Calculation.

ıe *Globe* continuing in the ſame Poſition you left it, you diſcover upon it an *Oblique Spherical Triangle* Z P ☉, in the Figure) conſtituted by the Interſections of Z P, Arch of the *Braſs Meridian*, Z ☉, a Part of the *Quadrant ltitude:* And of P ☉, an Arch of a *Meridian*, paſſing ıgh the 29 d. of *Taurus* in the *Ecliptick*, or *Hour-Circle* or 3 a Clock. Fig. XXXII.

In which *Triangle* there is given,

The Side Z P, (the Complement of the *Latitude*) 38 d. 30 m.

Fig. XXXII. 2. The Side P ⊙ (the Complement of the Sun's *Declination*, or his Diſtance from the Pole) 70 d.

3. The Angle Z P ⊙, (the Hour-Circle of *9* or *3* a Clock) 45 d.

To find,

1. The Side Z ⊙ (the Complement of the Sun's *Altitude.*)
2. The Angle ⊙ Z P, (the Sun's *Azimuth* from the *North* Part of the *Meridian.*)
3. The Angle of Poſition, Z ⊙ P.

The Canons for Calculation.

By *Caſe* III. and *Caſe* IX. of *O. A. S. T.*

(1.) As the Sine of half the *Sum* of the Two given Sides, Z P, 38 d. 30 m. and P ⊙ 70, *viz.* 54 d. 15 m.

Is to the Sine of half the *Difference* of thoſe Sides, *viz.* 15 d. 54 m.

So is the Co-tangent of half the given *Angle*, Z P ⊙, *viz.* 22 d. 30 m.

To the Tangent of half the *Difference* of the unknown *Angles*, at Z and ⊙, *viz.* 38 d. 48 m.

Then,

(2.) As the Co-ſine of half the *Sum* of the given Sides Z P and P ⊙, *viz.* 35 d. 45 m.

Is to the Co-ſine of half the *Difference* of thoſe Sides, *viz.* 74 d. 15 m.

So is the Co-tangent of half the given *Angle* Z P ⊙, *viz.* 22 d. 30 m.

To the Tangent of half the *Sum* of the Two unknown *Angles* P Z ⊙ and Z ⊙ P, *viz.* 75 d. 49 m.

Then,

	D.	M.
The half Sum before found is	75	49
The half Difference before found is	38	48
The Sum of them is	114	37
The Difference of them is	37	01

The one equal to the greater *Angle* ⊙ Z P 114.37, which is the Sun's *Azimuth* from the *North* Part of the *Meridian*: From the *South* Part 65 d. 23 m. and from the *Eaſt* or *Weſt* 24 d. 37 m.

The other, equal to the leſſer *Angle* Z ⊙ P 37 d. 1 m. which is the *Angle* of the *Sun's Poſition*.

PROB.

PROB. XI.

Fig. XXXII.

The Latitude *of the* Place (51 *d.* 30 *m.*) *the* Sun's Place *in the* Ecliptick (29 *d. of* Taurus; *or* 29 *d. of* North Declination) *and the* Sun's Altitude (12 *d.*) *being given*; *To find*,

1. *The Sun's* Azimuth, *from the* Eaſt, Weſt, North *or* South *Points of the* Horizon:
2. *The* Hour *of the* Day, *from* Noon *or* Midnight.
3. *The* Angle *of the* Sun's Poſition.

I. *By the* Cœleſtial Globe.

1. *For the* Sun's Azimuth.

THE *Globe* being Rectified, &c. and the *Quadrant of Altitude* fixed, and brought to the *Horizon*; turn 29 d. of *Taurus* toward the *Eaſt*, if in the Morning; or towards the *Weſt*, if in the Evening, till it come to lye juſt under 12 d. of the *Quadrant of Altitude*; and then note at what Degree in the *Horizon* the *Quadrant of Altitude* reſteth; which will be at 17 d. 8 m. from the *Eaſt*, if in the Morning, or 17 d. 8 m. from the *Weſt*, if in the Afternoon, which is the *Azimuth* from the *Eaſt* or *Weſt* towards the *North*. And this *Azimuth*, if reckoned by the Points of the Compaſs upon the *Horizon*, will be *E.* by *N.* 5 d. 35 m. Northward, if in the Morning; or *W.* by *N.* 5 d. 53 m. Northward, if in the Evening, when the *Sun* is in 29 d. of *Taurus*, and hath 12 d. of *Altitude*. Now if you count the Degrees of the *Horizon* between the *Quadrant of Altitude*, and the *North* Part of the *Meridian*, you ſhall find them to be 72 d. 52 m. which is the *Azimuth* from the *North*: And if you count them from the South Part of the *Meridian*, you ſhall find them to be 107 d. 8 m. which is the *Azimuth* from the *South*.

In like manner,

	D.	*M.*
The *Latitude* being	51	30
The *Sun*'s Place	1	00 *Aquarius*.
The *Sun*'s *Altitude*	12	00

Then will the *Azimuth* be found to be 56 d. from the *Eaſt* or *Weſt*, towards the *South*; which (by the Points of the Compaſs upon the *Horizon*) will appear to be S. E. by S. if in the Morning, or S. W. by S. if in the Evening.

2. *For the Hour of the Day.*

Fig. XXXII. Being 29 d. of *Taurus* to the *Meridian*, and ſet the *Index* to 12 a Clock: Then, if it be in the Forenoon, ſet the *Quadrant of Altitude* on the *Eaſt* Side of the *Meridian*; but on the *Weſt* Side, if it be in the Afternoon. And turn the *Globe* about, till the 29 d. of *Taurus* meet with 12 d. of the *Quadrant of Altitude*; and then ſhall the *Index* of the *Hour-Circle* point at 5 a Clock, and 36 m. if it be in the Morning; or at 24 m. after 6 of the Clock, if it be at Night. And that is the true Hour of the Day.

In like manner,

	D.	*M.*	
The *Latitude* being	51	30	
The *Sun*'s Place	1	00	*Tuurus.*
The *Altitude*	36	00	

Then will the Hour of the Day be found to be either 9 in the Morning, or 3 in the Afternoon. And which of theſe Hours it is, may beſt be known by a ſecond Obſervation of the *Altitude*: For if the *Altitude* do increaſe, it is the Forenoon; but if it decreaſe, it is the Afternoon.

Again,

	D.	*M.*
The *Latitude* being	52	30
The *Sun*'s Place in the beginning of *Taurus.*		
The *Sun*'s *Altitude*	25	56

The Hour of the Day would be found to be either 8 m. paſt 4 in the Afternoon. Or if it were in the Forenoon, 52 m. after 7 in the Morning.

3. *For the* Angle *of the* Sun's Poſition.

This may be found by the 26th Element, and as is taught in the Prœme before this Second Part: And in this Example will be found to be 39 d. 16 m.

II. *By* Trigonometrical Calculation.

Fig. XXXIII. The *Globe* reſting in the ſame *Poſition* you left it, when you wrought theſe *Problems* upon it, you will diſcover upon it an *Oblique-angled Spherical Triangle*, (ſuch as in the *Figure*, is noted with Z ☉ P: Which *Triangle* conſiſts of, (1.) Z P, an Arch of the *Braſs Meridian*, equal to the Complement of the given *Latitude*, 38 d. 30 m. (2.) Of Z ☉, an Arch of the *Quadrant of Altitude*,

Fig.
XXXIII.

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XXXII

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XXXIII

Altitude, croſſing the *Ecliptick* in 29 d. of *Taurus*, where the Sun is 12 d. high, and is equal to the *Complement* of the Sun's *Altitude* 78 d. (3.) An Arch of a *Meridian* (or *Hour-Circle*) P ⊙, paſſing through 29 d. of *Tturus*, and croſſing the *Quadront of Altitude* in 12 d. thereof, and is equal to the *Complement* of the *Sun's Declination* (or his Diſtance from the *Pole*) 70 d. So that the *Three Sides* of this *Triangle* are *Given*, and the *Three Angles* are *Required*. Fig. XXXIII.

The Canons for Calculation.

1. For the Sun's *Azimuth*, the *Angle* ⊙ Z P. By *Caſe* IX. of *O. A. S. T.*

	D.	*M.*
The Side ⊙ P, the *Co-declination*, is	70	00
The Side ⊙ Z, the *Co-altitude*, is	78	00
The Side Z P, the *Co-latitude*, is	38	30
Their *Sum* is	186	30
Their half *Sum* is	93	15
The *Difference* between the half *Sum*, and 70 d. ⊙ P	23	15

Being thus prepared, the Proportions are,

(1.) As the Radius, Sine 90 d.
Is to the Sine of Z P (one of the *Sides* containing the enquired *Angle* Z) 38 d. 30 m.
So is the Sine of Z ⊙ (the other Side containing the enquired *Angle* Z) 78 d.
To the Sine of 37 d. 31 m.

(2.) As this Sine of 37 d. 31 m.
Is to the Sine of the half *Sum*, 93 d. 15 m. (or 86 d. 45 m.)
So is the Sine of the *Difference*, 23 d. 15 m.
To the Sine of 40 d. 20 m.

(3.) To this Sine of 40 d. 20 m. add the Radius, and it will be 19.811063; the half whereof 9.905531 is the Sine of 53 d. 34 m. Whoſe Complement 36 d. 26 m. doubled, makes 72 d. 52 m. And that is the Quantity of the *Angle* ⊙ Z P: Or the Sun's *Azimuth* from the *North* Part of the *Meridian*. And that taken from 90 d. leaves 17 d. 8 m. for the Sun's *Azimuth* from the *Eaſt* or *Weſt*. And that added to 90 d. gives 107 d. 8 m. for the Sun's *Azimuth* from the *South*.

2. For

Fig. XXXIII. 2. For the *Hour*, the *Angle* Z P ☉. By *Caſe* I. of *O. A. S. T.*

As the Sine of the Side P ☉ (70 d.)
Is to the Sine of the *Angle* ☉ Z P, (72 d. 52 m.)
So is the Sine of the Side Z ☉ (78 d.)
To the Sine of the *Angle* Z P ☉, (84 d. 7 m.)
And that is the *Hour*, counted from the *North* Part of the *Meridian* (which in Time is 5 h. 36 m. that is, 36 m. after 5 in the Morning, or 95 d. 53 m. (or 6 h. 24 m.) from the *South*.

3. For the *Angle* Z ☉ P, the *Angle* of *Poſition*. By *Caſe* I. of *O. A. S. T.*

As the Sine of the Side P ☉, (70 d.)
Is to the Sine of the *Angle* ☉ Z P, (72 d. 52 m.)
So is the Sine of Z P, (38 d. 30 m.)
To the Sine of the *Angle* Z ☉ P, 39 d. 16 m.
Which is the *Angle* of the *Sun's Poſition*.

PROB. X.

To find the Longitude *and* Latitude *of any* Star.

THE *Longitude* of any *Star*, is an *Arch* of the *Ecliptick*, contained between the beginning of *Aries*, and the Interſection of an *Arch* of a great Circle, which paſſeth through both the Poles of the *Ecliptick*, and alſo through the Body of that *Star*.

The *Latitude* of a *Star* is that Part of an *Arch* of a great Circle, which paſſeth through both the Poles of the *Ecliptick*, and through the Body of the *Star*, and is contained between the *Ecliptick* Line and that *Star*.

I. For the *Longitude* ☉, Skrew the *Quadrant of Altitude* over that Pole of the *Ecliptick* which is neareſt to the *Star*, whoſe *Longitude* you ſeek. Then laying the *Quadrant* juſt over the Centre of the Star, look what Degrees of the *Ecliptick*, are cut by the (counting them from the beginning of *Aries*) *Quadrant of Altitude*, and thoſe Degrees are the Degrees of the *Star*'s *Longitude*. So the *Quadrant of Altitude* skrewed over the *North* Pole of the *Ecliptick*, and laid upon the bright Star *Capella*, the *Quadrant* ſhall cut 77 d. 16 m. of the *Ecliptick* Circle, counted from the beginning of *Aries*; and that is that *Star's Longitude*. Fig. XXXIII.

Note, That the Poles of the *Ecliptick* are diſtant from the Poles of the World 23 ½ d. on either ſide.

For the *Latitude*, the *Quadrant* fitted as before, and laid over the Centre of *Capella*, the *Star* ſhall cut 22 d. 50 m. of the *Quadrant of Altitude*; and ſuch is the *Latitude* of that *Star*, *North*, for that it lyes on the *North* Side of the *Ecliptick* Line.

This needs no Trigonometrical Calculation.

PROB. XI.

To find the Right Aſcenſion *and* Declination *of a* Star.

THE *Right Aſcenſion* of a *Star* is that Arch of the *Æquinoctial*, which is contained between the beginning of *Aries*, and that Point which comes to the *Meridian* with that *Star*.

The *Declination* of a *Star* is an Arch of the *Meridian* contained between the *Æquinoctial* and any *Star*.

For the *Right Aſcenſion*, (the *Globe* being rectified) bring *Capella* to the *Meridian*, and then ſhall you find 73 d. 7 m. of the *Æquinoctial* contained between the beginning of *Aries* and the *Meridian*; and that is the *Right Aſcenſion* of *Capella*.

For the *Declination*, bring *Capella* to the *Meridian*, ſo ſhall you find 45 d. 37 m. of the *Meridian* contained between the *Æquinoctial* and *Capella*; and that is the *Declination* of that *Star*. And in this manner you may find the *Longitude*, *Latitude*, *Right Aſcenſion*, and *Declination*, of any other *Star* upon the *Cæleſtial Globe*, as in this following Table of the principal Fixed Stars of the firſt Magnitude you ſhall find.

This needs no Trigonometrical Calculation.

Fig. XXXIII.

Stars Names.	*Longit.* D.	M.	*Latit.* D.	M.		*R. Aſcen.* D.	M.	*Declin.* D.	M.	
Arcturus	119	35	31	2	B	210	13	20	58	B
Lucida Lyra	280	43	61	47	B	276	27	38	30	B
Algol	51	37	22	22	B	41	46	39	39	B
Capella	77	16	22	50	B	73	7	45	37	B
Aldebaran	65	12	35	31	A	64	17	15	48	B
Regulus	145	17	0	26	B	147	43	13	33	B
Cauda Leonis	167	3	12	18	B	173	4	16	25	B
Spica Virgin.	199	16	1	59	A	196	56	9	31	A
Antares	245	13	4	27	A	242	23	25	37	A
Fomahant	329	11	21	00	A	339	46	31	17	A
Regel	72	17	1	11	A	74	44	8	37	A
Syrius	99	33	39	30	A	97	42	16	14	A
Procyon	111	18	15	57	A	110	34	6	3	B

PROB. XII.

To find the Diſtance of Two Stars.

1. IF the Two *Stars* be both of them under the ſame *Meridian*, Bring them under the General (or Braſs) *Meridian*, and ſee what Degrees of the *Meridian* are contained between them, for that is their Diſtance.

2. If they lye not under the ſame *Meridian*, but have the ſame Declination, or lye in the ſame Parallel, Bring one of them to the *Meridian*, and ſee what Degrees of the *Æquinoctial* are cut thereby: Then bring the other *Star* to the *Meridian*, and count what Degrees of the *Æquinoctial* are contained between the *Meridian* and the Degrees before found; for that is the Diſtance of thoſe Two *Stars*.

3. If the Two *Stars* do neither lye under the ſame *Meridian*, nor in the ſame Parallel, Then lay the *Quadrant of Altitude* (it being looſe) to both the *Stars*, and the Degrees of the *Quadrant* contained between the Two *Stars* is their Diſtance. And if the *Quadrant* be too ſhort, you may uſe the Circle of Poſition, or take their Diſtance with a pair of Calope-Compaſſes, and meaſure their Diſtance upon the *Æquinoctial*, or any other great Circle.

Thus,

Fig. XXXIII.

The Right Shoulder of *Auriga*, and the Right Shoulder of *Orion*, being under the ſame *Meridian*, their Diſtance will be found to be 37 d. 38 m.

Alſo,

Arcturus and the Lion's Neck, being near in the ſame Parallel, their Diſtance will be found to be 57 d.

Likewiſe,

Lyra the Harp, and *Marchad* in the Wing of *Pegaſus*, will be found, to be diſtant 63 d.

The finding of the Diſtance of *Stars* upon the *Cœleſtial Globe* is the ſame as the finding of Diſtance of Places upon the *Terreſtrial*, and conſequently the *Triangles* made upon the *Globe* are the ſame alſo: So that the Canons for Calculation for finding them will be the ſame alſo, and needs not be here again repeated.

PROB. XIII.

To know what Stars *will be upon the* Meridian *at any Hour of the Night.*

THE *Sun* being in 29 d. of *Taurus*, what *Stars* will be upon the *Meridian* at 10 a Clock, and 12 m. at Night. Bring 29 d. of *Scorpio* (which is the oppoſite Sign to *Taurus*) to the *Meridian*, and ſet the *Index* of the Hour-Circle to 12. Then turn the Globe about Weſtward till the *Index* point at 12 m. after 10 a Clock, and there hold the *Globe*; and all thoſe *Stars* which lye under the *Braſs Meridian* are then upon the *Meridian*, of which *Arcturus* is the Chief.

This needs no Trigonometrical Calculation.

PROB. XIV.

To know what Day in the Year any Star *ſhall be upon the* Meridian *at* 12 *a Clock at Night.*

BRing the *Star* to the *Meridian*, and mark what Degree of the *Ecliptick* is juſt under the *Meridian* at the ſame time: Then find that Degree of the *Ecliptick* in the *Horizon*, and note what Day of the Year ſtandeth againſt it, for that Day of the Year will that *Star* be upon the *South* Part of the *Meridian* at 12 at Night: And when the Sun is in the oppoſite Point of the

Fig. XXXIII. the *Eccliptick*, the ſame *Star* will be upon the *North* Part of the *Meridian* at 12 at Noon.

PROB. XV.

The Sun's *Place, and the* Altitude *of a known* Star *given; To find the Hour of the Night.*

THE *Sun* being in 21 d. of *Capricorn*, the *Altitude* of the *Great Dog* 14 d. I demand the Hour of the Night.

The *Globe* Rectified, &c. Bring 21 d. of *Capricorn* to the *Meridian*, and the *Index* to 12 a Clock. Then move the *Globe* and *Quadrant of Altitude* ſo together, that the *Great Dog* meet with 14 d. of the *Quadrant*; and then ſhall the *Index* point at 8 of the Clock, and 22 m. which is the true Hour of the Night.

And thus,

When the *Sun* is in	D.		and the *Altitude* of		D.	The ho. will be	D.	M.
	20	♐		*The Bull's Eye*	39		7	12
	20	♏		*The Bull's Eye*	30		9	2
	5	♊		*Arcturus*	50		11	3

PROB. XVI.

The Altitude *of* Aldebaran *(or any other Star) being given, in a known* Latitude; *To find the* Star's Azimuth.

THE *Quadrant of Altitude* being fixed in the *Zenith*, move it and the *Globe* till the Degrees of *Altitude* given do meet with the Centre of the *Star*; then ſhall the end of the *Quadrant of Altitude* ſhew you upon the *Horizon* the *Azimuth* in which the *Star* then is. And thus, if you bring the *Quadrant of Altitude* on the *Eaſt* Side of the *Globe*, moving it and the *Globe* both, till the Centre of *Aldebaran* do meet juſt with 42 d. of the *Quadrant*, you ſhall then find the *Quadrant of Altitude* to reſt at 33 d. of the *Horizon*, counted from the *Eaſt*; or at 57 d. if you count them from the *South*: And that is the *Azimuth* of *Aldebaran* when he hath 42 d. of *Altitude*; and that is near the S. E. by E. Point of the Compaſs.

The Latitude *of the* Place, (*51 d. 30 m.*) *and the* Declination *of a* Star, (*ſuppoſe the* Bull's Eye, Aldebaran) *given: To find* Fig. XXXIII.

PROB. XVII.

Its Right Aſcenſion.

THE *Globe* Rectified to the *Latitude*, &c. Bring *Aldebaran* to the *Meridian:* Then count how many Degrees of the *Æquinoctial* are contained between the *Meridian* and the beginning of *Aries*; which will be 64 d. 17 m. and that is the *Right Aſcenſion* of that *Star*; which in time (by allowing 15 d. for an Hour, and 1 d. for 4 m. of time) in 4 h. 16 m. its *Right Aſcenſion* in time.

And in the ſame manner may you find

			D.	M.		H.	M.
The *Right Aſcenſion* of	*Arcturus*	to be	210	13	in time	14	1
	Sprius		97	42		6	30
	Algol		39	39		2	38

The ſame Trigonometrical Calculation, as for the *Sun*'s *Right Aſcenſion*, as in *Prob.* III. ſerves for this alſo.

PROB. XVIII.

Its Aſcenſional Difference.

BRing the *Star* to the *Meridian*, and the *Hour-Index* to 12. Then bring the *Star* either to the *Eaſt* or *Weſt* Side of the *Horizon*, and there you ſhall find 1 h. and 27 m. contained between the *Index* and 6 a Clock: And ſuch is the *Aſcenſional Difference* of *Aldebaran.*

In like manner you may find

			D.	M.		H.	M.
The *Aſcenſional Difference* of	*Arcturus*	to be	28	40	or in time	1	55
	Syrius		21	28		1	26
	Algol		—	—		—	—

Algol his *Declination* being more than the Complement of the *Latitude*, never riſes nor ſets, but is always above the *Horizon.*

The ſame Trigonometrical Calculation, as for the *Sun.*

PROB.

Fig. XXXIII.

PROB. XIX.

Its Amplitude.

BRing *Aldebaran* to the *Horizon* on either Side of the *Globe*, and you ſhall find it to touch the *Horizon* at 25 d. 56 m. from the *Eaſt* or *Weſt* Northward; which is the *Amplitude* of the *Bull's Eye* riſing or ſetting. And according to the Points of the Compaſs it riſeth E. N. E. 2 d. 26 m. Northerly, and ſets W. N. W. 2 d. 26 m. Northerly.

And thus may you find

				D.	M.
That	*Arcturus*	Riſeth from the *Eaſt* or *Weſt*	Northward	35	6
	Syrius		Southward	26	41
	Algol		Never riſes or ſets.		

The ſame Trigonometrical Calculation as for the *Sun*.

PROB. XX.

The Semidiurnal Arch, *and the time that* Aldebaran *(or any other* Star) *continues above the* Horizon.

BRing the *Aldebaran* to the *Meridian*, and ſet the Hour-Circle to 12. Then turn the *Globe* Weſtward, till *Aldebaran* touch the *Horizon*; then ſhall the Hour-Index point at 7 h. 27 m. And ſo long time is *Aldebaran* above the *Horizon*, before he comes to the *Meridian*, and continues ſo many Hours and Minutes above the *Horizon*, after he hath paſt the *Meridian*, and ſets in the *Weſt*. And thoſe 7 h. and 27 m. is the *Semidiurnal Arch* of that *Star*; which doubled, is 14 h. 54 m. And ſo long doth that *Star* continue above the *Horizon* after the time of his riſing.

And in this manner you find

			D.	M.		H.	M.
The Semidiurnal Arch of	*Arcturus*	to be	7	55	And his Continuance above the *Horizon*	15	50
	Syrius		4	34		9	8
	Algol		12	00		24	00

The ſame Trigonometrical Calculation as for the *Sun*.

PROB. XXI.

Fig. XXXIII.

At what Hour (any time of the Year) Aldebaran *comes to the* Meridian.

LET the time be the Firſt of *January*, at which time the *Sun* is in 22 d. of *Capricorn*. Bring 22 d. of *Capricorn* to the *Meridian*, and ſet the Hour-Index to 12. Then turn the *Globe* about till *Aldebaran* be under the *Meridian*, and then you ſhall find the Index to point at 42 m. after 8 of the Clock, at which time *Aldebaran* will be upon the *Meridian* that Night.

In like manner you may find that

				H.	M.
Upon	*October* 28	*Arcturus*	will be upon the *Meridian* at	11	10
	January 21	*Syrius*		9	33
	January 1	*Algol*		7	12

The ſame Trigonometrical Calculation as for the *Sun*.

PROB. XXII.

At what Hour (at any time of the Year) Aldebaran *(or any other* Star) *riſeth or ſetteth.*

LET the time be *January* 1. By the laſt before-going, you found that *Aldebaran* came to the *Meridian* at 8 h. 42 m. And by the laſt but one you found his Semidiurnal Arch to be 7 h. 27 m. This being taken from 8 h. 42 m. the time of his being *South*, leaveth 1 h. 15 m. the time of its riſing; ſo that upon the Firſt of *January Aldebaran* did riſe at 15 m. after 1 in the Afternoon. Again, if you add his Semidiurnal 7 h. 27 m. to the time of its being *South* 8 h. 42 m. the Sum will be 16 h. 9 m. from which take 12 h. and the Remainder will be 4 h. 9 m. So that *Aldebaran* did ſet at 9 m. after 4 of the Clock the next Morning.

And in like manner you may find that

				H.	M.		H.	M.
Upon	*October* 28	*Arcturus*	Riſes at	3	8	Sets at	7	12
	January 21	*Syrius*		5	3		2	3
	January 1	*Algol*		——			——	

Algol never Riſes nor Sets.

The ſame Trigonometrical Calculation as for the *Sun*.

Fig. XXXIII.

PROB. XXIII.

At what Horary Diſtance from the Meridian, Aldebaran *will be due Eaſt or Weſt: And what* Altitude *he ſhall then have.*

BRing *Aldebaran* to the *Meridian*, and the Hour-Index to 12, and the *Quadrant of Altitude* to the *Weſt* Point of the *Horizon:* Then turn the *Globe* Eaſtward, till the Centre of the *Star* be juſt under the Edge of the *Quadrant*; then ſhall the Index point at 5 h. and 40 m. So that when *Aldebaran* is due *Eaſt* or *Weſt*, he will be 5 h. 40 m. of time ſhort of, or gone beyond, the *Meridian.*

And when the Centre of *Aldebaran* is juſt under the Edge of the *Quadrant of Altitude*, you ſhall find it to touch 20 d. 21 m. And ſuch is the *Altitude* of *Aldebaran* when he is upon the *Eaſt* or *Weſt Azimuth.*

In like manner may you find that

		H.	*M.*		*D.*	*M.*
Arcturus	will be upon the *Eaſt* or *Weſt Azimuth*, when he is diſtant from the *Meridian.*	4	49	and his *Altit.* will be	27	13
Syrius		5	04		20	56
Algol		3	15		54	37

The ſame Trigonometrical Calculation as for the *Sun.*

PROB. XXIV.

What Altitude *and* Azimuth Aldebaran *(or any other* Star*) ſhall have, when Six Hours diſtant from the* Meridian.

BRing *Aldebaran* to the *Meridian*, and the Index to 12. Then turn the *Globe* about till the Index point at 6; then lay the *Quadrant of Altitude* over the Centre of the *Star*, and you ſhall find it to lye under 12 d. 18 m. of the *Quadrant:* And ſuch is the *Altitude* of *Aldebaran*. At the ſame time look what Degrees of the *Horizon* are cut by the *Quadrant of Altitude*, and you ſhall find 8 d. between it and the *Eaſt* or *Weſt* Points Northwards. And ſuch is the *Azimuth* of *Aldebaran*. And according to this Rule you ſhall find that when

		Altit.		*Azim.*		
		D.	M.	D.	M.	
Arcturus	is 6 h. diſtant from the *Meridian* his	16	15	76	36	from the *North.*
Algol		12	38	79	43	

Syrius is never 6 h. diſtant from the *Meridian*, nor any other *Star* that hath *South* Declination.

Fig. XXXIII.

The Trigonometrical Calculation as for the *Sun*.

PROB. XXV.

To find what Altitude *and* Azimuth *any Star hath when he is at any horary Diſtance from the* Meridian.

THIS is no other than the laſt. For having brought the *Star* to the *Meridian*, and the Index to 12, move the *Globe* till it come to the deſigned Hour. Then the *Quadrant of Altitude* being laid over the *Star*, ſhall at the ſame time ſhew you both the *Altitude* and *Azimuth* thereof as before. This needeth no Example.

The ſame Trigonometrical Calculation as for the *Sun*.

PROB. XXVI.

Having the Azimuth *of a* Star, *to find at what Horary Diſtance that* Star *is from the* Meridian, *and what* Altitude *that* Star *then hath.*

BRing the *Star* to the *Meridian*, the *Index* to 12, and the *Quadrant of Altitude* to the *Given Azimuth*; then turn the *Globe* about till the Centre of the Star lye juſt under the *Quadrant of Altitude*; the *Index* at that time ſhall give the *Horary Diſtance*, and *Quadrant* the *Altitude* of the *Star*.

Example. *Aldebaran* being ſeen upon 80 d. of *Azimuth* from the North Weſtward, that is, near upon the W. by N. Point of the Compaſs; the *Star* brought to the *Meridian*, and the *Quadrant of Altitude* to 80 d. and the *Hour-Index* to 12. If you bring the *Star* to the *Quadrant of Altitude*, you ſhall find the *Index* to point at 6 h. which is the *Star's Horary Diſtance* from the *Meridian*. And the *Quadrant of Altitude* will ſhew 12 d. 18 m. the *Altitude* of *Aldebaran* at that time.

Theſe Twelve laſt *Problems* may be reſolved by Trigonometrical Calculation in all Reſpects as thoſe of the *Sun*. if (in all of them) inſtead of the Word [*Sun*] there, read [*Star*] here. And the Canons for Calculation will be the ſame in both.

Fig. XXXIII.

PROB. XXVII.

The Longitude *and* Latitude *of a* Star, *which is ſituate out of the* Ecliptick, *being given; To find the Right Aſcenſion and Declination of that* Star.

HERE is given the Situation of the *Star*, in reſpect to the *Ecliptick*, altho' its Situation be out of the *Ecliptick*; and his Place, in reſpect of the *Æquinoctial*, is required

I. *By the* Cœleſtial Globe.

Screw the *Quadrant of Altitude* over the Pole of the *Ecliptick*, and lay it to the Degree of the *Star's Longitude* in the *Ecliptick*. —Then count the *Star's Latitude* given upon the *Quadrant of Altitude*: And obſerve what *Meridian* paſſeth through that Point: For the Degrees of that *Meridian*, comprehended between that Point, and the Pole of the World, are the Degrees of the *Star's Declination*: And the ſame *Meridian* continued, will croſs the *Æquinoctial* Circle in the Point of the *Star's Right Aſcenſion*.

II. *By* Trigonometrical Calculation.

Fig. XXXIV. XXXV.

When the *Longitude* of the *Sun* falls to be in any Sign {Aſcending, as in Fig. XXXIV. / Deſcending, as in Fig. XXXV.} make choice of the *Triangle* P F S, for the *Latitude* of the given *Star* if it be {North, / South,} the which the Letters *b* and *m* written within them will determine. In which *Triangle* there is

Given,
1. The Side P F, the Diſtance of the Pole of the *Æquator* from the Pole of the *Ecliptick*, 23 d. 30. m.
2. The Angle F, whoſe Meaſure is the Arch of the *Ecliptick*, intercepted between the Sides F P and F S, from whom the *Longitude* of the *Star* is produced, and the Solſtitial Point found out.
3. The Side F S, the Complement of the given *Star's Latitude*.

Required,
1. The Side P S, the Complement of the *Star's Declination.*
2. The Angle P, which the Ark of the *Æquator* intercepted between the Sides P F and P S; of which, this *Right Aſcenſion* of the *Star* is made, whither from the Degree of the *Ecliptick* 270, or 90 d.

Fig. XXXIV. XXXV.

Example. Let there be given the *Head of Andromeda*, which is a *Star* of the Second Magnitude; whoſe *Longitude* is in 9 d. 38 m. ♈, and *Latitude* 25 d. 42 m. *b.* The *Longitude* of the *Star* falls out to be in an Aſcending Sign *Aries*; therefore, make choice of the Triangle P F S, Fig. XXXV. the *Latitude* being North, in which there is given, (1.) P F, the Diſtance of the Pole of the *Æquator* from the Pole of the *Ecliptick*, 23 d. 30 m. (2.) The Angle F, which the Ark of the *Ecliptick* intercepted between the Sides F P and F S, being produced, meaſureth, and gives the *Longitude* of the *Star* to be 9 d. 38 m. ♈, ſo that the Solſtitial Point ♋ is neareſt; ſubſtract therefore 9 d. 38 m. ♈ from 90 d. the beginning of ♋, the Remainder will be 80 d. 22 m. which is the Angle at F. (3.) F S, the Complement of the *Latitude* of the given *Star*, 64 d. 18 m. —And let there be ſought, (1.) P S, the Complement of the *Declination* 62 d. 45 m. therefore the *Declination* of the *Star* is 27 d. 15 m. (2.) The Angle P, 87 d. 47 m. Which added to 270 d. makes the *Right Aſcenſion* of the *Star* to be 357 d. 47 m.

The Canons for Calculation.

I. For the Angles S and P.

(1.) As the Sine of half the Sum of the Sides S F and F P, 43 d. 54 m.
Is to the Sine of half the Difference of thoſe Sides, 20 d. 24 m.
So is the Co-tangent of half the Angle at F, 40 d. 11 m.
To the Tangent of half the Difference of the Angles at S and P, 30 d. 46 m.

(2.) As the Co-ſine of half the Sum of the Sides F S and F P, 46 d. 6 m.
Is to the Co-ſine of half the Difference of thoſe Two Sides, 39 d. 36 m.

So is the Co-tangent of half the Angle at F, 40 d. 11 m.

Fig. XXXIV. XXXV. To the Tangent of half the Sum of the Angles at S and P, 57 d. 1 m.

Which 57 d. 1 m. added to the aforefound Difference, gives 87 d. 47 m. for the greater Angle at P; and 30 d. 46 m. the Difference aforefound ſubſtracted from 57 d. 1 m. leaves 26 d. 15 m. for the leſſer Angle at S.

Then,

(3.) As the Sine of the Angle at S, 26 d. 15 m.
Is to the Sine of the Side F P, 23 d. 30 m.
So is the Sine of the Angle at F, 80 d. 22 m.
To the Sine of the Side S P, 62 d. 45 m.
Whoſe Complement, 27 d. 15 m. is the *Star's Declination.*

PROB. XXVIII.

The Declination *and* Right Aſcenſion *of a* Star *being given: To find the* Longitude *and* Latitude *of that* Star.

THIS is the Converſe of the former *Problem*, and may be reſolved as followeth.

I. *By the* Cœleſtial Globe.

Screw the *Quadrant of Altitude* over the Pole of the *Ecliptick:* Then count upon the *Æquinoctial* Circle the Degrees of *Right Aſcenſion*; and upon that *Meridian* which paſſes through that Point, count the *Star's Declination* from the *Æquinoctial*, towards the *Pole*, and to that Point bring the *Quadrant of Altitude*; then will the *Quadrant* cut the *Ecliptick* in the Degrees of the *Star's Longitude*; and the Degrees of the *Quadrant*, comprehended between the *Ecliptick* Circle and its *Pole*, will give the Degrees of the *Star's Latitude.*

II. *By* Trigonometrical Calculation.

When the *Right Aſcenſion* of the *Star* falls out to be in the *First* or *Fourth Quadrant* of the *Æquinoctial*; as in *Fig.* XXXIV. or in the *Second* or *Third:* As in *Fig.* XXXV. make uſe of the Triangle P F S for the *Star's Declination* North or South. In which there is

Fig. XXXIV. XXXV.

Given,
1. P F, the Diſtance of the Pole of the *Ecliptick* from the Pole of the *World*, 23 d. 30 m.
2. P S, the Complement of the *Star's Declination*, 64 d. 13 m.
3. The Angle P, whoſe Meaſure is the Arch of the *Æquator* intercepted between the Sides P F and P S, 87 d. 47 m. from whence the *Right Aſcenſion* of the *Star* is produced, and will be found to be 357 d. 47 m.

Required,
1. The Angle F, whoſe Meaſure is the Arch of the *Ecliptick* intercepted between the Sides F P and F S, from whence the *Longitude* of the *Star* from the next *Solſtitial* Point is found.
2. F S, the Complement of the *Star's Latitude.*

The Canons for Calculation.

(1.) As the Sine of half the Sum of the given Sides P F and P S, 43 d. 8 m.
Is to the Sine of half the Difference of thoſe Sides, 19 d. 37 m.
So is the Co-tangent of half the given Angle F, 43 d. 54 m.
To the Tangent of half the Difference of the Angles at F and S, *viz.* 27 d. 3 m.

(2.) As the Co-ſine of half the Sum of the Sides F S and F P, 43 d. 8 m.
Is to the Co-ſine of half the Difference of thoſe Sides; 19 d. 37 m.
So is the Co-tangent of half the given Angle at F 43 d. 54 m.
To the Tangent of half the Sum of the Angles at F and S; *viz.* 53 d. 18 m.

The half Difference before found 27 d. 3 m. added to the half Sum now found, gives 80 d. 21 m. for the Angle at F, the Complement of the *Star's Longitude.* And ſubſtracted from the half Sum 53 d. 18 m. leaves 26 d. 15 m. for the Angle at S.

Then ſay,

(3.) As the Sine of the Angle at F, 80 d. 21 m.
Is to the Sine of the Side S P, 62 d. 45 m.
So is the Sine of the Angle at P, 87 d. 47 m.
To the Sine of the Side S F, 64 d. 18 m.
Whoſe Complement 25 d. 42 m. is the *Star's Latitude.*

A Table of the Right Aſcenſion and Declination of 100 *Eminent Fixed Stars, Calculated for the Year of Chriſt* 1700, *compleated according to* Ricciolus. *And their Difference of Right Aſcenſion and Declination in* 100 *Years.*

Names of the STARS.	*Right Aſcenſion for the Year* 1700.			*Declination for the Year* 1700.				*Diff. of Aſcen. in* 100 *Y.*		*Declin. in* 100 *Years.*	
	De.	*Mi.*	*Se.*	*De.*	*Mi.*	*Se.*		D.	M.	*Mi.*	
Head of *Andromeda.*	358	14	08	27	27	06	B	1	17	34	A
Girdle of *Andromeda.*	13	11	20	34	2	49	B	1	23	33	A
South Foot of *Andromeda.*	26	21	51	40	52	30	B	1	29	30	A
Fomalhant of ♒.	340	11	00	31	08	10	A	1	25	31	S
Right Shoulder of ♒.	327	56	55	1	43	44	A	1	20	29	S
Left Shoulder of ♒.	318	55	54	6	48	46	A	1	21	26	S
Left Hand of ♒.	307	45	54	10	33	44	A	1	26	19	S
Bright Star in the Eagle.	294	2	47	8	6	32	B	1	27	13	A
Firſt in Horn of ♈.	24	17	2	17	48	24	B	1	23	31	A
Second in ♈ Horn.	24	30	3	16	19	24	B	1	22	31	A
Bright one in *Aries.*	27	35	58	22	1	30	B	1	25	30	A
Goat of *Auriga.*	73	35	56	45	40	00	B	1	49	10	A
Right Shoulder of *Auriga.*	84	29	42	44	51	30	B	1	58	$4\frac{1}{2}$	A
Arcturus in *Bootes.*	210	33	2	20	48	2	B	1	11	$26\frac{1}{2}$	S
Left Shoulder of *Bootes.*	215	2	33	39	35	12	A	1	2	27	S

Names

Names of the STARS.	Right Aſcenſion for the Year 1700.			Declination for the Year 1700.				Diff. of Aſcen. in 100 Y.		Declin. in 100 Years.	
	De.	Mi.	Se.	De.	Mi.	Se.		D.	M.	Mi.	
Præſepe in *Cancer*.	125	46	2	20	43	4	B	1	28	19	S
Northern *Aſinego* in ♋.	126	26	0	22	3	0	B	1	30	20	S
Southern *Aſinego* in ♋.	126	54	3	19	15	0	B	1	27	20	S
Great Dog *Cyrius*.	97	57	6	16	18	6	A	1	7		A
Little Dog *Porcyon*.	110	54	33	5	59	12	B	1	20	12	S
Upper Horn of ♑.	300	24	34	13	22	6	A	1	25	16	S
Lower Horn of ♑.	301	7	29	15	38	2	A	1	27	17	S
Firſt in the Tail of ♑.	320	56	29	17	54	21	A	1	26	26	S
Second in the Tail of ♑.	322	43	40	17	22	22	A	1	25	27	S
Bright one in *Caſſiopea*'s Chair.	358	14	33	57	32	16	B	1	15	34	A
Scheder. or the Breaſt of *Caſſiopea*.	5	56	0	54	55	16	B	1	22	34	A
In the Flexture of *Caſſiopea*.	9	45	58	59	7	36	B	1	27	34	A
In *Caſſiopea*'s Knee.	16	36	0	58	40	22	B	1	35	33	A
Cephus his Girdle.	321	6	20	69	17	10	B	1	22	26	A
Bright one in the *Whale*'s Jaw.	41	38	7	2	50	50	B	1	15	25	A
Northern in the *Whale*'s Belly.	24	12	0	11	44	50	A	1	15	31	S
Southern in the *Whale*'s Belly.	7	5	8	19	35	40	A	1	18	34	S
Northern in the *Whale*'s Tail.	1	4	12	10	24	54	A	1	17	34	S

Names of the S T A R S.	*Right Aſcenſion for the Year* 1700.			*Declination for the Year* 1700.				*Diff. of Aſcen. in* 100 Y.		*Declin. in* 100 *Years.*	
	De.	*Mi.*	*Se.*	*De.*	*Mi.*	*Se.*		*D.*	*M.*	*M.*	
Bright one in the North. Crown.	230	39	0	27	45	20	B	1	5	21	S
In the Beak of the Swan.	289	39	48	27	22	40	B	1	1	11	A
In the Swan's Breaſt.	302	55	52	39	20	5	B	0	53½	18	A
In the Swan's Tail.	307	47	17	44	14	51	B	0	51½	20½	A
Upper Wing of the Swan.	293	56	2	44	26	21	B	0	48	14	A
Lower Wing of the Swan.	308	29	10	32	51	24	B	0	0	21	A
Bright one of the *Dragon.*	267	25	20	51	35	2	B	0	35	2	S
In the Head of *Caſtor.*	108	50	46	32	30	26	B	1	44	11	S
In the Head of *Pollux.*	111	43	36	28	43	2	B	1	34	12	S
Bright one in the Twins Foot.	95	3	32	16	37	32	B	1	28	2	S
Hercules Head.	255	21	37	14	46	48	B	1	8	8	S
Hercules Right Shoulder.	244	19	35	22	41	40	B	1	5	15	S
Hercules Left Shoulder.	255	31	33	25	15	48	B	0	52	8	S
Heart of *Hydra.*	138	12	22	7	21	30	A	1	15	25	A
Lion's Heart *Regulus.*	148	4	15	13	25	16	B	1	22½	28½	S
Lion's Tail.	175	25	34	16	14	4	B	1	19	24	S
Bright one in *Juba Leonis.*	150	48	47	21	21	0	B	1	25½	29	S
Bright one in *Lumbis Leonis.*	164	32	20	22	7	44	B	1	27	34	S
Uppermoſt in the Neck.	149	58	52	24	53	54	B	1	28	29	S
Loweſt in the Neck.	147	47	52	18	13	33	B	1	28	28	S

Names

Names of the STARS.	*Right Aſcenſion for the Year 1700.*			*Declination for the Year 1700.*				*Diff. of Aſcen. in 100Y.*		*Declin. in 100 Years.*	
	De.	*M.*	*Se.*	*De.*	*Mi.*	*Se.*		*D.*	*M.*	*M.*	
Thigh of the Hare.	78	51	30	20	59	0	A	1	5	7	S
Northern Scale of *Libra.*	225	15	26	8	14	46	A	1	21½	24	A
Southern Scale of *Libra.*	218	38	12	14	45	18	A	1	25	27	A
Bright one in *Lyra.*	276	39	32	38	32	16	B	0	50	4	A
Head of	260	15	38	12	49	22	B	1	11	7	S
Left Hand of	239	47	37	2	52	2	A	1	23	18	A
Right Knee of *Ophiucus.*	252	39	40	9	53	30	A	0	50	10	A
Left Knee of	245	11	37	15	16	30	A	1	23	15	A
Right Shoulder of	162	8	38	4	44	40	B	1	13	5	S
Uppermoſt in the Head of *Orion.*	79	41	10	9	34	38	B	1	22	7	A
Right Shoulder of *Orion.*	84	43	4	7	18	20	B	1	22	4	A
Left Shoulder of *Orion.*	97	16	40	6	3	2	B	1	19	8	A
Foot of *Orion Rigel.*	75	2	50	8	33	42	A	1	15½	9½	S
1	79	9	48	0	32	50	A	1	17½	7	S
2 In the Belt of *Orion.*	80	12	54	1	25	46	A	1	17	6	S
3	81	18	25	2	8	20	A	1	16	5	S

Names of the STARS.	Right Aſcenſion for the Year 1700.			Declination for the Year 1700.				Diff. of Aſcen. in 100 Y.		Declin. in 100 Years.	
	De.	Mi.	Se.	De.	Mi.	Se.		D.	M.	M.	
Mouth of *Pegaſus*.	322	27	36	8	32	14	B	1	18	26	A
Sa'd Alpharis in the Leg.	342	20	36	26	28	38	B	1	12	32	A
Marcab in the joining of the Wing.	342	28	10	13	35	58	B	1	15	32	A
End of *Pegaſus* Wing.	359	27	25	13	32	56	B	1	16	34	A
Bright one in the Side of *Perſeus*.	45	32	18	48	44	54	B	1	28	12	A
Ras Al Gol of *Perſeus*.	42	12	42	39	46	30	B	1	37	25	A
The hindermoſt in the Head of the Southern Fiſh.	345	24	5	1	40	2	B	1	17	33	A
In the Knot in the Line of ♓.	26	38	5	1	19	0	B	1	18	30	A
Bright one in the Head of ♐.	283	1	5	21	22	48	A	1	31	8	S
Antares, Heart of *Scorpius*.	242	47	28	25	39	54	A	1	32	16	A
Northern Front of *Scorpius*.	236	58	15	18	53	36	A	1	28	16	A
Middlemoſt *Scorpius*.	236	14	34	21	41	40	A	1	30	20	A
Southern Front of *Scorpius*.	235	18	0	25	9	54	A	1	37½	21	A
Brighteſt in the Neck of *Scorpius*.	232	24	0	7	24	36	B	1	15	21	S
Aldebaran, or Southern Eye of ♉.	64	41	35	15	52	10	B	1	26½	15	A
In the Northern Horn of ♉.	76	51	18	28	19	2	B	1	37	8	A
Southern Horn of ♉.	79	55	20	20	55	58	B	1	31	7	A
Northern Eye of ♉.	62	43	36	18	30	28	B	1	24	17	A
Loweſt of the *Hyades* ♉.	60	39	35	14	55	38	B	1	25	17	A
Bright one of the *Pleyades* ♉.	52	27	35	23	9	24	B	1	29	21	A

Fig. XXXIV. XXXV.

Names of the STARS.	Right Aſcenſion for the Year 1700.			Declination for the Year 1700.				Diff. of Aſcen. in 100 Y.		Declin. in 100 Years.	
	De.	Mi.	Se.	De.	Mi.	Se.		D.	M.	M.	
Spica Virginis.	197	22	55	9	33	30	B	1	19½	32½	A
Girdle of *Virgo.*	190	10	22	5	2	54	B	1	18	34	S
Vindemiatrix in *Virgo.*	191	52	20	12	34	58	B	1	17	33	S
The Bright one in the Shoulder of the Greater Bear.	161	17	5	63	22	2	B	1	41	32	S
The Bright one in its Side.	160	52	20	57	59	2	B	1	37	32	S
The Bright one in the hindermoſt Thigh.	174	23	34	55	23	42	B	1	23	34	S
On the Back near the Tail.	180	8	2	58	41	42	B	1	20	34	S
The Firſt in the Tail.	190	7	56	57	36	58	B	1	9	33	S
The Second in the Tail.	197	55	2	56	30	52	B	1	3	32	S
The Laſt in the Tail.	203	53	50	56	50	56	B	1	2	31	S
The laſt in the Tail of the leſſer Bear, now the Pole Star.	9	52	10	87	42	51	B	3	10	34½	A
The Bright one in the Shoulder, heretofore called *Cynoſura.*	222	39	20	75	37	30	B	1	15	2½	A

PROB. XXIX.

The Latitude *of a* Place, *(or Elevation of the* Pole) *and* Declination *of the* Sun, *or of a* Star, *being given: To find the* Aſcenſional Difference *of that* Star: *And from thence, (the* Right Aſcenſion *of the ſame* Star *being known) to find its* Oblique Aſcenſion: *The* Semidiurnal Ark, *the Continuance of it above and below the* Horizon: *Its* Ortive *and* Occaſive Amplitude: *And the* Oriental Angle, viz. *that of the Point of the* Ecliptick.

I. *By the* Cœleſtial Globe.

Fig. XXXVI. THE *Globe* being rectified to the *Latitude*, and the Place of the *Sun* brought to the *Meridian*; and the *Quadrant of Altitude* ſcrew'd to the *Zenith*, and brought to the *Eaſt* or *Weſt* Points of the *Horizon*; the ſeveral Arches and Angles made by the Poſitions and Interſections of theſe Circles being meaſured or counted upon the *Globe*, will reſolve the ſeveral Demands: But with ſome additional Works; which I here omit. Wherefore,

II. *By* Trigonometrical Calculation.

Here the Situation of a *Star* is given, in reſpect of the *Æquinoctial*; and Enquiry is made where its *Place* ſhould be, in reſpect of an *Oblique Horizon* given.

The *Globe* continuing in the ſame Poſition as when you wrought the *Problem* upon it, you will find conſtituted upon it *Two Spherical Triangles*; ſuch as in the *Figure* are noted with R *Or* S, for the *Declination* of the *Star* Northern or Southern. In either of which *Triangles*, there is (beſides the *Right Angle* at R)

Given,
- 1. The *Angle Or*, which is the *Angle* made by the *Æquator* and the *Horizon* of *London*, 38 d. 30 m.
- 2. The *Perpendicular* R S, the *Star's Declination* North.

And

Fig. XXXVI.

And there is Required,

- 1. *Or* R, the *Aſcenſional Difference*; which, when a *Star* declineth towards
  - An *Elevated Pole*,
    - *Subſtract* the *Decl.* from the R. *Aſcenſion*, it gives the *Obl. Aſcenſion.*
    - Add the *Decl.* to the R. *Aſcenſion*, it gives the *Obl. Deſcenſion.*
    - Add 90 d. it gives the *Semidiurnal Ark.*
  - A *Depreſſed Pole*,
    - Add the *Decl.* to the R. *Aſcenſion*, it gives the *Obl. Aſcenſion.*
    - Subſtract the *Decl.* from the R. *Aſcenſion*, it gives the *Obl. Aſcenſion.*
    - Subſtract 90 d. it gives the *Semidiurnal Ark.*
  - The *Semidiurnal Ark* being turned into *Hours*, gives half the Time of the *Stars* Continuance above the *Horizon:* Whoſe Complement to 12 *Hours* is its *Seminocturnal Ark*, or half his Continuance below the *Horizon.*
- 2. *Or* S, the *Amplitude* of the *Star*, at his *Riſing* or *Setting*, having either *North* or *South Declination.*
- 3. The *Angle* S; which *Subſtracted* in *Signs*,
  - *Aſcending*, from the *Angle* of the *Ecliptick* and *Meridian*,
  - *Deſcending*, from the Complement of the *Angle* of the *Ecliptick*; adding 180 d.
  - Gives the *Oriental Angle*

EXAMPLE I. *Let there be given an Elevation of the Pole (as at* London*) 51 d. 30 m. And the Sun's Declination, 0 d. 50 m. South.*

Fig. XXXVI. In the *Triangle* R *Or* S, (the *Declination* being Southward) there is given, (beſides the *Right Angle* at R,) (1.) The *Angle* R *Or* S, the Complement of the given *Latitude*, 38 d. 30 m. (2.) The *Side* R S, the *Declination* of the *Sun*, 1 d. 2 m. by which may be found,

(1.) The *Side Or* R, the *Aſcenſional Difference*, 1 d. 2 m. Which, when the *Sun* declines (as here) towards the *Depreſſed Pole*, being added to the *Right Aſcenſion* before found to be 358 d. 3 m. gives the *Oblique Deſcenſion*, to be 359 d. 7 m. But being taken from the ſaid *Right Aſcenſion*, 358 d. 3 m. gives the *Oblique Deſcenſion* to be 356 d. 59 m. And being taken likewiſe from 90 d. gives 88 d. 56 m. for the *Semidiurnal Ark:* Which being turned into *Time*, gives 5 h. 55 m. for the half continuance of the *Sun* above the *Horizon*, whoſe Complement to 12 h. is 6 h. 5 m. the half continuance of the Sun below the *Horizon:* But double the continuance of the Sun above the *Horizon*, the *Length* of the *Day* will be 11 h. 50 m. And double the continuance below, and the *Length* of the *Night* will be 12 h. 10 m.

(2.) The *Side Or* S, the *Ortive Amplitude* of the *Sun*, 1 d. 20 m.

(3.) The *Angle* R S *Or*, 51 d. 30 m. the given *Latitude*.

Which being ſubſtracted (becauſe, in this *Example*, the *Sign* is *Aſcending*, *viz.* ♓) from the *Angle* of the *Ecliptick* and *Meridian*, before found, to be 66 d. 31 m. gives 15 d. 1 m. for the *Oriental Angle* of the *Point* of the *Ecliptick*.

This *Example* was of the *Sun*, I will here give another *Example* of a Fixed Star, *viz.* The Head of *Andromeda*.

EXAMPLE II. *The* Declination *of the* Head *of* Andromeda *is* 27 *d.* 15 *m. repreſented by the* Side R S.

The Complement of the given *Latitude* is 38 d. 30 m. repreſented by the *Angle* R *Or* S.

Theſe being given, we are to find,

1. The *Side* R *Or*, the Star's *Aſcenſional Difference*.
2. The *Side Or* S, the *Amplitude* of the Star.
3. The *Angle* at S.

III. *The Canons for Calculation.*

1. For the *Side* R *Or*, by *Caſe* VII. of *R. A. S. T.*

As the Radius,
Is to the Tangent of R S, the Star's *Declination*, 27 d. 15 m.

So is the Co-tangent of the *Angle Or*, 38 d. 30 m. Fig. XXXVI.
To the Sine of the Side R *Or*. The Star's *Aſcenſional Difference*, 40 d. 21 m.

2. For the *Side Or* S. By *Caſe* IX. of R. *A. S. T.*

As the Sine of the Angle R *Or* S, 38 d. 30 m.
Is to the Radius:
So is the Sine of the Side R S, 27 d. 15 m.
To the Sine of the Side S R, (the Star's *Amplitude* at his *Riſing*) 47 d. 22 m.

3. For the *Angle* R S *Or*. By *Caſe* VIII. of R. *A. S. T.*

As the Co-ſine of the Side R S, 27 d. 15 m.
To the Radius:
So is the Sine of the Angle R *Or* S, 38 d. 30 m.
To the Sine of the Angle R S *Or*, 61 d. 41 m.

By which the *Oriental Angle* may be found.

	D.	M.
The *Right Aſcenſion* of *Andromeda*'s Head was found by (*Pr.* 27.) to be	357	47
And his *Declination* given *North*	27	15
His *Aſcenſional Difference* (by the (1.) hereof) is found to be	40	21
Which ſubſtracted from the Star's *Right Aſcenſion*, gives the *Oblique Aſcenſion* to be	317	26
And the *Declination* added to the *Right Aſcenſion*, gives	398	08
From which ſubſtract the whole Circle	360	00
There will remain for the *Oblique Deſcenſion*	38	08
To which add	90	00
And it gives the *Semidiurnal Arch* to be	128	08

	H.	M.
Which turned into Time, is	8	32
Which doubled, is	17	04

And ſo long the *Star* continues above the *Horizon* of *London*.
And that ſubſtracted from 24 h. leaves 6 h. 56 m. for the Star's continuance below the *Horizon* of *London*.

By the (2.) hereof, the *Amplitude* of the Star's Riſing from the *Eaſt* Northward, is found to be	47	22
And by the (3.) hereof, the *Angle* at S,	61	41

Which { Subſtracted in Signs Aſcending, from / Added in Signs Deſcending, to } the *Angle* of the *Ecliptick* and *Meridian*, gives the *Oriental Angle*.

Fig. XXXVI.

PROB. XXX.

The Declination *of the* Sun, *and the* Length *of a Day being given, to find the* Latitude *or* Height *of the* Pole.

I. *By the* Cœleſtial Globe.

BRing the *Solſtitial Colure* to the *Meridian*, and count the *Sun's Declination* upon it, either *North* or *South*, (ſuppoſe 23 d. 30 m. *North*) then let the *Sun's Declination* be 23 d. 30 m. *North*, and the Length of the Day 16 h. 26 m.

Bring the *Æquinoctial Colure* to the *Meridian*, and upon it count 23 d. 30 m. *North*, and there fix the Body of the *Globe*. Then, upon the Parallel of 23 d. 30 m. of *Declination*, count 8 h. 13 m. the half Length of the Day, and there make a Mark. Then move the *Braſs Meridian* up and down in the Frame, till the Mark you made do juſtly touch the *Horizon*; for then, the Degrees of the *Braſs Meridian*, comprehended between the *Horizon* and the *Pole*, will be the *Latitude* ſought, *viz.* 31 d. 30 m.

II. *By* Trigonometrical Calculation.

This being (in effect) but the Converſe of the foregoing *Problem*, the *Spherical Triangle* will be the ſame upon the *Globe* as in that; namely, The *Triangle* R *Or* S: In which there is given, (beſides the *Right Angle* at R) (1.) The *Side* R S, the *Sun's Declination*, 23 d. 30 m. *North*. (2.) The *Side* R *Or*, the *Aſcenſional Difference*, which the *Sun* hath when the Length of a Day given is either *Greater* or *Leſs* than 12 Hours.

1. Let the *Sun's Declination* be 23 d. 30 m.
2. The longeſt Day at *London*, 16 h. 26 m.
3. The *Aſcenſianal Difference*, 2 h. 13 m. which is half the Time by which the longeſt Day exceeds 12 h. *viz.* 4 h. 26 m. the half whereof, 2 h. 13 m. turned into Degrees, is 33 d. 52 m.

III. *The Canons for Calculation.* By *Caſe* XIII. of *R. A. S. T.*

As the Tangent of the Side R S, (the *Sun's Declination* given) 23 d. 30 m.

Is to the Radius:

So is the Sine of the Side R *Or*, (the *Aſcenſional Difference*) 33 d. 10 m.

To the Tangent of 51 d. 30 m. the *Latitude* ſought. Whoſe Complement, 38 d. 30 m. is the Angle R, *Or* S.

This *Problem* is of good uſe in *Geography*, where the Length of the *Longeſt Day*, of every *Clime* is given, to find what *Elevation* of the *Pole* anſwers thereunto. *Fig.* XXXVI.

PROB. XXXI.

The Latitude *of a* Place, *or Elevation of the* Pole, *and the* Oblique Aſcenſion *of a* Star *being given: To find the* Coſmical, Achronical *and* Heliacal Riſing *of that* Star.

THE *Coſmical Riſing* or *Setting* of a *Star*, is that which happens in the *Morning*, but the *Achronical* in the *Evening*; the one commencing at the *Sun's Riſing*, the other at his *Setting*. The *Heliacal Riſing* is, when a *Star* begins to get from under the *Sun Beams*; the *Setting*, when it begins to go under them.

But for the better ſeeing of the *Stars*, there muſt be a certain Depreſſion of the *Sun* below the *Horizon*, which is called The *Arch of Viſion*; for the Light of a *Star* is *Greater* or *Leſs*. In *Ptolomy*'s Opinion, *Venus* ſhould have an *Arch* of 5 Degrees, tho' it is viſible even in the Day-time, at a very great Diſtance.

	D.	M.
Jupiter and *Mercury*	10	00
Saturn	11	00
Mars	11	30
The Fixed Stars of the Firſt Magnitude	12	00
The Fixed Stars of the Second Magnitude	13	00
The Fixed Stars of the Third Magnitude	14	00
The Fixed Stars of the Fourth Magnitude	15	00
The Fixed Stars of the Fifth Magnitude	16	00
The Fixed Stars of the Sixth Magnitude	17	00
The Fixed Stars of the Nubilus Magnitude	18	00

But in this and the following *Problem*, the Situation of a *Star* is given in reſpect of the *Horizon* and *Æquator* together; and his Situation is ſought for in reſpect of the *Sun*'s being in, or near, the *Horizon*, and in the *Ecliptick*.

When therefore the *Oblique Aſcenſion* of a *Star* falls in the		Quadrant of the *Ecliptick*, as in	
	Firſt		Fig. XXXVII.
	Second		Fig. XXXVIII.
	Third		Fig. XXXIX.
	Fourth		Fig. XL.

I. *By the* Cœleſtial Globe.

Fig. XXXVII. XXXVIII. XXXIX. XL.

1. For the *Coſmical Riſing* and *Setting.*

Rectifie the Globe to your Latitude, and bring the Place of the Sun (ſuppoſe 11 d. of ♒) to the *Eaſt* Part of the *Horizon:* Then look what *Stars* are about the Verge of the *Eaſtern Semicircle* of the *Horizon*; for all thoſe *Stars* do at that time *Riſe Coſmically.* And thoſe *Stars* which touch, or are near the Rim of the *Weſt Semicircle* of the *Horizon*, do *Set* at that time *Coſmically.* So ſhall you find

May 27.	*Aldebaran* or the *Bull's Eye*, with divers other ſmaller *Stars*,	Riſing, and	The Right Leg of *Serpentarius*, and ſeveral other ſmaller *Stars*,	Setting *Coſmically.*

2. For the *Acronical Riſing* and *Setting.*

Rectifie the Globe to the *Latitude*, bringing the Place of the *Sun*, ſuppoſe 5 d. of *Scorpio* to the Weſt Part of the *Horizon*, then ſhall all thoſe *Stars* which you ſee on the Verge of the Eaſt Side of the *Horizon*, be Riſing *Acronically.* And all thoſe that are about the Verge of the Weſtern Part of the *Horizon*, are then Setting *Acronically.* And ſo upon the forementioned Time, you ſhall find

May 27.	A *Star* in the *Whale's Tail*, and ſeveral other ſmaller *Stars*	Riſing, and	The *Tail* of the *Lion*, the South Ballance, and ſeveral other ſmaller *Stars*	Setting *Acronically.*

3. For the *Heliacal Riſing* and *Setting.*

Rectifie the *Globe* to the *Latitude*, and the *Quadrant of Altitude* in the *Zenith*; then bring the given *Star* (ſuppoſe *Regulus*, or the *Lion*'s *Heart*) to the *Eaſt Side* of the *Horizon*, and the *Quadrant of Altitude* to the *Weſt Side*; then *Regulus* being a *Star* of the *Firſt Magnitude* (by the Rule of the Ancients) may be ſeen when the *Sun* is but 12 d. below the *Horizon*; wherefore, ſee what Degree of the *Ecliptick* doth cut the *Quadrant of Altitude* in 12 d. which you will find to be 9 d. of *Piſces*; the oppoſite Degree to which is 9 d. of *Virgo.* To which Sign and Degree, when the Sun cometh (which will be about the 23th of *Auguſt*) then will *Regulus*, or the *Lion*'s *Heart*, *Riſe Heliacally.*

Then, for his *Heliacal Setting*, bring the *Star* to the *Weſt Side* of the *Horizon*, and turn the *Quadrant* of *Altitude* to the *Eaſt Side*, and ſee what Degree of the *Ecliptick* is elevated upon the *Quadrant*, as the *Magnitude* of the *Star* you are to deal with doth require. For when the *Sun* comes to the oppoſite Degree of the *Ecliptick*, that *Star* ſhall *Set Heliacally*. So,

Fig. XXXVII. XXXVIII. XXXIX. XL.

The	*Pleiades*	ſhall *Riſe Heliacally* upon	*June* 4.	And *Sets Heliacally* upon	*April* 20.
	Aldebaran		*June* 26.		*April* 22.
	Arcturus		*Sept.* 26.		*Novem.* 19.

And ſo for others, as in the following *Table*.

A Table ſhewing the Time of the Year when 50 Eminent Fixed Stars do Riſe, both Coſmically and Achronically.

Names of the Stars.	*Coſmical Riſing.*		*Achronical Riſing.*	
Marchab. Pegaſ.	*January*	1	*March*	9
Right Shoulder of *Aquarius*		6	*Febr.*	14
Extream Star in the Wing of *Pegaſus*		28		19
Following Tail of the Goat	*February*	5	*Janu.*	26
Bright Star in the Ram's Head	*March*	2	*April*	20
The following Horn of the Ram		5		15
The former Horn of the Ram	*April*	10		30
North Tail of the Whale		12	*March*	19
Brighteſt of the Pleiades		22	*May*	10
The Knot in the Net of *Piſces*	*May*	1	*April*	25
North Horn of the Bull		15	*June*	7
North Eye of the Bull		20	*May*	12
In the Belly of the Whale		23	*March*	16
The Bull's Eye, *Aldebaran*		27	*May*	11
North Horn of the Bull	*June*	5	*June*	17
Lower Head of *Gemini*		21	*July*	20
Bright Foot of *Gemini*		25	*June*	5
Middle Star in *Orion*'s Girdle	*July*	9	*May*	4
North Aſſellus		12	*July*	25
Preſepe		14		19
South Aſſellus		17		17
Leſſer Dog, *Porcyon*		18	*June*	6
Great Dog, *Palilicium*		30	*May*	3
Lion's Heart	*Auguſt*	8	*Auguſt*	9

Fig.
XXXVII.
XXXVIII.
XXXIX.
XL.

Names of the Stars.	*Coſmical Riſing.*		*Achronical Riſing.*	
Lion's Back	*Auguſt*	10	*Octob.*	19
Hydra's Heart		20	*June*	16
Lion's Tail		22	*Octob.*	16
Hare's Thigh	*Septem.*	9	*April*	14
Vindemiatrix		13	*Nov.*	8
Arcturus		15	*Decem.*	13
Virgin's Girdle		19	*Octob.*	18
Bright Star of the Crown		28	*Jan.*	7
Virgin's Spike	*Octob.*	3	*Sept.*	24
Right Shoulder of *Hercules*		6	*Jan.*	9
Left Shoulder of *Hercules*		10		21
Head of *Hercules*		21		8
North Ballance		21	*Nov.*	17
South Ballance		23	*Octob.*	24
Swan's Bill		31	*Febr.*	16
Right Shoulder of *Ophiucus*	*Novem.*	5	*January*	4
Left Knee of *Ophiucus*		6	*Decem.*	6
Lower Wing of the Swan		4	*March*	11
Vulture's Tail		11	*Jan.*	27
Right Knee of *Ophiucus*		16	*Decem.*	6
Scorpion's Heart		22	*Novem.*	4
The Eagle		25	*Decem.*	31
Pegaſus Scheat	*Decem.*	11	*March*	24
Andromeda's Girdle		22	*April*	29
Andromeda's Head		25		6
Upper Horn of the Goat		25	*Jan.*	14

II. *By* Trigonometrical Calculation.

The *Latitude* of *London* is 51 d. 30 m.

The *Oblique Aſcenſion* of the Head of *Andromeda* is 317 d. 26 m. and therefore in the Fourth *Quadrant* of the *Eclipick.* Wherefore in the *Oblique-angled Spherical Triangles* ♈ *Or.* A, or ♎ *Or.* A.

In

Fig. XXXVII. XXXVIII. XXXIX. XL.

In which there is Given
1. The Angle at ♈ or ♎, (the greateſt *Declination* of the *Sun*, or *Obliquity* of the *Ecliptick*, 23 d. 30 m.
2. The Angle at *Or.* (which is at the Interſection of the *Æquator*, and the *Horizon* of *London*) 38 d. 30 m.
3. The Side *Or.* ♈, or *Or.* ♎, (the Diſtance of the *Oblique Aſcenſion* from the neareſt *Æquinoctial Point*, 42 d. 34 m.

To find 1. The Side ♈ A, or ♎ A; for, A is that Point of the *Ecliptick*, which *Aſcends* with a Star.

When therefore the Sun is in that Point, a *Star Riſeth Coſmically:* But when the Sun is in that Point of the *Ecliptick* that is oppoſite to A, a *Star Riſeth Achronically.*

To find 2. The *Oriental Angle* at the Point A, *viz.* the Angle ♈ A *Or.* or ♎ A *Or.*

Moreover, in the Triangle R A ☉, (beſides the Right Angle at R)

There is Given
1. The Side R ☉, the *Ark of Viſion*, agreeing with the Star, *viz.* 13 d. for the ſecond Magnitude.
2. The Angle R A ☉ (the ſame as ♈ A *Or.* or ♎ A *Or.* before found, 26 d. 50 m.

To find the Side A ☉, the Arch which muſt be added to the Point A, to find out the Point ☉; which, when the Sun is in, a *Star Riſes Heliacally.*

The Canons for Calculation.

1. For the Sides A *Or.* and *Or.* ♈. By *Caſe* III. of *O. A. S. T.*

The *Angles* Given are A *Or.* ♈, 141 d. 30 m. and *Or.* ♈ A, 23 d. 30 m. And the Given Side is *Or.* ♈ comprehended by them, 42 d. 34 m.

The *Sum* of the Given Angles is 164 d. Their *Difference* 118 d. The *half Sum* 82 d. The *half Difference* 59 d. And half the Given *Side* is 21 d. 17 m.

Then,

(1.) As the Sine of half the *Sum* of the Angles A *Or.* ♈, and *Or.* ♈ A, 82 d.
Is to the Sine of half the *Difference* of thoſe Angles:
So is the Tangent of half the Given *Side Or.* ♈, 21 d. 17 m.
To the Tangent of 18 d. 38 m. Which is half the *Difference* of the Two unknown *Sides Or.* ♈, and *Or.* A.

(2.) As

Fig. XXXVII. XXXVIII. XXXIX. XL.

(2.) As the Co-ſine of half the *Sum* of the Angles A *Or.* ♈, and *Or.* ♈ A. 82 d.
Is to the Co-ſine of half the *Difference* of thoſe Angles:
So is the Tangent of half the Given *Side Or.* ♈, 21 d. 17 m.
To the Tangent of 55 d. 19 m. Which is half the Sum of the Two unknown *Sides Or.* A, and *Or.* ♈. —The half Difference 18 d. 38 m. ſubſtracted from 55 d. 19 m. (the half Sum) leaves 36 d. 41 m. for the *Leſſer Side* A *Or.* —And the half Sum 55 d. 19 m. added to the half Difference, gives 73 d. 57 m. for the *Greater Side* A ♈, but (becauſe the Angle *Or.* oppoſite thereto, is above a *Quadrant*) add 90 d. to 73 d. 57 m. and the Sum 163 d. 57 m. is the Side A ♈.

2. For the Angle *Or.* A ♈. By *Caſe* II. of R. *A. S. T.*

(3.) As the Sine of the Side A *Or.* 36 d. 41 m.
Is to the Sine of *Or.* ♈ A, 23 d. 30 m.
So is the Sine of the Side *Or.* ♈, 42 d. 34 m.
To the Sine of the Angle *Or.* A ♈, 26 d. 50 m.

Then in the Triangle A R ☉, Right-angled at R, to find the Side A ☉; by *Caſe* IX. of *R. A. S. T.*

(4.) As the Sine of the Angle at A, 26 d. 50 m.
Is to the Sine of the Side R ☉ (the Angle of *Viſion*, 13 d. it being a Star of the Second Magnitude).
So is the Radius,
To the Sine of the Side A ☉, 29 d. 53 m. Which added to the Side A ♈, 163 d. 57 m. the Sum is 193 d. 50 m. from which ſubſtract 180 d. there will remain 13 d. 50 m.

Wherefore, the Head of *Andromeda* Riſeth *Coſmically* at *London*, with 13 d. 50 m. of ♑ *Capricorn*, in which Sign and Degree the Sun is about the 24th of *December*: And *Achronically* with 13 d. 50 m. of ♋ *Cancer*, in which Sign and Degree the Sun is about the 25th Day of *June*. —The *Oriental Point* of *Riſing* is at A, or the Angle ♈ A *Or.* 25 d. 50 m. And the Head of *Andromeda Riſeth Heliacally*, when the Sun is in 13 d. 50 m. of ♒ *Aquarius*, in which Sign the Sun is about the 22th of *February*, and *Sets Heliacally* when in the oppoſite Point, *viz.* 13 d. 50 m. of ♌ *Leo*, that is about the 26th of *Auguſt*.

PROB.

Fig. XLI. XLII. XLIII. XLIV.

PROB. XXXII.

The Elevation of a Pole, and the Oblique Deſcenſion of a Star given; To find the Coſmical, Acronical *and* Heliacal Setting *of that Star.*

I. *By the* Cœleſtial Globe.

THE Practical Performance of this and the following *Problems* is ſhewed before in *Prob.* XXXI. and needs not be here again recited. Wherefore,

II. *By* Trigonometrical Calculation.

When the Oblique Deſcenſion of a Star falls

In the	Firſt Second Third Fourth	Quadrant of the Ecliptick, as in	Figure XLI. Figure XLII. Figure XLIII. Figure XLIV.	Then,

By Trigonometrical Calculation.

In the Oblique-angled Triangle ♈ *Oc.* D, or ♎ *Oc.* D, there is given,

(1.) The Angle ♈ *Oc.* ♎, 23 d. 30 m.

(2.) The Angle *Oc.* the Angle of the *Æquator* and *Horizon* of *London*, 38 d. 28 m. its Comp. to 180.

(3.) ♈ *Oc.* or ♎ *Oc.* the Diſtance of the Oblique Deſcenſion, from the neareſt Part of the *Æquinoctial.*

To find, 1. The Side ♈ D or ♎ D.

For, D is the Point of the Ecliptick, which deſcends with the Star: When therefore the Sun is in that Point, where D is, a Star Sets *Achronically:* But when the Sun is in the Point of the Ecliptick, oppoſite thereto, the Star then Sets *Coſmically.*

2. The Angle ♈ D *Oc.* or ♎ D *Oc.*

Moreover, in the Triangle R D ⊙ (beſides the Right Angle at R) there is given,

(1.) The Side R ⊙, the Arch of Viſion.

(2.) The Angle R D ⊙, the ſame with the Angle ♈ D *Oc.* or ♎ D *Oc.* before found.

To find the Side D ⊙, the Arch which muſt be taken from the Point D in the Ecliptick, to get the Point ⊙ in the Ecliptick; to which Point, when the Sun comes, the Star Sets *Heliacally.*

EXAM-

EXAMPLE.

Fig. XLI. XLII. XLIII. XLIV. The Head of *Andromeda* (at *London*) ſets *Achronically* with 26 d. 46 m. of ♈: — *Coſmically*, with 26 d. 46 m. of ♎. —But, the Angle ♈ D *Oc.* is found to be 121 d. 32 m. and its Complement to 180 d. the Angle R D ☉, 58 d. 28 m. Wherefore, the Head of *Andromeda* ſets Heliacally, when the Sun is in 11 d. 31 m. ♈.

PROB. XXXIII.

The Latitude *of a Place, the Oblique Aſcenſion or Deſcenſion of a Star, and the* Longitude *of the Sun being given: To know whether a Star may be ſeen* Heliacally.

I. *By the* Cœleſtial Globe.

TO perform this by the Globe, ſee before *Prob.* XXXI.

II. *By* Trigonometrical Calculation.

This is but the Converſe of the Two foregoing *Problems*: And the Spherical Triangles upon the Globe will be the ſame; and therefore, in the Triangle R A ☉ or R D ☉

There is given, beſides the Right Angle at R,

1. The Oriental Angle at the Point A, or the Occidental at the Point D: The one being found in the Triangle ♈ *Or.* A, or ♎ *Or.* A, as in the *Problem*: The other in the Triangle ♈ *Oc.* D, or ♎ *Oc.* D, as in the *Problem.*
2. The Side A ☉, or D ☉; the Arch which is intercepted between the *Longitude* of the Sun, and the Aſcending or Deſcending Point of the Ecliptick.

To find out the Arch R ☉.

Which Arch, if it be greater, or equal, to the viſible Arch of the Given Star, that Star doth appear: But, if it be leſs, it lyes hid under the Sun's Beams.

[*Note.*] Sometimes the Oriental Angle is leſs than the Arch of Viſion; and then (even at Midnight) ſuch a Star cannot be ſeen. But, if you ſubſtract the Arch of Viſion from the greateſt Depreſſion of the *Æquator* under the *Horizon* (which is the Complement of the Latitude (or Elevation of the Pole at *London* 38 d. 30 m.) there remains the Northern Declination of a Degree of the Ecliptick, unto which the Sun muſt firſt come, before that Star will begin to appear at Midnight.

As

Fig. XLI. XLII. XLIII. XLIV.

As for Example. All Stars of the Fifth Magnitude, have an Arch of Viſion of 16 d. therefore ſubſtract 16 from 38 d. 30 m. the Depreſſion of the *Æquator*, and there will remain 22 d. 30 m. the Northern Declination of the Sun, *viz.* as much as is required that it be depreſſed 16 d. below the *Horizon*; that the Stars of the Fifth Magnitude may appear. But the Sun hath as great Declination in 13 d. 26 min. of *Gemini* ♊, and in 16 d. 33 m. of *Cancer* ♋: Therefore, whilſt the Sun goes through that Arch of the Ecliptick (which will be between the 24th of *May*, and the 28th of *June*) the Stars of the Fifth Magnitude (are not viſible at *London* (even at Midnight) becauſe the Twilight hinders.

Here needs no Canons for Calculation.

PROB. XXXIV.

The Elevation of the Pole, and the Longitude of the Sun: To find the Beginning of the Morning, and End of the Evening, Twilight.

I. *By the* Cœleſtial Globe.

IN this and the following *Problem*, the Situation of the Sun is given, in reſpect of the *Æquator*, and its Situation is ſought for in reſpect of the Meridian.

But, becauſe, at every Ceſſation of the Twilight, there is commonly required a Depreſſion of the Sun, of 18 d. below the *Horizon.*

Now, in the Latitude of 48 d. 30 m. the Sun being in the beginning of *Cancer* ♋, at Midnight, is depreſs'd below the *Horizon*, preciſely 18 d. Hence it follows, that in Places, having a greater Elevation of the Pole than 43 m. the Sun being in the *Tropick*, muſt (at Midnight) be depreſſed leſs than 18 d. from whence it follows, that Twilight will neither Begin nor End, but will laſt all Night. Therefore, to know when this *Problem* is in Uſe, ſubſtract 18 d. from the greateſt Depreſſion of the *Æquator*, (that is, at *London*, 38 d. 30 m.) and there remains the Northern Declination 20 d. 30 m. (as by the *Problem* converſed): So the Sun, when he hath 20 d. 30 m. of North Declination, which he will have when he is in 1 d. 20 m. of *Gemini* ♊, and in 28 d. 40 m. of *Cancer* ♋: Therefore, whilſt the Sun goes through that Arch of the Ecliptick (which is between the 12th Day of *May*, and the 11th Day of *July*) the Twilights laſts all Night at *London*, all which Time there is no Uſe of this *Problem* there.

At other Times of the Year it will be in Uſe, and may be reſolved as followeth.

II. *By* Trigonometrical Calculation.

Fig. XLV. Upon the *Globe*, the *Triangle* reſolving this *Problem*, is the *Oblique-angled Spherical Triangle* P N ⊙ in Fig. XLV. for the Declination of the *Sun*, Northern or Southern. In which there is

Given,
1. The Side P N, the Diſtance of the *Pole* of the *Æquator*, and the *Horizon* of *London*, 38 d. 30 m.
2. The Side N ⊙, the Complement of the Depreſſion of the Sun under the *Horizon*, which the Sun muſt have when the *Twilight Begins* or *Ends*, *viz.* 72 d.
3. The Side P ⊙, the Complement of the *Declination* of the Sun, which will be
   - Greater Than 90 Degrees, When in Northern Signs.
   - Leſſer Than 90 Degrees, When in Southern Signs.

Required, The Angle at P. Which muſt be turned into Time, the which, being

Counted from Midnight
- Shews the *Beginning* of the *Morning Twilight.*
- And ſubſtracted from 12 Hours, ſhews the *End* of the *Evening Twilight.*

The Canon for Calculation.

By *Caſe* XI. of *O. A. S. T.*

	D.	*M.*
The *Sum* of the Three Sides of the Triangle is	199	18
The *Half Sum* is 99 d. 24 m. *Or.*	89	36
The *Difference* between the *Half Sum* 99 d. 24 m. And the *Side* oppoſite to P is	27	24

Then ſay,

(1.) As the Radius,
To the Sine of P N, (one of the Sides containing the enquired Angle P) 38 d. 30 m.
So is to the Sine of P ⊙ (the other Side containing the enquired Angle P) 88 d. 48 m.
To the Sine of 38 d. 29 m.

(2.) As

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Fig.
LVI.
LVII.

Fig.
XLV.

(2.) As the Sine of 38 d. 29 m. laſt found. *Fig.*
Is to the Sine of the *Half Sum* of the *Sides*, 99 d. 14 m. (or XLV.
89 d. 36 m.)
So is the Sine of the *Difference*, 27 d. 24 m.
To the Sine of 47 d. 41 m.
To which Sine add the *Radius*, and half thereof will be the Sine of 59 d. 19 m. Whoſe Complement 30 d. 41 m. is *Half* the *Angle* at P.

Thus the Angle at P, being 61 d. 22 m. that reduced into *Time*, makes 4 h. 6 m. *fere*.

At which Time the *Twilight Begins* in the *Morning*. And that ſubſtracted from 12 h. leaves 7 h. 54 m. for the *Time* that *Twilight* will *End* in the *Evening*.

PROB. XXXV.

The Elevation of the Pole, *and the* Altitude *of a* Star; *whoſe* Right Aſcenſion *and* Declination *are both known: To find out the Moment of* Time, *and the* Azimuth *of the* Star *at that* Time.

I. *By the* Cœleſtial Globe.

REctifie the *Globe* to the *Latitude*, (ſuppoſe *London*) 51 d. 30 m. Bring the Place of the *Sun* (ſuppoſe 27 d. 54 m. of *Piſces*) to the *Meridian*, and the Hour-Index to 12, and the *Quadrant of Altitude* to the *Zenith*: Then turn the Body of the *Globe* about, till the *Star* (ſuppoſe *Caput Andromedæ*) meet with 5 d. 24 m. (the given *Altitude* thereof.) Then will the *Hour-Index* point at the Time required, *viz.* 3 h. 47 m. *Morn.*

II. *By* Trigonometrical Calculation.

The *Sun* at Noon is at the higheſt. Take away therefore the *Fig.*
Sun's *Right Aſcenſion*, from the *Right Aſcenſion* of the *Star*; and XLVI.
the Remainder being turned into *Time*, ſhews how many *Hours* XLVII.
Afternoon the *Star* is at the higheſt. Therefore,

When a Star is in the { Eaſtern / Weſtern } Parts, as in { Fig. XLVI. / Fig. XLVII.

Fig. XLVI. XLVII.

In the Triangle P Z S there is

Given,
1. The Side P Z, the Diſtance of the *Pole* of the *Æquator*, and the *Horizon* of *London*, 38 d. 30 m.
2. The Side Z S, the Complement of the *Star's Altitude*, 84 d. 36 m.
3. The Side P S, the Complement of the *Star's Declination*, 62 d. 42 m.

Required,
1. The Angle P, and muſt be turned into Time:
And In Fig. XLVI. / In Fig. XLVII. muſt be *Taken* from / *Added* to the *Time* of *Culmination*,
that the *Time* of *Obſervation* may appear.
2. The Angle at Z, the *Azimuth* of the *Star*, at the Time of *Obſervation*.

The Canon for Calculation. By *Caſe* XI. of *O. A. S. T.*

1. For the Angle at P.

(1.) As the Radius,
Is to the Sine of P Z, 38 d. 30 m. (the Complement of the *Latitude.*)
So is the Sine of Z S. (the Complement of the *Star's Altitude*, 84 d. 36 m.
To the Sine of 38 d. 18 m.

(2.) As the Sine of 38 d. 18 m.
Is to the Sine of the half Sum of the *Three Sides* of the *Triangle*, 92 d. 54 m. (or 87 d. 6 m.)
So is the Sine of the *Difference* (between the *Half Sum*, and the *Side* Z S, oppoſite to the Angle at P) 8 d. 18 m.
To the Sine of 61 d. 10 m. The Double whereof 122 d. 20 m. is the Quantity of the *Angle* at P. Which, turned into *Time*, gives 8 h. 10 m.

2. For the Angle at Z.

As the Co-ſine of the *Star's Altitude* Z S, 84 d. 36 m.
Is the Sine of the Angle at P, 122 d. 20 m. *Or*. (comp. to 180 d.) 57 d. 40 m.
So is the Sine of the Side S P, (the Comp. of the *Star's Declination*) 62 d. 42 m.
To the Sine of the Angle at Z, 48 d. 57 m. And that is the *Star's Azimuth* from the *North-Eaſtward.*

Example

Fig. XLVI. XLVII.

Example in *Caput Andromedæ.*

In the *Latitude* of 51 d. 30 m. the *Altitude* of the *Head* of *Andromeda*, was *Obſerved* in the *Eaſtern Part* to have 5 d. 24 m. of *Altitude* above the *Horizon*: The *Sun's Longitude* at that time being in 27 d. 54 m. of *Piſces* ♓. At which

	D.	M.
Time alſo the *Right Aſcenſion* of the *Sun* is	358	04
The *Right Aſcenſion* of *Cap. Andromedæ*, found as before	357	45
To which add 360 d. it makes	717	45
From which ſubſtract the *Sun*'s *Right Aſcenſion* 358 d. 4 m. there remains	359	41

Which turned into *Time*, gives 23 h. and 57 m. which is as many *Hours* as the *Star* is coming to the *Meridian* after the *Sun*.

Thus the Angle at P, being found to be 122 d. 20 m. which (the *Star* being in the *Eaſtern* Part) muſt be ſubſtracted from the Time of *Culmination*, 23 h. 57 m. and then there will remain 15 h. 47 m. for the Time of *Obſervation* Afternoon, which is 3 h. 47 m. in the Morning.

PROB. XXXVI.

The Time *of Obſervation, and* Latitude *of the Place, being given: To find out the* Altitude *and* Azimuth *of a* Star *given.*

I. *By the* Cœleſtial Globe.

REctifie the *Globe* to the *Latitude*, the *Quadrant of Altitude* to the *Zenith*, the Place of the *Sun* to the *Meridian*, and the *Index* to 12. Then turn the Body of the *Globe* about, till the *Index* points at the *Time* given, and there keep it: Then the *Quadrant of Altitude* laid over the *Star* will give you upon it the *Star's Altitude*; and when it cuts the *Horizon*, the *Azimuth* thereof alſo.

II. *By* Trigonometrical Calculation.

This is but the Converſe of the former *Problem*: And the Triangle made upon the *Globe* the ſame, *viz.* the Triangle Z P S, in Fig. XLVI, and XLVII. In which Triangle there is

Given,

Fig. XLVI. XLVII.

Given,
1. The *Side* Z P (the *Diſtance* of the *Poles* of the *Æquator*, and *Horizon* of *London*) 38 d. 30 m.
2. The *Side* P S, the Complement of the *Star's Declination (Cap. Andromeda)* 62 d. 42 m.
3. The *Angle* S P Z (the *Time* between the Time of *Obſervation*, and the Time of the *Stars* next *Culmination*, *viz.* 8 h. 10 m) which is 122 d. 20 m.

Required,
1. The *Side* Z S, (the *Complement* of the *Star's Obſerved Altitude*).
2. The *Angle* at Z (the *Azimuth* of the *Star*, from the *North* Part of the *Meridian*.)

The Canons for Calculation. By *Caſe* IX. and *Caſe* I. of *O. A. S. T.*

1. For the Angle at Z.

(1.) As the *Sine* of half the *Sum* of the *Sides* Z P and Z S, 50 d. 36 m.
Is to the *Sine* of their half *Difference*, 12 d. 6 m.
So is the Co-tangent of half the *Given Angle*, Z P S, 61 d. 10 m.
To the *Tangent* of 8 d. 30 m. the half *Difference* of the Two *Angles*, Z and S.

(2.) As the Co-ſine of half the *Sum* of the *Given Sides*, 50 d. 36 m.
Is to the Co-ſine of their *half Difference*, 12 d. 6 m.
So is the Co-tangent of half the given *Angle* Z, 61 d. 10 m.
To the Tangent of 40 d. 22 m. Which is the *half Sum* of the Two *Angles*.

		D. M.		D. M.		
Then 8 d. 30 m.	Added to	40 22	gives	48 52	For the Angle	Z.
	Subſtracted from			31 52		S.

So that the *Azimuth* of the *Star* is 48 d. 52 m. from the *North* Part of the *Meridian*.

2. For the Side Z S.

(3.) As the Sine of the *Angle* at Z, (laſt found) 48 d. 52 m.
Is to the Sine of the *Side* S P (the Complement of the Star's *Declination*) 62 d. 42 m.
So is the Sine of the *Angle* at P, 122 d. 20 m. (or 57 d. 40 m.)
To the Sine of 84 d. 36 m. for the *Side* Z S. Whoſe Complement, 5 d. 24 m. is the *Altitude* of the Star at the Time of Obſervation.

Aſtrono-

Aſtronomical Problems
Relating to
ASTROLOGY.

INTRODUCTION.

A*Strology* conſiſteth principally of Two Parts, *viz.* the one *Mathematical*, as is the *Aſtronomical Part*; the other *Judiciary*, as is the *Aſtrological Part.*

The *Mathematical Part* teacheth how in a *Scheme* or *Figure* (as they call it) to repreſent the *Face* of the Heavens in *Plano*, for any Hour of the Day or Night, at all Times of the Year, and in all Parts of the World.

The *Aſtrological Part* teacheth how (from the Sight of the ſaid Poſition of the *Scheme* or *Figure* of the Heavens at the Time of its Erection) to give a determinate *Judgment* of what was demanded upon that *Erection* of the *Scheme* or *Figure*; as of *Annual Revolutions*, *Elections*, the *Nativity* of a Perſon, &c.

The principal Authors that have given their Opinions concerning the dividing of the Heavens into 12 *Manſions* or *Houſes*, are, 1. *Ptolomy*, 2. *Alcabitius*, 3. *Campanus*, and 4. *Regiomontanus*: Which laſt Way is now generally received and practiced among the *Aſtrologers* of theſe Times, and by them termed the *Rational Way of* Regiomontanus.

Now, becauſe (as I ſaid before) that the Erection of a *Figure* of the Heavens is the *Mathematical Part* of *Aſtrology*, I ſhall therefore ſhew how by the *Globes* to erect a *Figure* of the Heavens according to the *Rational Way of* Regiomontanus.

PROB. I.

How to Erect a Figure *of the Heavens in the* Latitude *of* London, 51 *d.* 30 *m.* N. *for the* 10th *Day of* March, *at* 49 *m. after* 9 *in the* Forenoon; *at which Time the* Sun *entred into the firſt Scruple of* Aries, *in the Year* 1675.

THE Heavens are divided in XII *Houſes* or *Manſions*, by 12 *Semicircles* of *Poſition*; for which Purpoſe, to ſome *Globes*, there is made one of *Braſs*, which is fixed in the Interſections of the *Meridian* and *Horizon*, by the *Elevation* or *Depreſſion* whereof the Heavens may be divided into Parts or Houſes through each Degree of the *Ecliptick.*

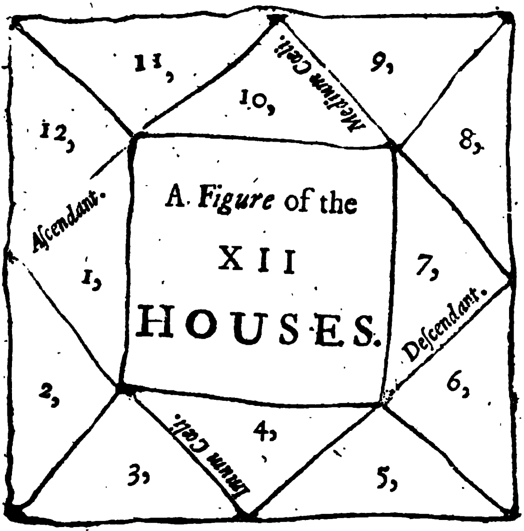
Of theſe XII *Houſes*, or *Manſions* of Heaven, Four are called *Cardinal*, as, (1.) The *Horoſcope*, or *Aſcendent*, or *Cuſpis* of the *Firſt Houſe.* (2.) The *Medium Cæli*, or Angle of the *South*, or *Cuſpis* of the *Tenth Houſe.* (3.) The *Deſcendent*, or *Angle* of the *Weſt*, or the *Cuſpis* of the *Seventh Houſe.* (4.) The *Imum Cæli*, or the *Angle* of the *North*, or the *Cuſpis* of the *Fourth Houſe.*

Regiomontanus divides the Heavens into XII *Houſes* according to his Way, by the *Circle* of *Poſitions* paſſing through every 30th Degree of the *Æquinoctial*, and cutting the *Ecliptick* at ſeveral Points, which are the *Cuſpiſes* of the ſeveral *Houſes*: —So that when the *Globe* is ſet to the *Latitude*, and the Hour-Wheel *Rectified* and brought to the Given Hour, you have the *Cuſpiſes* of the Four *Cardinal Houſes*: For,

The Degree of the *Ecliptick* cut by the	Eaſt Side of the *Horizon*,	Gives the *Cuſpis* of the	Firſt	Houſe.
	South Part of the *Meridian*,		Tenth	
	Weſt Side of the *Horizon*,		Seventh	
	North Part of the *Meridian*,		Fourth	

The *Cuſpiſes* of the other 8 Houſes are found by the Motion of the *Circle* of *Poſition*, as ſhall be ſhewed by and by.

The *Houſes* are *denominated* by 1, 2, 3, 4, *&c.* to 12, from the *Aſcendent* downwards to the *Imum Cæli*, up again to the *Deſcendent*, and again by *Medium Cæli*, down to the *Aſcendent.* As in the following *Scheme.*



t this ſuffice for *Definition*, and now we will come to the
ice by the *Globes*.

ſt, To the Day propoſed, the 10th of *March*, find (by the
Aſtronomical Problem) the Sun's Place in the *Ecliptick* at
n, which you ſhall find to be in 0 d. 5 m. of *Aries*.
condly, Set the *Globe* to the *Latitude* 51 d. 30 m.
irdly, Bring the *Sun's Place* at *Noon* (0 d. 5 m. of *Aries*) to
Meridian.
urthly, Turn the *Globe* about till the *Hour-Index* point to
lour given, *viz.* to 49 m. after 9 in the *Morning*.
ftly, The *Globe* being fixed in this *Poſition*, you ſhall find
he *Eaſt-Semicircle* of the *Horizon* doth cut the *Ecliptick* in
29 m. of *Cancer*, which is the *Sign* then *Aſcending*, and muſt
aced upon the *Cuſpis* of the *Firſt Houſe*.
ien caſt your Eye upon the Interſection of the *South* Part of
Meridian and the *Ecliptick*, and there you ſhall find the *E*-
ck cut by the *Meridian* in 25 d. of *Aquarius*, and that Point
e *Ecliptick* is then in the *Medium Cœli*, and muſt be ſet upon
uſpis of the *Tenth Houſe*.
ſo, you find that the *Weſt Semicircle* of the *Horizon*, cuts the
tick in 00 d. 29 m. of *Capricorn*; which Point is then *De*-
ing, and muſt be placed upon the *Cuſpis* of the *Seventh Houſe*.

Laſtly, You ſhall find that the *North* Part of the *Meridian* doth cut the *Ecliptick* in the 25th ⊙ d. of *Leo*, which Point is then upon the *Imum Cæli*, and muſt be placed upon the *Cuſpis* of the *Fourth Houſe*.

Thus have you found the Points of the *Ecliptick*, which do occupy the *Cuſpiſes* of the Four *Cardinal Houſes*: Now for the other *Eight Houſes*.

Let the *Globe* ſtill reſt in its former *Poſition*, and then,

Firſt, Bring the *Circle of Poſition* to its Place on the *Eaſt* Side of the *Horizon*; and being there fixed, raiſe it upwards towards the *Meridian*, till 30 d. of the *Æquinoctial* be intercepted between the *Horizon* and the *Circle of Poſition*; and then you ſhall find that the *Circle of Poſition* will interſect the *Ecliptick* in 20 d. of *Taurus*; which Degrees muſt be ſet upon the *Cuſpis* of the *Twelfth Houſe*.

Secondly, Move the *Circle of Poſition* yet higher towards the *Meridian*, till 30 d. more of the *Æquinoctial* be intercepted between it and the *Horizon*, (in all 60 d.) and when in doth ſo, you ſhall find the *Circle of Poſition* will cut the *Ecliptick* in 36 d. of *Piſces*; which Point muſt be ſet upon the *Cuſpis* of the *Eleventh Houſe*.

The *Meridian* gives the *Cuſpis* of the *Tenth Houſe* in 25 d. of *Aquarius*, as before.

Thirdly, Move the *Semicircle of Poſition* from the *Eaſt* Side of the *Horizon* to the *Weſt* Side, and move it downwards from the *Meridian*, till 30 d. of the *Æquinoctial* be intercepted between the *Meridian* and *Circle of Poſition*, and then you ſhall find the *Circle of Poſition* will interſect the *Ecliptick* in 7 d. of *Aquarius*; which Point muſt be ſet upon the *Cuſpis* of the *Ninth Houſe*.

Fourthly, Move the *Circle of Poſition* yet lower by 30 d. *i. e.* 60 d. from the *Meridian* downwards, and then you ſhall find the *Poſition-Circle* to cut the *Ecliptick* in 21 d. of *Capricorn*; which Point muſt be ſet upon the *Cuſpis* of the *Eighth Houſe*.

The *Deſcendant* or *Cuſpis* of the *Seventh Houſe*, is the Interſection of the *Weſt* Side of the *Horizon* and *Ecliptick*, which is in 00 d. 29 min. of *Capricorn*, as before.

And thus have you found the *Cuſpiſes* of the *Four Houſes* above the *Horizon*, beſide the *Aſcendant* and the *Medium Cæli*, *viz.* of the 12, 11, 9 and 8 Houſes. Now the *Cuſpiſes* of the Four other Houſes under the Earth have the ſame Degrees of the oppoſite Signs upon them. For,

		Houſe.			*Houſe.*
20 d. ♉	Being upon the *Cuſpis* of the	12	20 d. ♏	will be on the *Cuſpis* of the	6
26 d. ♓		11	26 d. ♍		5
7 d. ♒		9	7 d. ♌		3
21 d. ♑		8	21 d. ♌		2

For the Six Signs
Aries, *Taurus*, *Gemini*, *Cancer*, *Leo*, *Virgo*,
are oppoſite to
Libra, *Scorpio*, *Sagitt.* *Capric.* *Aquar.* *Piſces.*

And this is the manner how (by the *Globe*) to erect a *Figure* according to the (reputed) *Rational Way* of *Regiomontanus.*

Now if you would inſert the Places of the *Planets* into your *Figure*, (for it is them that the *Aſtrologer* principally giveth *Judgment* by) your beſt Way will be to have Recourſe to ſome good *Ephemeris*, or *Calculate* them from *Aſtronomical Tables*).

Having in the foregoing *Precepts* ſhewed how to erect a *Scheme* (or *Figure* of the *Heavens*) by the *Globe*, I ſhall now (to retain the ſame Method throughout this whole *Treatiſe*) ſhew how to effect the ſame *Trigonometrically*; the which I ſhall do in the *Three* following *PROBLEMS.*

PROB. I.

The Time *of the* Day, *and* Latitude *of the* Place *given: How to Erect a* Scheme *or* Figure *of the* Heavens *for that* Time *and* Place.

By Trigonometrical Calculation.

YOU muſt firſt find (either by *Tables* already Calculated, or by the foregoing *Aſtronomical Problems*) the true *Place* of the *Sun* in *Longitude*; and alſo, his *Right Aſcenſion*, both for the *Moment* of *Time* given. *Fig.* XLVIII.

Then turn the *Time* given, (counted from the preceding Noon) into *Degrees* and *Minutes* of the *Æquator*; and add them (being ſo turned into *Degrees* and *Minutes*) to the *Right Aſcenſion* of the *Sun* before-found; the *Agregate* (caſting away 360 d. if need

Fig. XLVIII. need be) is the *Right Aſcenſion* of the *Mid-Heaven:* To which you muſt find what Degree of the *Ecliptick* doth anſwer; for that will be the *Sign* of the Xth *Houſe*.

Then, To the *Right Aſcenſion* of the *Mid-Heaven*, add 30 d. then 60 d. again 90 d. and 120 d. and 150 d. And thus you will find the *Oblique Aſcenſion* of the XIth, XIIth, and Iſt *Houſe*; or of the IId and IIId *Horoſcope*.

By the Erection of the *Circle* of *Poſition* above the *Horizon* at every 30 Degrees, there are made ſeveral *Spherical Triangles*, as H Æ 11, *&c.*

In which Triangle (beſides the Right Angle at Æ,) there is

Given,
1. The Side H Æ, (the Elevation of the *Æquinoctial* above the *Horizon* of *London*) 38 d. 30 m.
2. The Side Æ 11, 30 d. (and Æ 12, 60 d. *&c.*

To find, The Angle Æ 11 H, — Æ 12 H, *&c.* By *Caſe* XIII. of *R. A. S. T.*

As the Tangent of Æ H, 38 d. 30 m.
Is to the Radius:
So is the Sine of Æ 11, 30 d.
To the Tangent of 32 d. 9 m.
Whoſe Complement 57 d. 51 m. is the Quantity of the Angle H 11 Æ.

So will the Angle	Be found to be D.	M.	to which the Angle Is equal.
H 11 Æ	57	51	Q 3 O
H 12 Æ	42	32	Q 2 O
H 1 Æ	7	10	Q 1 O
H 2 Æ	42	32	Q 12 O
H 3 Æ	57	51	Q 11 O

Theſe Angles being once found will be always in Uſe in the *Latitude* of 51 d. 30 m. For that they ſhew the Quantity of the Angle, that the *Ecliptick* makes with every *Circle* of *Poſition*.

Moreover,

In the Triangle 11 ♈ X, — 12 ♈ Y, *&c.* there is

Given,
1. The Angle X 11 ♈, which is a contiguous Angle to the before-found Angle H 11 Æ.
2. The Side ♈ 11, the Complement of the *Oblique Aſcenſion* of the XIth *Houſe*, to a whole Circle, or 360 d.
3. The Angle 11 ♈ X, 23 d. 30 m.

To find the Side ♈ X, or the Diſtance of the *Sign* of the XIth Houſe from the *Vernal Æquinox.* By *Caſe* IV. of *R. A. S. T.* Fig. XLVIII.

But it will be ſufficient to count the *Signs* of the *Oriental Houſes:* For thoſe which are oppoſite to them, (as the IVth to the Xth, — the Vth to the IXth, *&c.*) are counted as many *Degrees* and *Minutes* as the others; but of an *Oppoſite Sign.*

Example, *Let it be required to* Erect *a* Scheme *of the* Heavens, *(in the* Latitude *of* London, 51 *d.* 30 *m.) for the 6th Day of* March, *the 16 h.* 5 *m. before* Noon.

The *Place* of the *Sun* then was in 27 d. 54 m. ♓.

The *Sun's Right Aſcenſion*, 358 d. 4 m.

The *Time* 16 h. 5 m. turned into Degrees, gives 241 d: 15 m. Which added to the *Right Aſcenſion* of the *Sun*, make 599 d. 19 m. from which ſubſtract the Circle (360 d.) there remains 239 d. 19 m. for the *Right Aſcenſion* of the *Mid-Heaven.* To which anſwer 1 d. 27 m. of ♐, which is the *Sign* of the Xth *Houſe.* And from thence I collect the *Oblique Aſcenſion* of the reſt of the *Houſes* as followeth, *viz.*

						D.	*M.*	
The *Oblique Aſcenſion* of the	XI	Houſe to be	269	19	The *Sign* thereof is	15	15	♐
	XII		299	19		1	09	♑
	I		329	19		29	27	♑
	II		359	19		28	46	♓
	III		29	29		11	45	♉

But becauſe the IVth *Houſe* is oppoſite to the Xth *Houſe*, it will have as many Degrees and Minutes upon the *Cuſpis* thereof, but of the oppoſite *Sign*, *viz.* ♊, and the Vth the ſame as the XIth, *viz.* 15 d. 15 m. of the oppoſite *Sign*, ♊. The VIth 1 d. 00 m. ♋. The VIIth 29 d. 27 m. of ♋. The VIIIth 28 d. 46 m. of ♍. The IXth 11 d. 45 m. of ♏. As may be ſeen in the *Erected* Fig. XLIX.

PROB. II.

How to Direct *any* Point *of an* Erected Scheme *to any other* Point.

1. THE *Point* that is *Directed* is called, otherwiſe, the *Firſt Place*, or *Significator:* But that to which we direct it, the *Second Place*, or the *Promiſſor:* But the *Significators* are directed whither they tend: *Directed*, that is, *Going* according to the *Series* of the *Signs*; but *Retrograde* in *Antecedens*, or againſt the *Succeſſion* of the *Signs.* Fig. XLIX.

2. Let the *Poſition* made by any direct *Significator*, or by the *Promiſſor* to which the *Retrograde* is *Directed*, be called the *Horizon* of a *Star:* And it is either a right one, paſſing through both the *Poles*, as the *Meridian*; or an *Oblique* one, having One of the *Poles* elevated.

3. The *Oblique* one is, ſometimes, the ſame as the *Horizon* of the *Place*; and conſequently, the *Elevation* of the *Pole* is the ſame: But, when it is *different* (which moſt commonly happens) you muſt firſt enquire how much One of the *Poles* is *Elevated* above the *Horizon* of the *Star:* But if a Star be

Fig. L. I. II. III. IV.

Oriental,	With Declination,	*North*,	As in	Fig. I.
		South,		Fig. II.
		North,		Fig. III.
Occidental,		*South*,		Fig. IV.

In the Triangle P O S, or P H S, for the Situation of the *Star* above or below: There is

Given,
1. The Side P O, or P H, the *Elevation* of the *Pole* at *London*, 51 d. 30 m.
2. The Side P S, the Complement of the *Star's Declination.*
3. The Angle S P O, comprehended between them (whoſe *Meaſure* is the *Arch* of the *Æquator*, which is intercepted between the Sides P O (P H) P S: One of which being produced, ſhews the *Right Aſcenſion* of the *Star*; the other, the *Right Aſcenſion* of *Imum Cœli.*

Required, The Angle P O S, or P H S. By *Caſe* III. of *R. A. S. T.*

Again, In the Triangle R P O, or R P H, (which are in different *Hemiſpheres*, when O or H (before found) are greater than a *Quadrant* or 90 d.)

There is Given, beſides the Right Angle at R,
1. P O or P H, the *Elevation* of the *Pole.*
2. The Angle O or H, before found, (which if it exceeds a *Quadrant*, is Complement to 180 d.

To find R P, The *Elevation* of the *Pole* above the *Horizon* of the *Star*. By *Caſe* III. of *R. A. S. T.*

4. Having found the *Elevation* (by the Third beforegoing) then look

when the *Horizon* hath any Star that is {Oriental, the Aſc. / Occidental, the / Deſcenſion} {Obl.} *Fig.* I. II. III. IV. both of the *Significator* and *Promiſſor*.

5. Having found out the {*Significators Direct*, / *Promiſſor*, to whom the / *Retrograde* is *Directed*,} *Aſcenſion* or *Deſcenſion* for the *Horizon* of the *Star*: Let the *Right* one, or the *Oblique* one, be ſubſtracted from the like *Aſcenſion* or *Deſcenſion* of the other *Star* (adding 360 d. if Need be) the *Remainder* is the *Ark* of *Direction* ſought for.

PROB. III.

The Ark of Direction *of any* Significator *being given; To find how far it will reach.*

THIS is but the Converſe of the foregoing: Therefore look for the *Elevation* of the *Pole* above the *Horizon* of a *Direct Significator*, as before; and at that *Elevation* you will get (by the 3d.) the Oblique {*Aſcenſion* / *Deſcenſion*} of the ſame *Significator* {*Oriental*, / *Occidental*,} to which {*Aſcenſion* / *Deſcenſion*} add the given *Ark* of *Direction*, ſo will you get the *Oblique* {*Aſcenſion.* / *Deſcenſion.*} To which find what Degree of the *Ecliptick* anſwers (being counted from the *Elevation* of the *Pole* above the *Horizon* of the *Significator*) which is the very *Place* to which the *Ark* is *Directed*, and to which the *Significator* will come.

But ſubſtract the *Ark* of *Direction* given, from the *Right Aſcenſion* of the *Retrograde Significator*, and there will remain the Degree of the *Æquator*, which is found out by the *Retrograde Significator*, by *Direction*, tending to the *Promiſſor*: The *Circle* of *Poſition* paſſing through this *Degree* (which ought well to be obſerved) will be the *Horizon* of the *Promiſſor*: Above which, find the *Elevation* of the *Pole*, as before: For, if that Degree fall on the {*North*, Fig. I. / *South*, Fig. II.} the {Eaſtern / Weſtern} Part of Heaven, the *Significator* inclining unto the {*North*, Fig. III. / *South*, Fig. IV.} in the Triangle POS, or PHS.

For

Fig. L. I. II. III. IV. For the Situation of the Degree obſerved, whether Above or Below.

From the given Angles, as above, (the Angle P being counted in the *Æquator*, not from the *Right Aſcenſion* of the *Significator*, but from that Degree which hath, oftentimes, been taken notice of, as that at the *Medium* or *Imum Cœli*) I ſeek for the Angle O or H.

And moreover, in the other *Triangle*, R P, the very *Elevation* of the *Pole* above the *Horizon* of the *Promiſſor*: At this *Elevation* of the *Significator*, if you obſerve that Degree $\left.\begin{array}{l}\textit{Oriental}\\ \textit{Occidental}\end{array}\right\}$ you will gain the *Oblique* $\left\{\begin{array}{l}\textit{Aſcenſion.}\\ \textit{Deſcenſion.}\end{array}\right\}$ From which take the given *Ark* of *Direction*, and the *Remainder* will be the *Oblique* $\left\{\begin{array}{l}\textit{Aſcenſion,}\\ \textit{Deſcenſion,}\end{array}\right\}$ to which the Degree of the *Ecliptick* anſwers, (but it muſt be counted from the *Elevation* of the *Pole* above the *Horizon* of the *Promiſſor*) and is the very *Place*, to which the *Significator* will come, by this *Ark* of *Direction* given.

The End of the Aſtronomical Problems.

Fig. L.
I. II.
III. IV.

ANCILLA MATHEMATICA.

VEL,

Trigonometria Practica.

SECTION V.

OF

SCIOGRAPHIA,

OR

DIALLING.

CHAP. I.

Of Dialling *in general.*

OF the *Mathematical Sciences*, I know none more Ingenious and Uſeful then This of the Deſcription of *Sun-Dials* upon all ſorts of *Plains* howſoever ſituate: Neither is there any thing that draws more Admiration from all knowing Men; then to ſee ſtreight Lines drawn upon a *Plain* at *Unequal Diſtances* to meaſure out exactly the *Equal Diviſions* of the *Time* of an *Artificial Day*: For, although the Sun appears in different places of Heaven, according to the different *Seaſons* of the *Year*; yet, the ſame *Streight Lines*, do ſtill determine the ſame *Hour*, at all thoſe different *Seaſons*. And as this *Science* is of ſuch

Excellency and *Uſe*, I ſhall in this *Section*, briefly ſhew (and that by an *Artifice* not uſually practiced) how ſuch *Hour-Lines* may be deſcribed upon all *Plains* in any part of the World: And that by *Trigonometrical Calculation*, as well as by projecting the *Circles* of the *Globe* upon a *Plain*.

Now, All *Plains* have a particular reſpect to ſome *Point*, in the *Latitude* of that Place in which they ſtand: Wherefore, ſome particular *Place* muſt be aſſigned for *Exemplifying* the ſame, in all the Varieties. We will therefore take *London*, the *Metropolis* of *England* (which is ſituate in 51 deg. 30 m. of *North Latitude*) for our *Examples* following.

CHAP. II.

Of the Diverſity of Plains, *upon which* Hour-lines *may be deſcribed.*

ALL *Plains* upon which *Sun-Dials* may be made, in any *Latitude* or Place of the World, are ſituate either

Parallel
Perpendicular } To the *Horizon* of that
Oblique

Place (or *Country*) in which they are made. And, of theſe ſeverally.

I. A *Plain* that lies *Parallel* (or *Level*) to the *Horizon*, Is called an *Horizontal Plain*, for the Latitude of that *Place* in which it is to ſtand.

II. Of *Plains* that are *Perpendicular*, or *Erect*, to the *Horizon*, there are ſeveral *Varieties*. For, of ſuch *Plains*, if the *Face* thereof doth directly behold the

Eaſt
Weſt } Point of the *Horizon* of the *Place*
North
South

wherein they ſtand, They are then called,

Eaſt
Weſt } *Erect, Direct* Plains.
North
South

But if ſuch *Erect Plains* do not behold the *Direct Eaſt*, *Weſt*, *North* or *South* Points of the *Horizon*, they then will behold either

The
South-Eaſt
South-Weſt
North-Eaſt
North-Weſt
And then they are called

North
or
South
Erect Plains; *Declining*
Eaſt
or
Weſt.

III. Of *Plains* that lie *Obliquely* to the *Horizon*: (ſuch as the *Roofs* of *Houſes*, the *Cooping* of *Walls*, &c.) are called *Reclining Plains*. And if ſuch *Plains* do behold the direct

Eaſt
Weſt
North
South
Points of the *Horizon*: They are then

called *Direct Eaſt*, *Weſt*, *North* or *South* Plains: *Reclining* from the *Zenith* of the Place in which ſuch Plain ſtands.

But if ſuch *Reclining Plain* doth not reſpect the true *Eaſt*, *Weſt*, *North*, or *South* Points, then they will lie open either to the

South-Eaſt
South-Weſt
North-Eaſt
North-Weſt
And then they are called

South
or
North
Reclining Plains: *Declining*
Eaſt
or
Weſt.

And theſe are all the *Varieties* of *Plains*, upon which *Hour-Lines* may be deſcribed.

Only note,] That all *Reclining Plains* whatſoever; whether *Direct* or *Declining*, have *Two Faces*; the *Upper-Face*, which beholds the *Zenith* of the Place, is called the *Reclining Plain*: And the *Under-Face*, which beholds the *Horizon* of the Place, is called the *Incliner*: And one and the ſame *Dial* ſerves for both Places.

CHAP. III.

How to find the Situation *of any* Plain, *in reſpect of* Declination *and* Reclination, *in any Latitude.*

I. To find the *Reclination.*

Fig. LI. *Definition.* THE Quantity of the *Reclination* of a *Plain*, is the *Arch* of that *Vertical* or *Azimuth* Circle which is perpendicular to the *Reclining Plain*, comprehended between the Zenith of the Place and the Plain.

To find which, let A B C D repreſent ſuch a *Reclining Plain*: Draw, firſt, thereon, by help of a Ruler and Quadrant, a Right Line G H, parallel to the *Horizon* of the place; which ſhall be the *Horizontal Line* of the Plain. Croſs this Line G H, with another Right Line K S at Right-Angles to it; which Line K S ſhall be the *Vertical Line* of the Plain.

To this Line K S, apply a ſtreight Ruler K L: And to that end of it which lyeth clear of the Plain, as at L; apply a *Quadrant* O E P, having a Thrid and Plummet hanging from the Centre. Then ſee what number of *Degrees* of the *Quadrant* are contained between O and E; for ſo much doth that *Plain Recline* from the *Zenith* of the Place; and is the *Reclination* of the Plain.

II. To find the *Declination.*

Definition. THE *Declination* of a *Plain*, Is an Arch of the *Horizon* comprehended between the *Pole* of the *Plain*, and the *Meridian* of the *Place.*—Or, It is the diſtance of the *Plain* it ſelf, from the *Prime Vertical Circle*, or *Azimuth* of *Eaſt* or *Weſt.*

To find out the *Declination* of any *Plain*, there are required two *Obſervations* to be made by the *Sun*, both at the ſame *Time*, as near as may be.—The *Firſt*, Of the *Horizontal Diſtance* of the *Sun*, from the *Pole* of the *Plain.*—And the *Second*, Of the *Sun's Altitude.*

I. To find the *Horizontal Diſtance.*

Apply one Edge of the *Quadrant* to the *Horizontal Line* of the *Plain*, ſo that the other may be *Perpendicular* to it; and let the *Limb* of the *Quadrant* be towards the *Sun*. The *Quadrant* thus applied to the *Plain*, and held *Level* (as near as you can conjecture) hold up a *Thrid* and *Plummet* at full Liberty, near the *Limb* of the *Quadrant*; ſo that the *Shadow* of the *Thrid* may paſs through the *Centre* and *Limb* of the *Quadrant*: And then obſerve the Degrees cut by the *Shadow* of the *Thrid* in the *Limb* of the *Quadrant*; and number them from that ſide of the *Quadrant*, that ſtandeth *Square* or *Perpendicular* to the *Plain*: For thoſe *Degrees* are the *Horizontal Diſtance* ſought for.

II. To find the Sun's *Altitude*.

Hold up a *Quadrant* with both your Hands, turning the *Left ſide* of your *Body* to the *Sun*; then raiſe up, or depreſs the *Quadrant*, ſo held, till the *Sun* ſhining through the *Hole* in that *Sight* which is neareſt the *Centre*, do caſt its Beam of *Light* upon the *Hole* in the other *Sight* fartheſt from the *Centre*: And at ſuch time mark exactly what *Degrees* of the *Quadrant* are cut by the *Thrid*, for thoſe Degrees are the *Sun's Altitude* at that time.

The *Horizontal Diſtance*, and the *Sun's Altitude* thus *Obſerved*, at the ſame inſtant (as near may be) will help you to the *Plain's Declination*; by the *Rules* following. For,

I. By having the Sun's *Altitude*; you may find the Sun's *Azimuth*, as in PROBL. IX. of the Uſe of the *Cœleſtial Globe* in *Aſtronomy*: And by CASE XI. of O. *A. S. T.* Then,

II. When you make your *Obſervation* of the *Horizontal Diſtance*; Mark whither the *Shadow* of the *Thrid* did fall between the *South*, and that *Side* of the *Quadrant* which was *Perpendicular* to the *Plain*.

Firſt. If the *Shadow fall between* them: Then the Sun's *Azimuth* from the *South*, and the *Horizontal Diſtance* added together, do give the *Declination* of the *Plain*: And (in this Caſe) the *Declination* is unto the *ſame Coaſt* with the *Sun's Azimuth*; that is, *Eaſtward*, if the *Obſervation* were made in the *Forenoon*; or *Weſtward*, if in the *Afternoon*.

Secondly. If the *Shadow* fall *Not between* them:

Then, The *Difference* between the *Sun's Azimuth* and *Horizontal Diſtance* is the *Declination* of the *Plain*: And in this Caſe)——If the *Azimuth* be the *Greater* of the two, then the *Plain Declines* to the *ſame Coaſt* whereon the *Sun* is; But if the *H. Diſtance* be the *Greater*; then the *Plain Declines* the *contrary Coaſt*.

Fig. LI. And here *Note*] That the *Declination* thus found, is always accounted from the *South*; and that all *Declinations* are counted from either *North* or *South*, towards either *Eaſt* or *Weſt*: And muſt never exceed 90 Degrees.

I. If therefore, The Degrees of *Declination* do exceed 90 deg. you muſt take the reſidue of that Number to 180 deg. and that ſhall be the *Plain's Declination* from the *North*.

II. If the *Degrees of Declination* exceed 180; then the *Exceſs* above 180, gives the *Plain's Declination* from the *North*, towards *that Coaſt*, which is *Contrary* to the *Coaſt* whereon the *Sun* was, at the time of *Obſervation*.

CHAP IV.

How Hour-Lines *may be deſcribed upon an* Horizontal Plain *in any* Latitude; viz. *of* London, 51 *d.* 30 *m.*

I. By the Globe.

ELevate the *Globe* to the *Latitude* of the place for which you would make your *Dial*, (ſuppoſe for *London*, in the *Latitude* of 51 deg. 30 min.) Then bring the *Vernal Equinoctial Colure* (which is the firſt point of *Aries* alſo) to the *Meridian*, and (if you will) the *Index* of the *Hour-Circle* to 12. This done,

1. Turn the *Globe* about *Weſtward*, till the *Hour-Index* points at 1 a Clock, or rather [till 15 degrees of the *Equinoctial* come to be juſt under the *Meridian*] and there keeping the *Globe*, look upon the *Horizon* how many degrees thereof are cut by the *Equinoctial Colure*; which you ſhall find to be 11 deg. 30 min. which ſet down in a little Table, as you ſee here is done; for this 11 deg. 50 min. is the diſtance that the *Hour-lines of* 11 and 1 a clock are diſtant from the *Meridian* upon the Dial Plain.

Latitude		d. m. 51. 30
		d. m.
12		00.00
11	1	11.50
10	2	24.20
9	3	38. 3
8	4	53.35
7	5	71. 6
6		90. 0

2. Turn the *Globe* more *Weſtward*, till 30 degrees of the *Equinoctial* comes to the *Meridian*, and then ſee what degrees of the *Horizon* are cut by the *Equinoctial Colure*; which you will find to be 24 deg. 20 min. which note down in a Table as before, for that is the *Hour-diſtance* of 10 and 2 a clock from the *Meridian*.

3. Turn the *Globe* ſtill more *Weſtward*, till 45 degrees of the *Equinoctial* come to the *Meridian*; and then ſhall the *Equinoctial Colure* cut 38 deg. 3 min. of the *Horizon* counted from the *Meridian*; which is the diſtance of 9 and 3 a clock.

Do thus with the other hours of 8 and 4, of 7 and 5, and ſo ſhall the *Colure* cut 90 degrees at 6 a clock, or when 90 degrees of the *Equinoctial* comes to the *Meridian*. And this being done, your Dial is ſo far made as the *Globe* can aſſiſt you.

II. *The Geometrical Conſtruction of this* Dial, *in order to the Trigonometrical Calculation.*

1. With 60 Degrees of a *Scale of Chords*, deſcribe a Circle repreſenting your *Dial-plain*, and *Horizon* of the *Place*, (viz. *London.*) Croſs it with 2 Diameters S N, repreſenting the *Meridian* of the *Globe*, and *Hour-line* of XII; and the Line E W, for the *Hour-line* of VI: then will Z repreſent the *Zenith* of the *Place*, and be the Centre of the Dial. *Fig.* LII.

2. The *Latitude* of the Place being 51 d. 30 m. Set them from S to *a*, and from W to *b*: Then a Ruler laid from W to *a*, will croſs the *Meridian* S N in P, the *Pole* of the *World*: And laid from E to *b*, it will croſs the *Meridian* N S in Æ, the interſection of the *Meridian* and *Equinoctial*. And now you have three Points, *viz.* W, Æ, E; through which you may deſcribe the *Æquinoctial* Circle W Æ E, whoſe *Centre* will be at *c*, always in ſome part of the Line N S, extended, if need be.

3. Divide the Semicirle W N E, into 12 equal parts, at the points ⊙ ⊙ ⊙, &c. And laying a Ruler to Z, and every of thoſe points ⊙ ⊙ ⊙, it will croſs the *Æquinoctial* Circle W Æ E, in the points * * *, &c. dividing that Semicircle into 12 unequal parts.

4. A Ruler laid to P, the *Pole* of the World, and to the ſeveral points * * *, &c. it will cut the Circle of your *Plain* in the points 1, 2, 3, &c. on the *Eaſt-ſide* of N, and 11, 10, 9, &c. on the *Weſt-ſide* of N.

Laſtly. If you lay a Ruler to the Centre Z, and the reſpective points 1, 2, 3, &c. and 11, 10, 9, &c. they ſhall be the true *Hour-lines* belonging to an *Horizontal Dial* for the *Latitude* of 51 de. 30 m. And their reſpective *diſtances* from N will be the ſame as in the *Table* they were found to be by the *Globe*.

III. By

Fig. LII.

III. *By Trigonometrical Calculation.*

In theſe Horizontal Dials, there is nothing to be found by *Calculation Trigonometrical*, but the *Hour-diſtances* upon the *Plain* from the *Meridian*; for which, This is

The Canon for Calculation.

As the Sine of 90 de.
  Is to the Sine of the *Latitude* P N, 51 de. 30 m.
So is the *Tangent* of 15 d. (the *Æquinoctial Diſtance* of *One* hour; of 30 d. for *Two* hours; of 45 d. for *Three* hours, *&c.*)
To the *Tangent* of 11 d. 50 m. for 11 and 1—— of 24 d. 20 m. for 10 and 2—— for 38 d. 3 min. for 9 and 3, *&c.* as in the Table, before found by the Globe.

So have you all the Hour-lines between 6 in the morning and 6 at night; and for the Hour-lines of 4 and 5 in the morning, and of 7 and 8 at night, draw the ſame Lines before and after 6, through the Centre, as in the Figure, and they ſhall be the true Hour-lines: And ſo is your *Dial* finiſhed.

The *Stile* muſt ſtand upright at 12 of the clock, not inclining on either ſide.

And in this manner may you deſcribe Hour-lines upon an Horizontal Plain in any Latitude.

CHAP. V.

How to deſcribe Hour-Lines *upon an* Erect direct South Plain, *in the* Latitude *of* London, 51 *deg.* 30 *m.*

AN *Erect Direct South Dial*, in any Latitude, is no other than an *Horizontal Dial* in that Latitude, which is equal to the Complement of that *Latitude* in which it is an *Erect Direct South Dial*: So that an *Erect Direct South Dial* in the Latitude of 51 d. 30 m. will be the ſame as an *Horizontal Dial* in the *Latitude* of 38 de. 30 m. which is the Complement of 51 de. 30 m.——So that the making of ſuch a *Dial*, both by the *Globe* and by *Tri. Calculation* is the ſame with the other, only inſtead of 51 de. 30 m. *Latitude* you ſet your Globe to 38 d. 30 m. and ſo in the Calculation alſo: But in theſe *Dials* there needs no Hour-

Lines to be drawn through the Centre; for that the Sun never Shines upon them before 6 in the *Morning*, nor after 6 at *Night*. *Fig.* LIII.

The *Stile* of theſe *Dials* muſt ſtand upon the *Hour-Line* of 12, and muſt point downwards towards the *South Pole*. As in Figure LIII.

CHAP. VI.

To make an Erect Direct North Dial, *in the Latitude of* London 51 *de.* 30 *m.*

THE *North Erect Direct Dial*, is the ſame with the *South*, only the *Stile* muſt point upwards towards the *North-Pole*, and the hours about Midnight, as 9, 10, 11, 12, at Night; and 1, 2 and 3 in the Morning muſt be left out, and 4 and 5 in the Morning; and 7 and 8 at Night muſt be drawn through the Centre: So is your *North-Dial* alſo finiſhed, as in Fig. LIV. *Fig.* LIV.

CHAP. VII.

To make an Erect Direct Eaſt *or* Weſt Dial *in the Latitude of* London, 51 *de.* 30 *m.*

I. *By the* Globe.

THE *Globe* rectified to the *Latitude*, the *Index* to 12, the *Quadrant of Altitude* in the *Zenith*: If you turn the *Quadrant of Altitude* ſo about till the graduated edge thereof do behold the direct *Eaſt* or *Weſt*-points of the *Horizon*, you ſhall find that it will lie in the very Plain of the *Meridian-Circle*, and ſo the *Pole* will have no elevation over it; for turning the *Globe* about, the *Equinoctial Colure* will not cut the *Quadrant of Altitude* in any particular degree, but it will cut all the degrees thereof at the ſame time; wherefore the Hour-lines of theſe Plains will make no *Angles* at the *Pole*, and therefore muſt be parallel one to the other, which the *Globe* evidently demonſtrates, but will not conveniently give the parallel diſtance of each from other, they being *Fig.* LV.

Fig. LV. nearer or farther off each other according as the *Stile* is proportioned to the Plain, which I ſhall now come to ſhew.

For the Reaſons aforeſaid, there is no *Trigonometrical Calculation* required in the making of theſe *Dials*; and therefore, I ſhall proceed to

II. *The Geometrical Conſtruction of theſe* Dials.

Let the Plain upon which you would make an *Eaſt or Weſt Dial*, be A B C D.

1. Upon D (or any where towards the lower part of the Line B D, for an *Eaſt Dial*, or of A C for a *Weſt*) with 60 degrees of your *Chord*, deſcribe an Arch F G, upon which ſet the Complement of the *Latitude* of the place, *viz.* 38 deg. 30 min. from F to G, and draw the Line D G E for the *Equinoctial*.

2. Towards the upper part of this Line, as at P, aſſume any point, and through it draw the Line 6 P 6 perpendicular to the *Equinoctial*, for the Hour-line of Six.——Alſo, towards the lower part of the ſame Line, aſſume another point as L, and through it draw the Line 11 L 11 perpendicular alſo to the Æquinoctial for the Hour-line of Eleven.

3. With 60 degrees of your *Chords*, upon the point L, deſcribe a ſmall Arch of a Circle, as H K, and upon it (always) ſet 15 degrees (or one hours diſtance) from H to K, and draw the Line L K M, cutting the Hour-line of Six in M.

4. Upon M as a Centre, with 60 degrees of your *Chord*, deſcribe an Arch of a Circle N O, which divide into five equal parts in the points ⊙ ⊙ ⊙ ⊙.

5. Lay a Ruler upon M, and each of theſe points ⊙ ⊙ ⊙ ⊙ and the Ruler will cut the *Equinoctial-line* E D in the points ****, through which points, if you draw Right Lines parallel to the Hour-line of 6, they ſhall be the Hour-lines of 7, 8, 9, and 10 of the Clock, the Hour-lines of 6 and 11 being drawn before.

6. For the Hour-lines of 4 and 5 in the Morning, before 6, they retain the ſame diſtance from 6, as do the hours of 7 and 8; and thus is your *Dial* finiſhed.

The *Stile* muſt ſtand upon the Hour-line of 6, and be elevated ſo high as is the length of the Line M P, and may either be a Pin of Wyre, or a Plate of Braſs or Iron.

The *Weſt Dial* is the ſame with the *Eaſt*, only changing the names of the Hours.

For

Fig. LV.

Fig. LVI.

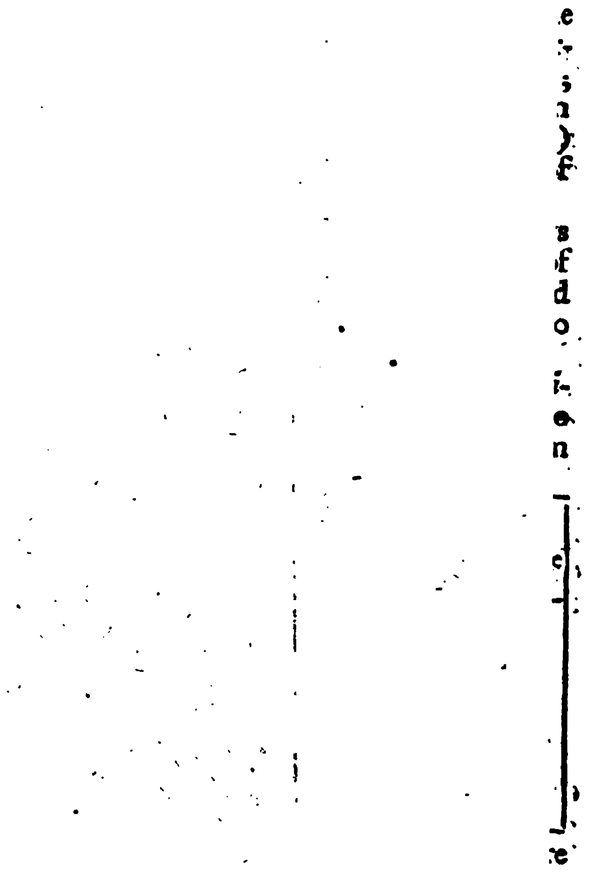


Fig. LV.

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For Fig. LV.

4, 5, 6, 7, 8, 9, 10, 11 in the Morning, in the *Eaſt-Dial.*
Muſt be changed to
8, 7, 6, 5, 4, 3, 2, 1 in the Afternoon in the *Weſt Dial*:
Which is all the difference.

CHAP. VIII.

To make an Erect Dial, *declining from the* South, Eaſtward, *or* Weſt-ward; 30 *degrees in the* Latitude *of* 51 *deg.* 30 *min.*

1. *By the* Globe.

Fig. LVI. THE *Globe* being Rectified to the *Latitude* of the place, the *Quadrant of Altitude* in the *Zenith*, the *Index* of the Hour-Circle at 12, and the *Equinoctial Colure* brought under the *Meridian*;

1. Count the Declination of the Plain upon the *Horizon*, from the *Eaſt or Weſt*-points thereof (according as the Plain declines) towards the *South*: namely, 30 degrees; and to that point of the *Horizon* bring the *Quadrant of Altitude*, and there keep it.

2. Turn the *Globe* about till the *Index* of the Hour-wheel cuts 11 of the Clock, or rather (as I ſaid before) till 15 degrees of the *Equinoctial* have paſſed the *Meridian*, and then ſhall you find the *Equinoctial Colure* to cut the *Quadrant of Altitude* at 9 deg. 50 min. if you count the degrees from the *Zenith point* downwards.

3. Turn the *Globe* farther about, till 30 degrees of the *Equinoctial* be paſt the *Meridian*, and then ſhall yon find the *Colure* to cut the *Quadrant of Altitude* at 18 deg. 14 min. couuted from the *Zenith* downwards as before.

4. Do the like with all the reſt of the Hours, and you ſhall find that at the ſeveral 15 degrees of the *Equinoctial*, the *Equinoctial Colure* will cut ſuch degrees of the *Quadrant of Altitude* as are expreſſed in this Table, if you count them from the *Zenith* downwards, as is before directed.

Hours from Noon.		Hour-diſtances upon the Plain.	
		d.	*m.*
12		00	00
11	1	09	50
10	2	18	14
9	3	26	19
8	4	34	56
7	5	44	56
6	6	57	49
5	7	75	37

This done;

5. Bring the *Quadrant of Altitude* to the other ſide of the *Meridian*, and ſet it to 30 degrees, the Plains declination, counted from the *Eaſt* or *Weſt*-points *Northward*, as you did

*Fig.*LVI before towards the *South*, which will be in the juſt oppoſite point of the *Horizon* to which it was before; and alſo, bring the *Equinoctial Colure* under the *Meridian*. Then,

6. Turn the *Globe* about (the contrary way to what you did before) till 15 degrees of the *Equinoctial* be paſt the *Meridian*, and then ſhall you find the *Equinoctial Colure* to cut at 12 deg. 23. min of the *Quadrant of Altitude* counted from the *Zenith*.

And ſo continuing turning the *Globe* about till 30, 45, and 60 degrees of the *Equinoctial* have paſſed the *Meridian*, you ſhall find the *Equinoctial Colure* to cut the *Quadrant of Altitude* at ſuch degrees as are expreſſed in this Table.

Hours from Noon.		Hour-diſtances on the Plain.	
		d.	*m.*
12		00	00
1	11	12	23
2	10	29	19
3	9	52	42
4	8	80	07

The Hour-diſtances upon the Plain being thus attained, there are two other requiſites in all upright declining *Dials* alſo to be found by the *Globe*, before the *Dial* can be finiſhed. Namely,

1. The diſtance of the *Sub-ſtile* from the *Meridian*.
2. The height of the *Pole* above the *Plain*, or the height of the *Stile* above the *Sub-ſtile*.

To find both which,

Bring the *Equinoctial Colure* to the Plains declination 30 degrees counted upon the *Horizon* from the *South-Eaſtward*; and the *Quadrant* of *Altitude* to 30 degrees counted in the *Horizon* from the *Eaſt-Northward*: So ſhall the *Quadrant* cut the *Colure* at *Right Angles*. And

The number of degrees of the *Quadrant* contained between this *Interſection* and the *Zenith* (which here is 21 deg. 41 min.) is the diſtance of the *Sub-ſtile* from the *Meridian*. And the degrees of the *Colure* contained between this *Interſection* and the *Pole* (which here is 32 deg. 37 min.) is the height of the *Pole* above the Plain.

II. The *Geometrical projection* of this *Dial*, in order to the *Trigonometrical Calculation* of the *Hour diſtances* and other *Requiſites* belonging to ſuch an *Erect Declining Plains*.

1. Upon the Point Q, as a Centre, with 60 de. of a *Scale* of *Chords*, deſcribe a Circle, repreſenting your *Dial Plain*: And croſs it with two Diameters Z Q N for the *Vertical*; and H Q O for the *Horizontal Line* of the *Plain*.

2. Set

Fig. LVI.

2. Set 30 deg. the *Plains Declination*, from N to *c*, if the Plain Decline *Eaſtward*, as in this Example; or from N to *e*, if *Weſtward*: Then lay a Ruler from Z to *c*, and it will cut the *Horizontal Line* of the *Plain* in K, ſo have you three Points Z, K and N, by which to deſcribe the Arch Z K N, repreſenting the *Meridian* of the *Place*. And to find the *Pole* thereof, ſet 90 de. from *c*, to *d*, and then a Ruler laid from Z to *d*, will cut the Horizontal Line H O, in W, which is the Pole of the *Meridian Circle* Z K N: and the Weſt-point of the Horiz.

3. Set 51 deg. 30 m. the *Latitude* of the Place, from O to *a*, and from N to *b*: Then a Ruler laid from W to *a*, will croſs the *Meridian* in P, the *Pole* of the *World*: And laid from W to *b*, it will croſs the *Meridian* in Æ, the point through which the *Æquinoctial Circle* is to paſs: And now you have two points W and Æ, through which the Æquinoctial Circle æ Æ æ may be deſcribed (by the XXI *Geometrical Problem*, Lib. 1.)

4. Through P, the *Pole* of the *World*, and Q, the *Pole* of the *Plain*, draw the right Line P Q, for the *Axis* of the *World*, and *Sub-ſtilar Line* of your *Dial*: And in this Line (extended) will the *Center* of the *Æquinoctial Circle* æ Æ æ be found.

5. From P, the *Pole* of the *World*, lay a Ruler to Æ, the interſection of the *Meridian* and *Æquinoctial*, and it will cut the *Plain* B: At this point B, begin to divide the *Semicircle* H N O of the Plain, into 12 equal Parts, at the points ⊙ ⊙ ⊙, *&c.*

6. From Q, (the *Pole* of the *Plain*) lay a Ruler to every of the points ⊙ ⊙ ⊙, *&c.* and it will croſs the *Æquinoctial Circle* æ Æ æ, in the the Points ***, *&c.*

7. Lay a Ruler to P (the *Pole* of the *World*) and every of the points ***, *&c.* and it will cut the *Primitive Circle* repreſenting the *Dial Plain*, in the points N, 9, 10, 11, *&c.* on the *Weſt* ſide, and N, 1, 2, 3, *&c*, on the *Eaſt* ſide of the *Meridian*.

8. Laſtly, Lines drawn from the Centre Q through theſe points, ſhall be the true *Hour-lines* of an *Erect Plain Declining* from the *South-Eaſtward* 30 de. in the *Latitude* 51 de. 30 m. And now,

Concerning the other *Requiſites* belonging to this *Erect Declining Plain.*

Theſe are all of them repreſented to the Eye in the *Scheme* of the *Projection* of the *Plain*: Wherein, by the interſection of the *ſeveral Circles* there is conſtituted a *Right-angle Spherical Triangle* Z T P, in which,

The

Fig. LVI.

The ſide
Z T is the *diſtance* of the *Sub-ſtile* from the *Meridian.*
Z P the Complement of the *Latitude* of the Place.
T P the *height* of the *Pole* above the *Plain.*

The Angle
P Z T, the Complement of the *Plain's Declination.*
Z P T, the *Plain's difference* of *Longitude.*
Z T P, is a Right Angle.

And all theſe *Sides* and *Angles* may be meaſured upon the Projection it ſelf, by the *Precepts* deliver'd in the firſt *Book*, *Sect.* III. of *Spherical Trigonometry Geometrically performed* by *Projection.* And they will be found to be as here expreſſed, *viz.*

Side	*de.*	*m.*	Angle	*de.*	*m.*
Z T	21	49	P Z T	60	00
Z P	38	30	Z P T	36	25
T P	32	36	Z T P	90	00

And now I ſhall ſhew how all theſe may be found

By Trigonometrical Calculation.

Before any *Erect Declining Dial* can be made, there are *Three things* which muſt be found, beſides the *Hour-diſtances*, and thoſe are theſe:

1. The height of the *Pole* (or *Stile*) above the *Plain.*
2. The *diſtance* of the *Subſtile* from the *Meridian.*
3. The *Plain's difference* of *Longitude.*

All which may be found in the *Triangle* Z T P: In which there is given (beſides the Right Angle at T,)

(1.) The *Angle* T Z P, the *Complement* of the *Plain's Declination*, 60 deg.

(2.) The *Side* Z P, the Complement of the *Latitude* of the Place 38 de. 30 m.

By which may be found,

I. The *Side* T P: The *Height* of the *Pole*, or *Stile*, above the *Plain*:
By CASE I. of R. A. S. T.

As the Radius, S. 90 deg.
Is to the Side Z P, 38 d. 30 m. the Co-Latitude.
So is the Sine of T Z P: the Co-declination 60 de.
To the Sine of T P, 32 deg. 26 m.
Which is the *height* of the *Pole* (or *Stile*) above the *Plain.*

II. The

The *Side* Z T, The *Diſtance* of the *Sub-ſtile* from the *Meridian*: By CASE II of R. S. T. *Fig.* LVI.

As the Radius Sine 90 de.
to the Tangent of Z P, the Co-Latitude 38 d. 30 m.
So is the Co-ſine of T Z P, 30 deg.
o the Tangent of Z T, 21 de. 40 m.
Which is the *Diſtance* of the *Sub-ſtile* from the *Meridian*.

I. The *Angle* Z P T, The *Plains difference of Longitude*: By Caſe III. of *R. A. S. T.*

As the Radius,
to the Tangent of the Declination T Z P, 60 deg.
So is the Co-ſine of Z P, the Co-Latit. 51 de. 30 m.
o the Co-Tangent of T P Z, 35 de. 25 m.
Which is the *Plain's difference* of *Longitude.*

Theſe three Requiſites being thus found by Trigonometrical lculation; The Plain's Difference of Longitude 36 de. 25 m. lling between 30 and 45 deg. (which are the Second and Third quinoctial Hour-diſtances) there will be contained therein two mpleat Hours, and 6 deg. 25 m. over: which ſhews that the b-ſtilar Line of the Dial will fall between the Hour-Lines of and X in the Morning (in this Eaſt Dial) but between the our of II and III in the Afternoon if the Plain had Declined eſtward:

Having proceeded thus far. Prepare a Table of Hours fit for e Plain, ſuch as is here done.

A Ta-

Fig. LVI.

A Table of the Hour Diſtances, for a South-Dial, Declining Eaſt or Weſt, 30 deg. In the Latitude of 51 deg. 30 min.

	d.	m.
Stiles height	32.	36
Diſtance of the Sub-ſtile and Meridian.	21.	40
Difference of Long.	36.	25

Hours from the		*Equinoctial Hour Diſtances.*		*True Ho. Diſtances upon the Plain fro. the Subſt.*	
Eaſt	Weſt	D.	M.	D.	M.
IV	VIII	83	35	78	12
V	VII	68	35	53	57
VI	VI	53	35	36	08
VII	V	38	35	23	16
VIII	IV	23	35	13	14
IX	III	8	35	4	36
			Sub-	ſtile.	
X	II	6	25	3	28
XI	I	21	25	11	56
XII	XII	36	25	21	40
I	XI	51	25	34	03
II	X	66	25	51	00
III	IX	31	25	74	21

Then againſt XII, ſet the *Plain's Difference of Longitude* 26 de. 25 min. (in the ſecond Column) and from it ſubſtract 15 deg. and there will remain 21 d. 25 m. which ſet againſt XI. and I: And from 21 d. 25 m. ſubſtract 15 de. and there will remain 6 d. 25 m. which ſet againſt X and II. And (becauſe it is leſs than 15 de.) write the Word Sub-ſtile over it, and ſubſtract it from 15 deg. and there will remain 8 deg. 35 m. which write over Sub-ſtile, and againſt the Hours of IX and III. Then to theſe 8 d. 35 m. add 15 deg. and and it makes 23 de. 25 m. which ſet againſt VIII and IV: And thus, by the continual Addition of 15 deg. you ſhall have ſuch *Æquinoctial Diſtances* as in the Table: Which Table, thus prepared, the next thing will be

To find the true *Hour-diſtances* upon the *Plain*, from the *Sub-ſtile*. For which, This is

The Canon for Calculation.

As the *Radius*

To the Sine of the *Stiles height* 32 d. 36 m.

So is the Tangent of the *Æquinoctial Diſtance* 6 d 25 m.

To the Tangent of 3 deg. 28 min. Which is the *Diſtance* of the *Hour-lines* of X and II. upon the *Plain* from the Sub-ſtile.

And ſo will the Tangent of the next *Æquinoctial Diſtance* 21 d. 25 m. be, to the Tangent of 11 de. 56 min. for the *Diſtance* of the *Hour-lines* of XI and I, from the Sub-ſtile: And ſo for all the reſt, as in the Table. And ſo you will find them to be in the Figure alſo.

Fig. LVII. And in the making of this *Dial*, you have made four; as in *Fig.* LVII.

For, if you hold the Paper upon which the *South-Eaſt declining Dial* is drawn, againſt the Light, then ſhall you diſcover the *Stile* to ſtand on the *Right-hand* of the *Plain*; whereas it now ſtands on the *Left-hand*; ſo the ſame *Hour-lines*, *Sub-ſtile*, *Stile* and all, being drawn on the back-ſide of the Paper, and thoſe that are the *Forenoon-hours* in the *Eaſt-decliner* numbred as the *Afternoon-hours* in the *Weſt-decliner*; that is, call 11, 1, and 10, 2, and 9, 3, *&c.* as in the Tables; ſo ſhall the *South-Dial* declining *Eaſt* 30 degrees, become a *South-Dial* declining *Weſtward* 30 degrees. Fig. LVII.

And if you turn the *South-Eaſt-Dyal* upſide-down, ſo that the *Stile* may point upwards towards the *North-Pole*, (and leave out the Hours about 12, as 9, 10, 11, and 1, 2, and 3, which in *North Dials* repreſent 9, 10, and 11 at Night, and 1, 2, and 3 in the Morning; all which time (in thoſe middle *Latitudes*) the Sun is under the *Horizon*) it will become a *North-Dial* declining *Eaſtward* 30 degrees.

Alſo if you turn the *South* declining *Weſt-Dial* upſide-down, and leave out the hours about *Midnight*, as 9, 10, 11, 12, 1, 2, and 3, it will then become a *North-Dial* declining *Weſtward* 30 degrees.

Now for ſuch *South* or *North Dials* as do decline far towards *Eaſt* or *Weſt*, as 60, 70, 80, or 85 degrees, there you ſhall find that the Hour-diſtances will fall ſo near together, that they will be of no competent diſtance one from another, except they be extended very far from the Centre; and therefore the old way hath been (in ſuch Caſes) to draw the *Dial* upon the Floor of a Room, extending the *Sub-ſtile*, *Stile* and *Hour-lines* till they appear of a competent diſtance from each other, and then according to the bigneſs of your *Dial-plain*, to cut off the *Hour-lines*, *Stile* and *Sub-ſtile*; and ſo transfer them from the Floor to the *Plain* upon which the *Dial* is to be made: but this way being to Mechanical for an Artiſt to exerciſe, I ſhall therefore here inſert a more artificial way of performing this work *Geometrically*, by which (although the *Dial* ſhould decline 80 or 88 degrees) upon a quarter of a ſheet of Paper you may draw your *Dial*, and have the *Stile* of a competent height, and all the Hour-lines at a convenient diſtance one from another. And ſo let this ſuffice to be ſaid in this place concerning *Upright declining Dials*; for I intend not here to teach the *Art of Dialling*, but ſhew the *Uſe* of the *Globes*: and from thence to Calculate the Requiſites from them Trigonometrically.

Fig. LVII.

CHAP. IX.

Concerning ſuch Erect South *or* North Dial Plains, *which decline many degrees towards the* Eaſt *and* Weſt.

I. *By the* Globe.

Fig. LVIII. THE Operation by the *Globe* is altogether the ſame, as in the laſt *Chapter*, and therefore need not be here again repeated. Wherefore, I will proceed to

II. The *Trigonometrical* Calculation.

This Work alſo is to be performed in like manner as the Former: And therefore I ſhall not again repeat it, but give an *Example* ready wrought.

Suppoſe an *Erect Plain* in the *Latitude* of *London*, 51 de. 30 m. ſhould decline from the *South* towards the *Eaſt* 85 deg.

I. For finding the *Requiſite*, if you work according to thoſe *Canons* for *Calculation*, in the laſt *Chapter* delivered; you will find

			de.	m.
The	*Height* of the *Pole* (or *Stile*) above the *Plain*	to be	3	06
	Diſtance of the *Sub-ſtile* and *Meridian*		39	22
	Plain's Difference of *Longitude*		86	05

Hours *Morn.*	*Afł.*	*Æqui-noctial Diſtanc.* D.	M.	*Hour Diſtances on the Plain* D.	M.
XII		86	05	38	23
XI	I	71	05	8	58
X	II	56	05	4	36
IX	III	41	05	2	42
VIII	IV	26	05	1	31
VII	V	11	05	0	36
The Place	of	the	Sub-ſtile		
VI		3	55	0	13
V	VII	18	55	1	04
IV	VIII	33	55	2	06

Theſe *Requiſites* thus found, you may proceed to the making of a *Table* for the *Hour-diſtances* in all reſpects, as in the laſt Chapter. By ſetting down the *Hours* proper for the *Dial* in order, as in this *Table*.

Then, againſt XII, ſet the *Difference of Longitude*, 86 d. 05 m. from which Subſtract 15 deg. and there will remain 71 d. 05 m. which ſet againſt XI and I. And from 71 d. 05 m. ſubſtract 15 de. the remainder will be 56 d. 05 m. which ſet againſt X and II. And ſo by the continual *Sub-ſtraction*

ſtraction of 15 deg. you ſhall find 11 de. 05 m. to ſtand againſt *Fig.*
the Hours of VII and V; under which wriſe *The place of the Sub-ſtile:* And then, ſubſtract 11 d. 05 m. from 15 de. there will remain 3 de. 55 m. which muſt be ſet under the *Sub-ſtile*, againſt the *Hour* of VI: And then, by the continual *Addition* of 15 deg. thereto; you ſhall have ſuch *Æquinoctiol diſtances* as are expreſſed in the Table, againſt the *Hours* of V and VII, and IV and VIII. Then LVIII.

II. For the *Five Hour diſtances upon the Plain:* If you work by the *Canon* delivered in this laſt Chapter, you will find them to be ſuch as are exhibited in the Third *Column* of this *Table.*

By this *Table*, you ſee that the *Five Hour Diſtances* upon the *Plain* about the *Sub-ſtile* (and indeed all the reſt, except the extreme Hour of XII) do fall ſo near together, that without a very large extention of them from the *Centre*, there will be no competent diſtance between *Hour-line* and *Hour-line*: Wherefore, laying aſide your *Table*, proceed to make your *Dial Geometrically*, according to the following Precepts.

III. The *Geometrical Projection*, of *this* (or the like) *Dial*: when the *Pole* hath but ſmall *Elevation* above the *Plain.*

1. Draw a Right Line A B, perpendicular by one ſide of your *Fig.*
Plain, and towards the *Right Hand*, becauſe the Plain declineth LIX.
Eaſtward, And with 60 deg. of a *Scale* of *Chords*, deſcribe an obſcure Arch of a Circle C D E; and upon it ſet 38 de. 23 m. the *Sub-ſtiles diſtance* from the *Meridian*, from C to D, and draw the Line A D, for the *Sub-ſtile*.

2. Take 3 deg. 6 m. the *Stiles height*; and ſet them from D to E, drawing the Line A E for the *Stile* of the *Dial*.

3. Now (becauſe the *Stile* is but of ſmall *Elevation*, *viz.* but 3 de. 6 m.) draw another Line G H, *parallel* to the Line of the *Stile* A E, at ſuch convenient diſtance as in your judgment will beſt fit the *Dial Plain*, ſo that the Hours belonging to it, may come within the Limits of the Plain: So ſhall that Line, G H, ſo drawn, be the *Augmented Stile* of the Dial.

And, by the *Sub-ſtile*, and this *Augmented Stile*, the *Hour-Lines* may be deſcribed (at convenient diſtance) without any regard had to the *Centre* of the *Dial*. For,

4. Aſſume any two points in the *Sub-ſtilar* Line of the *Dial* A D, at ſome convenient diſtance from each other, as R and S: and through thoſe two points, draw two infinite Right lines, both

Fig. LIX. of them at *Right Angles* to the *Sub-ſtilar Line* A D ; as the Lines Z R Z and X S X.

5. Set one foot of your Compaſſes in the Point R, and take the neareſt diſtance to the new *Augmented Stile* G H ; and ſet that diſtance upon the *Sub-ſtilar* Line from R to K.——Alſo from the Point S, take the neareſt diſtance to the *Augmented Stile* G H, and ſet that diſtance alſo upon the *Sub-ſtilar Line*, from S to L.

6. Upon theſe two Points K and L, (with 60 de. of a *Scale* of *Chords*) deſcribe two *Semi-Circles* : and in either of them ſet off 86 d. 05 m. (the *Plains Difference of Longitude*) from R to M, and from S to M, both of them on the ſame ſide of the *Sub-ſtilar Line*, on which the firſt *Perpendicular* A B was drawn.

7. Divide either of the *Semicircle* into 12 equal parts (beginning at the points M) at the ſeveral points ⊙ ⊙ ⊙, *&c.*

8. Lay a Ruler to the point L, and to the ſeveral points ⊙ ⊙ ⊙, *&c.* the Ruler will croſs the Line X S X, in the points * * *, *&c.*——Alſo, Lay a Ruler to K, and the reſpective Points ⊙ ⊙ ⊙, *&c.* the Ruler will croſs the Line Z R Z, in the ſeveral Points * * *, *&c.*

Laſtly, Lines drawn from the ſeveral Points * * * in the Line Z R Z, to the ſeveral points * * * in the other Line X S X each to its Correſpondent, (which the *Sub-ſtilar Line* will direct you) thoſe Lines ſo drawn ſhall be the true *Hour-Lines* proper for the *Plain* : And will be at a competent diſtance one from the other, as by the *Figure* they do appear ; the ſight whereof will be more ſatisfactory than many Words.

Note, that in the making of this *Dial*, you have made four Dials, *viz.*

A	South Declining	Eaſt Weſt	85 Degrees.
	North Declining	Eaſt Weſt	

But you muſt change the *Names* of the *Hours*, and place the *Stile* on the contrary ſide of the Line A B for the *South Declining Weſt*——And by turning the Dial upſide downwards for the North-decliners, ſo that the Stile may point upwards to the North Pole ; and the Hours about Midnight omitted : As in the laſt Chapter is directed.

Fig. LIX.

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Fig. LIX.

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Fig. LIX.

C H A P. X.

Direct South *and* North Declining Plains, *and how* Hour-Lines *may be deſcribed upon them.*

UCH *Plains* as do directly behold the $\left\{\begin{matrix}\textit{North}\\ \textit{South}\end{matrix}\right\}$ point of the *rizon*, but do *Recline* (or fall backwards) from the *Zenith* towards e $\left\{\begin{matrix}\textit{South}\\ \textit{North}\end{matrix}\right\}$ are called $\left\{\begin{matrix}\textit{North}\\ \textit{South}\end{matrix}\right\}$ *Direct Plains Reclining*: So ma- r Dgrees as the *Reclination* is:. And of ſuch *Plains* there are x *Varieties*; Three of *South*, and Three of *North Recliners*: All uich may be *Reduced* to *New Latitudes*, wherein they will be- me *Horizontal Plains*: And conſequently *Dials* (or *Hour Lines*) ty be deſcribed upon them both by the *Globe*, *Spherical Pro- tion*, and *Trigonometrical Calculation*, according to the *Precepts* livered in the *Fourth Chapter* hereof.

I. Of South Recliners.

Examples of all theſe *Varieties* of *Reclining Plains*, in the *Latitude* of *London*, 51 deg. 30 m. To find the *New Latitudes*.

I. Variety. Let there be a *Direct South Plain* in the *Latitude* of *London*, which *Reclines* from the *Zenith* thereof 20 deg. In what *Latitude* will that be an *Horizontal Plain?*

The Plains *Reclination* 120 deg. being *leſs* then 38 de. 30 m. *Complement of the Latitude* of *London*; Subſtract 20 deg. from d. 30 m. the *Remainder* (or *difference*) 18 de. 30 m. is the *-Latitude*. So that an *Horizontal Dial* made for that *Lati- e*, ſhall be a *South Recliner* 20 deg. in the *Latitude* of *Lon-*

II. Variety. If a *South Plain* in the Latitude of 51 deg. 30 m. ſhould *Recline* 60 deg. from the *Zenith* thereof: In what Latitude will that be an *Horizontal Plain?*

The *Reclination* of the Plain 60 deg. being *Greater* than 38 deg. m. the *Complement of the Latitude* of *London*, Subſtract 38 de. m. from 60 deg. and the *Remainder* 21 deg. 30 m. is the *-Latitude*: And an *Horizontal Dial* made for the Latitude of

21

Fig. LIX. 21 de. 30 m. ſhall ſerve for a *South Dial Reclining* 60 deg. in the Latitude of *London*.

III. Variety. If a *South Plain* in the Latitude of *London* ſhould Recline from the Zenith thereof 38 d. 30 m. *Equal* to the *Complement of the Latitude* of *London*. Then

The *Difference* between the *Complement of the Latitude of London* and the *Reclination* being *nothing*; it ſhews, the *New-Latitude* to be no *Latitude*, that is neither *Pole* hath any *Elevation* above ſuch a *Plain*: And therefore a *Dial* for ſuch a *Plain* muſt be made in all reſpects as an *Erect Direct Eaſt* or *Weſt Dial* is made, by the *Precepts* in *Chapter* VII. hereof: Only, the *Hour-Line* of VI there, muſt be the *Hour-Line* of XII in this: And as the *Stile there* was equal to the diſtance between *Six* and *Three* or *Nine* a Clock. So in this, it muſt be equal to the diſtance between XII and IX or III. And may be either a *Plate* of that *breadth*; or a *Wyre* or *Pin* of that *Length*.

II. Of North Recliners

I. Variety. If a *North Plain* in the *Latitude* of 51 de. 30 m. ſhould *Recline* from the Zenith 20 deg. In what *Latitude* will that be an *Horizontal Plain*?

The *Reclination* 20 deg. being *leſs* than the *Complement of the Latitude* of *London*, 38 de. 30 m. Add the *Reclination* 20 de. and the *Co-Latitude* 38 deg. 30 m. together; their *Sum* 58 deg 30 m. is *The New Latitude*: And an *Horizontal Dial* for that Latitude ſhall be a *North Plain Reclining* 20 deg. in the Latitude of 51 de. 30 m.

II. Variety. If a *North-Plain* in the *Latitude* of *London*, 51 deg. 30 m. ſhould Recline from the Zenith 75 deg. In what *Latitude* will ſuch a Plain be *Horizontal*?

The *Reclination* 75 deg. being *greater* than 38 deg. 30 m. *Add* them together, and they make 113 deg. 30 min. which being above 90 deg. take the Complement thereof to 180 deg, which is 66 deg. 30 m. And that is the *New-Latitude*: So that an *Horizontal Dial* made for the Latitude of 66 de. 30 m. will be a *North Plain Reclining* 75 deg. in the *Latitude* of *London* 51 deg. 30 m.

III. Variety. If a *North Plain* in the Latitude of *London* 51 de. 30 m. ſhould Recline from the *Zenith* thereof 51 de.

30

30 m. In what Latitude will ſuch a Plain be Horizontal? *Fig.* LIX.

Here, the *Reclination* 51 de. 30 m. is *Equal* to the Latitude of *London*; And the *Sum* of the *Reclination* 51 de. 30 m. and the *Complement* of the *Latitude* 38 de. 30 m. Added together, their Sum is 90 deg. for *The New-Latitude*: And an *Horizontal Dial* made for that *Latitude* of 90 deg. (which is no other then a Circle divided into 24 *Equal Parts*, for the *Hours*; and a *Wyre* erected perpendicularly, of any *Length* for the *Stile*.)

Theſe are all the *Varieties* of *South* and *North Reclining Plains*, Reduced to *New Latitudes*, wherein they will become *Horizontal Plains*: So that neither *Trigonometrical Calculation*, *Geometrical Projection*, or *Operation* by the *Globe*, need be here Repeated, they being all the ſame as in the IVth *Chapter* hereof.

Note alſo: That in making of any of theſe *North* or *South Reclining Dials*, you have made alſo A *Direct North or South Dial inclining from the Zenith towards the Horizon* ſo many Degrees as is the $\left\{\begin{matrix}\text{Re-}\\ \text{In-}\end{matrix}\right.$ *clination*. So that when you have made a *South-Dial Reclining* from the *Zenith* 60 deg. (as is the *Second Variety* of *South Recliners* in this *Chapter*) you have made alſo a *North Dial inclining to the Horizon* 60 degrees, either by drawing of the Hour-lines and *Stile* through the *Centre*; or by turning the *Reclining Dial* about upon the *Hour-line* of VI. And then, as the *North Pole* is elevated upon the *South Recliner*; ſo much will the *South Pole* be elevated above the North Incliner, &c.

CHAP. XI.

Of Direct Eaſt or Weſt Reclining Dials, and how Hour-lines may be deſcribed upon them.

AS all *Direct North* and *South Reclining Dial Plains* were *Reduced* to *New-Latitudes* wherein they would be *Horizontal Plains*; and therefore made by the *Directions* given in the CHAP. IV. hereof: So all *Direct Eaſt* or *Weſt Reclining Dial*

Plains

Fig. LIX. *Plains* in any one *Latitude*, may be *Reduced* to *Erect*, or *Upright*, *Declining Plains* in another *Latitude*: and therefore may be made by the *Precepts* delivered in the VIIIth. CHAP. hereof. Either, By the *Globe*; By *Spherical Projection*; or, By *Trigonometrical Calculation*: So that the Work of this CHAP. ſhall be only to ſhew

I. How to *Reduce* any *Direct East* or *West Reclining Dial-Plain* in any Latitude, (ſuppoſe *London* 51 de. 30 m.) to a *New-Latitude*: wherein the *Reclining Plain* ſhall become an *Erect* (or Upright) *Plain*. And

II. What *Declination*, that *Upright Plain* ſhall have in that *New-Latitude*.

EXAMPLE.

Suppoſe then, that a *Direct East* or *West Plain*, in the *Latitude* of 51 de. 30 m. ſhould *Recline* from the *Zenith* 40 deg. In what *Latitude* will that be an *Upright Plain?* And what *Declination* ſhall it have in that *Latitude?*

RULE.

The *Complement* of the *known Latitude*, is (always) The *New-Latitude*. And

The *Complement* of the *Reclination* is (always) The *Declination* in that *New-Latitude*.

So that if an *East* or *West Dial*, ſhould *Recline* 40 deg. in the *Latitude* of 51 d. 30 m. That will be *An Upright Plain*, *Declining* 50 deg. in the *Latitude* of 38 de. 30 m

For, 38 de. 30 m. being the *Complement* of the *Latitude* of *London* 51 d. 30 m. is the *New-Latitude*: —— And 50 deg. being the *Complement* of 40 de. the *Plains Reclination*, is the *Declination* in the *New-Latitude*.

So that, if (according to the *Precepts* delivered in the VIIIth. CHAP. hereof) you make an *Upright Dial* for the *Latitude* of 38 de. 30 min. to *Decline* 50 deg. Such a *Dial* will ſerve for an *East* or *West-Dial Reclining* 40 de. in the *Latitude* of (*London*) 51 de. 30 m.

Thus for the *Making* of the *Dial*.

But in the *placing* of the *Dial* (thus made) upon the *Reclining Plain*, this *Difference* is to be obſerved: For, Whereas in all *Upright Declining Plains*, the *Meridian*, or *Hour-line* of XII is always *Perpendicular* to the *Horizon* of the *Place* for which it is made.

But this *Declining Plain* when it is applyed to the *Reclining Plain*, the *Hour-line* of XII. muſt lie *Parallel* to the *Horizon* of the *Place*, as in the *Figure*. Fig. LX.

And here Note, that all *Eaſt Recliners* in the known *Latitude* (as here *London*) are *North-Eaſt-Decliners* in the *New-Latitude*: And all *Weſt-Recliners*, are *North-Weſt-Decliners*.

And Note farther; That upon all *Eaſt* and *Weſt Reclining Plains* in *North-Latitudes*, that the *North Pole* is (always) *Elevated*: And upon the *Eaſt* and *Weſt inclining Plains*, oppoſite to them, the *South Pole* is *Elevated*.

And Laſtly, Note that when you have made an *Eaſt* or *Weſt Reclining Dial*, you have made Four, *viz.* An *Eaſt* and *Weſt Reclining*, and, An *Eaſt* and *Weſt Inclining*.

CHAP. XII.

Of Dial Plains *that do both* Decline *and* Recline: *How* Hour-Lines, *&c. are to be deſcribed on them.*

THESE *Plains* may alſo be *Reduced* to *New-Latitudes* and *New Declinations*, where they may ſtand as *Upright Decliners*: And ſo *Dials* may be deſcribed on them, by the Directions in the VIIIth. CHAP. hereof: And to find the *New-Latitude*, and *New Declination* of any ſuch Plain, The *Rules* following will direct.

EXAMPLE.

Suppoſe that in the *Latitude* of *London*, 51 d. 30 min: A *South Plain* ſhould *Decline* towards the *Eaſt* or *Weſt* 24 deg. 20 min. And alſo *Recline* from the Zenith 54 deg.

I. To find the *New-Latitude*

The *Canon* for Calculation.

As the Radius, Sine 90 de.	10.
Is to the Co-ſine of the *Old Decl.* 24 de. 20 m.	9.959596
So is the Co-Tangent of the *Reclination* 54 d.	9.861261
To a Fourth Tangent, *viz.* 33 de. 30 m.	19.820857

This *Fourth Tangent* being thus found, the Rules following are to be obſerved.

I. In *South* Recliners.

RULE I. This *Fourth Tangent* muſt be compared with the *Old Latitude*, and the *Complement* of their *Difference* is the *NEW LATITUDE*.

So, In this *Example*, The *Fourth Arch* 33 de. 30 m. being ſubſtracted from the *Old Latitude* 51 de. 30 m. their difference is 18 deg. whoſe Complement 72 deg. is the *NEW LATITUDE*.

RULE II. If the *Fourth Tangent* fall out to be *Equal* to the *Old Latitude*, Then the *Difference* will be nothing; And ſo the *Plain* will be a *Polar declining Plain*: For the *Pole* will have no *Elevation* over it.

So, in the *Latitude* of 51 de. 30 m. If a *South Plain* ſhould *Decline* towards the *Eaſt* or *Weſt* 65 deg. 40 m. And *Recline* 18 de. 9 m. By the former *Canon*, the *Fourth Tangent* will be found 51 de. 30 m. *Equal* to the *Old Latitude*: So that the *Difference* is *Nothing*. And the *Hour-Lines* will be *Parallel* one to another as in the *Direct Eaſt*, *Weſt*, and *Polar Dials*, in CHAP. VII. and CHAP. X.

RULE III. If the *Fourth Tangent* prove to be *Greater* than the *Old Latitude*, Then, The *North Pole* is *Elevated* in *South Decliners*: But if the *Fourth Tangent* be *Leſſer* than the *Old Latitude*, Then, The *South Pole* is *Elevated* in *North Decliners*.

II. In North Recliners.

RULE I. The *Fourth Tangent* found as before, is to be compared with the *Complement* of the *Old Latitude*, and their *Difference* is the *NEW LATITUDE*.

RULE II. If the *Fourth Tangent* prove to be *Equal* to the *Complement* of the *Old Latitude*: that *Declining Reclining Plain* will be an *Æquinoctial Plain Declining*.

So, In the Latitude of 51 de. 30 m. If a Plain ſhould *Decline* from the *North* towards the *Eaſt* or *Weſt* 60 deg. And alſo *Recline* from the *Zenith* 32 deg. 11 m. The *Fourth Tangent* will be found to be 38 de. 30 m. *Equal* to the *Complement* of the *Old Latitude*: And will, therefore, be An *Æquinoctial Declining Plain*.

II. To find the *New Declination.*

The *Canon* for Calculation.

As the Radius, Sine 90 d.	10.
To the Co-ſine of the Reclination 54 d.	9.769218
So is the Sine of the Old Declination, 24 d. 20 m.	9.614944
To the Sine of 41 de. 01 min.	*9.384162

Which 41 de. 1 min. is the *NEW DECLINATION.*

And now, if (according to the *Precepts* delivered in the VIIIth. CHAP. hereof) you do make an Upright *South Dial Declining* 14 deg. 1 m. in the *Latitude* of 72 deg. That *Dial* ſhall ſerve for A *South Plain*, in the Latitude of 51 de. 30 m. *Declining* 24 deg. 20 m, and *Reclining* 54 deg.

New Latitude	72	00
New Declination	14	01
Stiles heigth	17	26
Diſt. Sub. & Merid.	4	17
Difference of Long.	13	59

	Hours.		Eq. Di.		Tr. H D.	
41	VII	V	89	17	89	47
41	VIII	IV	74	17	46	48
41	IX	III	59	17	26	46
41	X	II	44	17	16	17
41	XI	I	29	17	9	32
41	XII		14	17	4	22
19	I	XI	00	43	0	12
19	II	X	15	43	4	49
19	III	IX	30	43	10	06
19	IV	VIII	45	43	17	05
19	V	VII	60	43	28	06
19	VI		75	43	49	39

And in ſuch a *Dial* you will find the *Requiſites* belonging thereunto, to be as the Table in the *Margin.* And thus much for the *Making* of the *Dial* according to its *New Latitude*, and *New Declination.*

But to apply this *Dial* to the *Reclining Plain* in the *Old Latitude*, you are not to place the *Hour-line* of XII *perpendicular* to the *Horizon*, as in *Upright Plains*, but it muſt make an *Angle* with the *Horizontal Line* of the *Reclining Plain*: And therefore the *quantity* of that *Angle* muſt be firſt found. Therefore,

III. To find the *Angle* made between the *Meridian* and the *Horizon.*

The *Canon* for Calculation.

As the Radius, Sine 90 d.	10.
Is to the Sine of the *Reclination* 54 d.	9.655347
So is the Tangent of the *Old Declination* 24 d. 20 m.	9.907957
To the Tangent of 20 deg. 6 m.	*9.563304

Whoſe *Complement* 69 deg. 54 min. is the *Angle* that the *Meridian* (or *Hour-line* of XII.) muſt make with the *Horizontal Line* of the *Reclining Plain*. Thus for the *Quantity* of the *Angle*. But

IV. To know which way (or to what Coaſt) the *Meridian Line*, *Aſcending* or *Deſcending Above* or under the *Horizontal Line* of the *Plain*, is to be drawn.

GENERAL RULES.

In *North* Incliners. / *South* Recliners.

- *Leſs* than Equinoctial, the Meridian muſt be drawn
  - Above / Below
  - That *End* of the *Horizontal Line*, which lies *contrary* to the Coaſt of the Plain's Declination.
- *More* than Equinoctial, the Meridian muſt be drawn
  - Below / Above
  - That *End* of the *Horizontal Line*, which lies *the ſame way* with the Coaſt of the Plain's Declination.

In { *North* Recliners / *South* Incliners }

- *Leſs* than a *Polar* the *Meridian* muſt be drawn. { Above (*North* Recliners) / Below (*South* Incliners) } That *End* of the *Horizontal Line*, that looks *the ſame way* with the Coaſt of the Plain's Declination—And this *Meridian* thus drawn, in *North Recliners*, repreſents 12 at *Midnight*.
- *Equal* to a *Polar*, the Meridian muſt be drawn below the *Horizontal Line*, at that end which is contrary to the *Coaſt* of *Declination*: And the *Six* a Clock *Hour-line*, is (always) the *Subſtilar*.
- *More* than a *Polar*, the Meridian muſt be drawn. { Below (*North* Recliners) / Above (*South* Incliners) } And from that *End* of the *Horizontal Line*, which lies *contrary* to the *Coaſt* of the *Plain's Declination* —— And, in *South Incliners*, is only ſerviceable to help to draw the reſt of the *Dial*.

V. How the *Dial* (being made according to the *New Latitude* and *New Declination*) is to be transferred from the *Paper Draught*, upon the *Reclining or Inclining Plain*.

Having drawn an *Horizontal Line* upon your *Dial Plain*, in the moſt convenient part thereof, and made choice of a Point therein for the *Centre* of your *Dial*; apply the *Centre* of your *Paper Draught* to this *Centre*; moving the *Paper Draught* about till the *Meridian Line* thereof do make an *Angle* with the *Horizontal Line* drawn upon the *Plain*, equal to what you found it to be by the *Third Section* of this CHAP. And to its *Proper Coaſt*, as you found it by the *Fourth Section*: Then (if you have not erred in any of your former workings) will the *Stile* of your *Paper Draught* (or rather a pattern of it cut in *Paſt-board*) being placed upon the *Sub-ſtile* of the *Paper Draught*, have direct reſpect to the *Elevated Pole*.—And thus due reſpect being had to what is here deliver'd delivered, you may eaſily transfer the *Stile* and the reſt of the *Hour-lines* to the *Plain*; putting thereon ſo many as the Plain is capable to receive at any time of the Year; and leaving out ſuch as are ſuperfluous.

CHAP. XIII.

Of the Inſcription of other Great *and* Small Circles *of the* Sphere *upon all ſorts of* Dial-Plains.

IN the foregoing Chapters is ſhewed how the *Meridians* (or *Hour-Circles*) of the *Sphere* may be deſcribed upon all ſorts of *Plain Superficies* howſoever ſituate: And as thoſe *Hour-Circles* were deſcribed, ſo may all other *Circles* of the *Sphere* be alſo delineated: And the manner how, I have largely treated of in my particular *Treaties* of *Dialling* already extant: So that in this place I ſhall be but brief therein, and yet ſufficiently *Plain*. And whereas the *Hour* of the *Day* in all *Sun-Dials* is ſhewed by the whole *Axis* of the *Stile*; ſo all other *Circles* ſo deſcribed, are ſhewed by one *ſingle point* taken in ſome convenient place of the ſaid *Axis*, from which Point a Perpendicular let fall to the *Plain* upon the *Sub-ſtilar Line* ſhall be called the *Perpendicular Stile*: And the point upon the Plain, The *Foot* of the *Perpendicular Stile*; and the *Point* in the Axis (before aſſumed) the *Gnodus* or *Apex* of the *Stile*. —— Now, for the performances of the Work following (beſides the *Trigonometrical Calculation*) the Practitioner muſt be provided with a *Scale* (or a *Sector* rather) which muſt have upon it *Scales* of *Sines*, *Tangents*, *Secants*, *Chords*, and *equal parts*: or if he have by him, *Tables* of *Natural Sines*, *Tangents*, and *Secants*, then a *Line of equal parts* only drawn from the Centre, will perform the work rather better than by the other *Scales*. Being thus provided; The firſt buſineſs muſt be

How to *proportion* the *Perpendicular Stile* to the *Plain*.

If your *Dial Plain* be ſmall, conſider whether it be *Direct*, or *Declining*; if *Direct* the *Sub-ſtile* may be placed in the *Middle*: If *Declining*, more to the ſide contrary to the *Coaſt* of *Declination*.

The Sub-ſtile well made choiſe of, and the *Perpendicular*

Stile thereon *Erected*: Make the *Perpendicular Stile* the *Radius* (or Equal to the *Tangent* of 45 degrees) and make that part of the *Sub-ſtile* which lies beyond the foot of the *Stile* and towards the Centre, equal to the *Tangent Complement* of the heighth of the *Pole* (or *Stile*) above the *Plain*: and the other part of the *Sub-ſtile*, below the *Foot* of the *Stile*, Equal to the Tangent Complement of the *Meridian Altitude* of the *Sun*, when he is in that *Tropick* which is to be moſt remote from the *Centre* of the *Dial*.

CHAP. XIV.

Of the Inſcription of the Signs *or* Parallels *of the* Sun's Courſe.

A *Sign* is the Twelfth part of the the *Ecliptick*, and therefore contains 30 degrees.

A *Parallel*, is the *Sun's Diurnal Motion* Day by Day; and becauſe there are 47 deg. between the two *Tropicks*, there may be ſo many *Parallels*, that is, *Circles*, which the Sun deſcribeth every 24 Hours: and although there be 47 of theſe, yet in the *Latitude* of 51 deg. 30 min. we account but *nine*, *viz.* thoſe which are the *Day* from *Sun* to *Sun*, when it is 8, 9, 10, 11, 12, 13, 14, 15, or 16 juſt hours long. The *Deſcription* of theſe *Parallels* and *Signs* is made the ſame way; only due reſpect muſt be had to the quantity of the *Sun's Declination*: For (in all *Direct Horizontals*) the *Perpendicular Stile* being made *Radius*, the *Tangent Complement* of the Sun's height, in any *Sign* or *Parallel*, at any hour of the *Day*, ſet off from the *Foot* of the *Stile*, and extended to that *Hour-line*, gives a *Mark* upon the *Hour-line*, by which the *Parallel* of that *Day* ſhall paſs: So that this *Work*, repeated ſo often as the number of *Parallels* to be inſcribed, and the *Hour-lines* require; ſhall give reſpective *Points* enough, in each *Hour*, to draw each *Parallel* by.

Example. In the Latitude of 51 de. 32 m. the Sun being in the beginning (or entring) of *Piſces*; The *Sun's height* above the *Horizon* at every *Hour* may be found (by CASE IX of O. *A. S. T.*) to be as followeth, *viz.*

At

At	or	Deg.	M.	The Complement	
12		27.	01	62.	59
1	11	25.	37	64.	23
2	10	21.	49	68.	11
3	9	15.	57	74.	03
4	8	8.	32	81.	28

Now, the *Perpendicular Stile* being *Radius*, the Tangents of the Complements of the reſpective *Altitudes*, as 62 deg. 59 m. the Complement of 27 de. 01 m. ſet from the foot of the *Per. Stile*, on the *Hour-line* of 12 (or *Sub-ſtile*) ſhall give a point thereon, by which the *Parallel* of *Piſces* muſt paſs: And ſo, the Tangent of 64 de. 23 m. ſet from the foot of the *Per. Stile*, upon the *Hour-lines* of 11 and 1 a Clock, ſhall give you two other points by which the ſaid *Parallel* ſhall paſs: And ſo for all the reſt of the *Hour-lines*, through which points found upon all the *Hour-lines*, a *Line* drawn by an even hand, ſhall be the *Parallel* required; for along that Line will the *Shadow* of the *Top* of the *Per. Stile* (as it creepeth along) paſs, when the Sun is in the beginning of *Piſces*, *viz.* about the 9th of *February*.

And therefore, generally in *Verticals*, as alſo in all *Recliners*; that is to ſay, upon all *Plains* whatſoever: Draw an *Horizontal Dial* proper to the *Plain*, and inſcribe the *Signs* or *Parallels* upon it, by ſetting off from the *Foot* of the *per. Stile*, the *Tangents Complements* of the *Sun's height* at every *hour* in the beginning of every *Sign* above that Plain (taken as an *Horizontal*, the Foot of the *per. Stile* being ever *Radius*) and at the end of theſe *Tangents* ſo ſet off upon every reſpective *Hour-line*, will be a Point; By which Points, *Lines* drawn with an even Hand, ſhall trace out upon the *Dial Plain*, the *Parallels* required.

Example. Suppoſe a Plain Decline 30 deg. and Recline 55 deg. the height of the *Pole* above the Plain 19 deg. 25 min. And the *Sun's height* at the beginning of *Taurus* to be at the ſeveral *Hours*, as in this Table.

At	or	De.	M.	Complements	De.	M.
12		38.	05		07.	55
11	1	73.	30		16.	30
10	2	60.	03		29.	57
9	3	46.	01		43.	59
8	4	31.	53		58.	07
7	5	17.	47		72.	13

Then, The Tangents of the Complements of theſe *Hour-diſtances* (as 7 deg. 55 m. for 12 : 16 deg. 30 m. for the Hours of 11 and 1) ſet off from the Foot of the *per. Stile* (the ſaid *Stile* being the *Radius* to theſe *Tangents*) to the obſcure *Horizontal Hours* of 12, 11, 10 : and 1, 2, 3, *&c.* give the true *diſtances* between the *Foot* of the *Stile*, and thoſe *auxiliary Hours*, for the *Parallel* of *Taurus*; and ſo points for the deſcribing of other *Parallels* of *Declination* : Having firſt (by *Trigonometrical Calculation*) found the *Horizontal Diſtances*, and the *Sun's Altitude* at his entrance into thoſe Parallels of *Signs* or *Declination*, in ſuch *Latitude* as you have need of. All which are taught how to do in the foregoing parts of this Book.

CHAP. XV.

Of the Inſcription of the Vertical Circles (*commonly called* Azimuths) *upon all* Dial Plains.

THESE are great *Circles* of the *Sphere*, whoſe *Poles* lie in the Horizon, and interſect one another in the *Zenith* and *Nadir* Points of the *Place* wherein the *Dial* is to ſtand.

The whole *Horizon* being divided into 32 equal parts; theſe *Circles* paſſing through thoſe *Diviſions*, are called *Points* of the *Compaſs*, and denominated accordingly, as *South*, *S by E*, *S S E*, &c. But the better way of accounting them is by 10, 20, 30, *&c. Degrees* from the *Meridian* on either ſide thereof.

Firſt, in all *Horizontal Dials*; the *Perpendicular Stile* being choſen, making the Foot thereof the *Centre*; at any convenient diſtance, deſcribe a *Circle*; and account from the *Meridian* both ways, *Arches* equal to 10, 20, 30, *&c. Degrees* : From which Diviſions, right Lines drawn to the *Foot* of the *Stile* aforeſaid, ſhall repreſent thoſe *Azimuths* upon that *Dial*.

Secondly, Upon a *Prime Vertical* (or *South*) *Dial* : Through the Foot of the *Per-ſtile*, draw a Right-Line Parallel to the *Horizon*; and making the ſaid *Stile Radius*; upon the *Parallel Line*, ſet off, both ways from the *Meridian Tangents* of 10, 20, 30, *&c. Degrees*; through which Diviſions, Right-lines drawn, all at *Right Angles* with the *Parallel Line*, ſhall be the *Azimuths*.

Thirdly, Upon any *Declining Vertical*, the ſame being done, ſhall give the *Azimuths* of 10, 20, 30, *&c.* degrees from the *Meridian* of the *Plain*; or from the *Meridian* of the *Place*, juſt allowance being made for the *Difference* of *Meridians*.

Fourthly, In *South Declining Reclining* Plains, the *Per. Stile* being choſen, and made the *Radius*, the *Tangent Complement* of the *Reclination*, applyed from the *Foot* of the *Per. Stile* to the *Meridian* of the *Place*, ſhall determine the *Zenith* of the *Place*: through which, and the *Foot* of the *Stile*, (that is the *Zenith* of the *Plain*) a right Line drawn, ſhall be a *Perpendicular* to the *Horizontal Line*, and ſhall concur with the *Æquator* in the *Hour-line* of 6; and therefore, if from the *Foot* of the *Stile* upon the ſaid *Perpendicular*, towards the *North* (for the former application was made towards the *South*) be ſet off the *Tangent* of the Reclination, a *Line* drawn from the end thereof, at *Right Angles* with it, ſhall be the *Horizontal Line*: Upon which, the *Tangents* of 10, 20, 30, *&c.* (the *Secant* of the *Reclination* being now made *Radius*) ſet from the ſaid *Right Angle*; *Lines* drawn from them to the *Zenith* of the *Place*, ſhall be the *Azimuths*.

Fifthly, The *Diſtance* between the *Meridians* being known, upon the *Horizontal Line*; the *Azimuths* which were accounted from the *Meridian* of the *Plain*, may be fitted for the account from the *Meridian* of the *Place*, with eaſe.——For *Example*, let that diſtance be the *Tangent* of 20 deg. Then that *Azimuth* which is 10, from the one; is 10 from the other alſo: And that which is 30 on the ſame ſide of the Sub-ſtile, is 10 on the other ſide of the *Meridian* of the *Place*: And the like method ſerves for any diſtance.

CHAP. XVI.

Of the Inſcription *of* Almicanters *or Circles of the* Sun's Altitude *upon* Dial Plains.

THESE are *leſſer Circles* of the *Sphere*; and may be called the *Parallels* of *Declination* from the *Horizon*; they having in all reſpects, the ſame relation and habitude to the *Azimuths*, as the *Signs* and *Parallels* of *Declination* have to the *Meridians*; although theſe be counted by 15 *deg.* and thoſe uſually by 10.

And therefore, as in the deſcription of the *Signs* and *Parallels*; ſo in theſe,

Let an *Horizontal Dial*, proper to the *Plain*, be firſt (obſcurely) deſcribed; and then, as it was there ſhewed, that the points through which the *Signs* or *Parallels* muſt paſs, upon every *Hour-line*, might be had by applying the *Tangents* of the *Complements* of the *Sun's height* of thoſe *Hours* in thoſe *Parallels*, from the *Foot* of the *Per. Stile*, to the reſpective *Hour-lines*: —— So here, making uſe of that *Azimuth* which is perpendicular to the Plain, (which in all *Plains* is that which paſſeth through the *Foot* of the *Per. Stile*) the reſt of the *Azimuths* being alſo inſcribed, the *Tangents Complements* of the *Sun's height* above the *Plain*, when he is in any *Azimuth*, applyed from the *Foot* of the *Per. Stile* to the ſaid *Azimuth*, gives a *Point*, through which that *Circle* or *Almicanter*, upon that *Azimuth* muſt paſs.

Now to know what *Altitude* the Sun will have, when he will be upon any *Azimuth*, in any *Parallel* of *Declination* (or degree of the *Ecliptick*) is taught in the *Section* of *Aſtronomical Problems*; Or, by the Reſolving of an *Oblique Angled Spherical Triangle*, wherein is always Given, *Two Sides*, and the *Angle* oppoſite to one of them to find the third *Side*, (By CASE V. of *O. A. S. T.*) Which *third Side* ſo found, is the *Complement* of the *Altitude* which is in this caſe required; and muſt accordingly be ſet from the *Foot* of the *Per. Stile* unto the *Azimuths*, &c.

CHAP XVII.

How to inſcribe the Jewiſh, Babyloniſh *and* Italian Hours, *upon all* Dial-Plains.

FOR the Inſcription of theſe *Hours* upon *Dial-plains*, there needs no *Trigonometrical Calculation*: For the two *Tropicks*, the *Æquator*, and other *Parallels* of *Declination* being already deſcribed (or ſuch of them as ſhall be needful) together with the *common Hour-lines* proper for the *Plain*, Points through which theſe *Hour-lines* may paſs, may be found by theſe following Directions.

The *Babyloniſh Hours* are accounted *Equal Hours* from *Sun-riſing*, and may be inſcribed upon any *Plain*, by help of thoſe two *Pa-*

rallels of *Declination* which ſhew the *Longeſt* and *Shorteſt Day*, conſiſting of *whole* (or entire) Hours: As with us 16 and 8; and the *Æquator*. For,

A Line drawn through	9	In the Parallel of	8 of Declination	Shall
	7		Æquinoctial	
	5		16 of Declination	

be the *Hour* of *One* from the *Sun's riſing*. And likewiſe in the ſame order; through 6, 8 and 10, ſhall give the *Second Hour* from *Sun-riſing*; and in the like *Order*, all the reſt.

In Winter, when the *Parallel* of 8 *Hours* ſhall fail, the other two Points will ſerve to draw it by, becauſe thoſe *Hours* are *Streight Lines*. But after the firſt *Six Hours* are inſcribed, the *Æquinoctial* alſo failing, ſome other *Diurnal Arch*, (as of 9 or 10 Hours) muſt be deſcribed to ſupply that want.

The *Italian Hours* are accounted by 1, 2, 3, &c. from *Sun-ſetting*: And for the Inſcription of theſe, the ſame *Parallels* of 8 and 16 *Hours*, with the *Æquator*, will ſerve: For a Line drawn through them, in the Hours of 9, 7 and 5, *Afternoon* (obſerving the ſame as before) ſhall be the *hour* of *One*; the like through 7, 5 and 3, ſhall be the *hour* 23: The *Night hours* of 9, 10, &c. are the *Morning hours* produced.

The *Jewiſh hours* are reckoned like the *Babyloniſh*, from *Sun-riſing*; but unequally; their *Sixth hour* being (always) *Noon*; and every *hour*, one *Twelfth* part of the *Artificial Day*, of what length ſoever that be.

For the *Inſcription* of them: The *Vulgar Hours* proper for the *Plain* being firſt drawn, and the *Diurnal Arches* of 15, 12, and 9 *hours*, divide the degrees in each by 12, and that Quotient by 15; or elſe (which is all one) *divide* the ſaid *Arches* by 180, the three *Quotients* ſhall give the juſt *Times*, in *hours*, and uſual parts of *hours* from 12 a Clock, upon the two *Parallels* and the *Æquator*: through which, *Lines* drawn by a *Ruler*, ſhall be the *Jewiſh hours* required.

Example: In Latitude 51 deg. 32 min. the Diurnal Arch of 15 hours is in Degrees 225, which divided by 180, the Quotient is 1 $\frac{1}{4}$ h. and ſo much the *Jewiſh hours* of 5 and 7 are diſtant from *Noon*; one hour and a quarter being a *twelfth part* of the *Diurnal Arch* of 15 hours: And this *hour* and *quarter* being *doubled*, gives the place for 4 and 8: *Tripled*, the place of 3 and 9, &c. from *Noon*, upon that *Parallel* of 15 hours.

In like manner, the *Diurnal Arch* of 9 hours is 135 deg. which divided by 180, the Quotient is $\frac{135}{180}$, that is 3 quarters of an hour: Which ſhews the place of the *Jewiſh hours* of 7 and 5, to be three quarters after, or before, *Noon*: and that doubled is *One hour and a half*, which gives the place of 8 and 4; all one with our $1\frac{1}{2}$ and and $10\frac{1}{2}$, and ſo *Tripling* and *Quadrupling* and *Quintupling* 3 quarters, you have the places of the *Jewiſh hours* upon this *Parallel* of 9 hours length of the Day.

And theſe parts *Doubled* and *Tripled*, as is ſaid, will always (in this *Parallel* and the former) fall upon *even hours*, *halves* and *quarters* of *hours*: And that is the only reaſon why theſe two *Parallels* of 15 and 9, are preferred; there being no neceſſity of uſing them, more than the *Tropicks* or other *Parallels*, only this conveniency of even parts.

Laſtly. In the *Diurnal Arch* of 12, that is, the *Æquator*, the *Common* and the *Jewiſh hours* concur; that is, the *Jewiſh hours* of 5 and 7, with our *hours* of 11 and 1: Their 4 and 8, with our 10 and 2, &c. So that a *Line* drawn from $1\frac{1}{4}$ in the *Parallel* of 15, to 1 in the *Æquator*, and from thence to $\frac{3}{4}$ in the *Parallel* of 9, is the 7th *Jewiſh hour*. And ſo are all the reſt to be inſcribed.

Trigo-

Trigonometria Practica.

SECTION VI.

OF NAVIGATION.

I Intend not here to treat of *Navigation* in the general; it being an Art that requires (for the true Underſtanding, either the Theory, or Practice of it) an inſpection into divers other *Sciences Mathematical*; of which, that of *TRIGONOMETRIA*, (or the *Doctrine* of *Triangles*) is the *Principal*; for that the ſolution of all ſuch *Problems* which are of daily Uſe at Sea, are performed thereby; and thoſe are ſuch as concern *Longitude*, *Latitude*, *Rumb* (or courſe) and *Diſtance*, &c. And therefore, I ſhall *Define*, Firſt, what is meant by *Longitude*, *Latitude*, *Rumb*, *Diſtance*, &c. And Secondly, any two of them being *known*; how to find the other two; and that by *Trigonometrical Calculation*: with ſome other *Problems* pertinent to that Art. And I ſhall perform them, (1) By Plain Sailing. (2) By Mercators Sailing; And (3) By the Middle Latitude.

DEFINITIONS.

I. *Longitude*, Is the *Diſtance* of a *Place* from ſome known *Meridian* to that *Place*; and is always counted upon the *Æquinoctial* Circle, from that known Meridian towards the *Eaſt* or *Weſt*.

II. *Latitude*, Is the *Diſtance* of any *Place* from the *Æquinoctial* Circle; counted upon that *Meridian* Circle which paſſeth over that *Place*, towards either of the *Poles*; either *North* or *South*: and accordingly the *Latitude* is *Denominated* either *North* or *South Latitude.*

III. *Rumb*, (or *Courſe*) Is that *Angle* which a *Ship* in her *Sailing* makes with the *Meridian* of the Place from whence the Ship came, and the Place where the Ship then is: But the *Complement* of the *Rumb*, is that *Angle* which the *Rumb* makes with that *Parallel* of *Latitude* in which the Ship is; and is the *Complement* of the *Rumb* to 90 deg.——The *Rumb* is made known to the *Mariners* at all times, by help of his *Compaſs.*

IV. *Diſtance*, Is the *Number* of *Leagues*, *Miles*, *Centeſms*, &c. that a Ship hath *Sailed* upon any *Rumb* or *Courſe.*——And this is known to the *Mariner* by the vering (or running out) of the *Log-Line* in any known quantity of *Time*.

V. If your *Rumb* (or *Courſe*) be directly *Eaſt* or *Weſt*, you alter not your *Latitude* at all:——If your *Courſe* be *Northward*, you continually *Raiſe* the *North Pole*; and you increaſe your *Latitude Northward*: Or the *South Pole*, if you *Courſe* be from the *Æquinoctial* Southward.

So that

The *Raiſing* of the *Pole* is, When you Sail from a *Leſſer* Latitude to a *Greater*: And the *Depreſſing* of the *Pole* is, when you Sail from a *Greater* to a *Leſſer* Latitude.

Theſe things known, before I proceed to the Solution of the ſeveral *Queſtions* or *Problems* in *Navigation*, unto which the *Doctrine* of Triangles is ſubſervient, it will be neceſſary to ſay ſomething concerning the *Situation* of *Places* upon the *Earth* or *Sea* in reſpect of *Longitude* and *Latitude*: And of their *Diſtances* one from another in *Leagues*, *Miles*, or *Minutes*: And then, ſhew how to lay down (upon a *Blank Chart*) any two (or more) *Places*, according to their reſpective *Latitudes* and *Difference* of *Longitudes*. All which ſhall be comprehended under theſe *Heads* following.

I. *Of the* Situation *of* Places, *in reſpect of* Longitude *and* Latitude.

IN this *Problem* there are variety of *Caſes*, according as the *places* may be ſituate one from the other, in reſpect of the *Firſt*, or *General Meridian*, as to their *Difference of Longitude*; and in reſpect of the *Æquinoctial*, as to their difference of *Latitude*: And for the better underſtanding of theſe *Varieties*, I ſhall exhibit all of them in one *Scheme*: In which,

Fig. LXI. N Æ S is the *Firſt Meridian*, or beginning of *Longitude*.
W Æ E the *Æquinoctial Circle*.
Æ E the *Eaſt part* thereof.
Æ W the *Weſt part* thereof.
N the *North Pole*.
S the *South Pole*.
D B and T A, two *Parallels* of *North Latitude*.
O R and Z X, two *Parallels* of *South Latitude*.
N I L M S a *Meridian* of *Weſt Longitude*.
N H C S a Meridian of *Eaſt Longitude*.

CASE I. If the *Places* lies in the *Æquinoctial Circle*, as the *Places* L, G, H, and K; and ſo have no Latitude—Or in the ſame Parallel of *Latitude*, as the *Places* D, V, X, and B, in the *Parallel* of 40 de. of *North Latitude*; and the Places O, M, C and R in the *Parallel* of 35. 85 deg. of *South Latitude*: Then, (1.) If the Two Places propoſed, do lie both on the *Eaſt-ſide* of the *Firſt Meridian*, as H, K — X, B — C R: Or, both on the *Weſt-ſide*, as D, V — L, G — O, M: Then, The *Leſſer Longitude ſubſtracted* from the *Greater*, gives the *Difference of the Longitudes* of thoſe *Places*. But, (2.) If of the two *Places* propoſed, one do lie on the *Eaſt-ſide*, and the other on the *Weſt-ſide* of the *Firſt Meridian*: As V and X — L and H, M and C: Then, the two *Longitudes added* together, gives the *Difference* of *Longitude* between thoſe two Places. But, Note,

If the *Sum* of the two *Longitudes* do exceed 180 deg. *ſubſtract* the *Sum* of them from 360 deg. and the *Remainder* is the *Difference* of *Longitude* of thoſe two *Places*.

CASE II. If One *Place* lie in the *Æquinoctial Circle*, as L, and the other under the ſame *Meridian* N L S, as the *Places* V, in the *Parallel* of 40 deg of *North*, and M, in the *Parallel* of 35. 85 deg. † of *South Latitude*: Then, the *Latitudes* themſelves are eſteemed (or are to be taken) for the *Difference* of *Latitude*.

Fig. LXI.

† *In all this* Section, *the* Degrees *are not divided into 60 min. but into* Centeſms, *or* hundred Parts.

CASE III. If Two Places propoſed differ both in *Latitude* and *Longitude*, do lie both of them on the *North*, as V in the *Parallel* of 40 deg. and I in the *Parallel* of 18. 25 deg. the *leſſer Latitude* taken from the *greater*, leaves 21. 75. deg. for the *Difference* of *Latitude* of thoſe Two *Places*: The like, if both the *Places* had lain on the *South-ſide* of the *Æquinoctial*, as at C and Y.—But if the Two *Places* propoſed had been V, in 40 deg. of *North Latitude*, and C, in 35. 85 deg. of *South Latitude*; then, the *Sum* of them two *Latitudes* 75. 85 deg. would be the *Difference* of *Latitudes*, which can never exceed 90 deg.

How to Reduce Degrees *and* Minutes, (*or* Degrees *and* Centeſms, *or hundred parts of* Degrees) *into* Miles, *or* Minutes.

THIS is done by multiplying the *Degrees* by 60. and adding the odd *Minutes*, if any be, and the Product will be *Miles*. Alſo *Degrees*, and *Centeſms* of *Degrees*, multiplied by 60, the Product will be *Degrees*, and *Centeſms*. So,

26 de. 33 m. multiplied by 60: The Product of 26 by 60, is 1560, to which add the 33 Minutes, and it makes 1593 Miles.

Or, 72. 75 deg. multiplied by 60, will produce 4365 *Miles*: And 26. 47 deg. by 60, will produce 1588. 20 *Miles*; that is 1588 *Miles*, and $\frac{1}{5}$ or $\frac{20}{100}$ of a *Mile*; and ſo of any other.

III. *How to lay down (upon a* Blank Chart*) any two* Places, *which differ both in* Longitude *and* Latitude.

LET the two *Places* propoſed be the *Harbour* of *Jamaica*; which lies in 18. 25 deg. of North *Latitude*, and 78. 35 deg. of Weſt *Longitude*—And the *Cape of Good Hope*; which lies in 35. 85 deg. of *South Latitude*, and 27. 50 deg. of *East Longitude*.

Fig. LXII.

The Geometrical Conſtruction of the *Figure.*

Firſt, Draw a Right-Line Æ Æ for the *Æquinoctial Circle* of the *Terreſtrial Globe:* And another Right Line N S, for the *Principal*, or *Firſt Meridian*, from whence the *Longitude* of *Places* begin to be accounted either *Eaſt* or *Weſtward* from it: which two Lines croſs each other at Right Angles in the Point *æ*.

Secondly, The *Longitude* of *Jamaica* being 78. 35 deg. Reduce them into *Miles* (by multiplying them by 60) and they make 4701 *Miles*, which taken from any *Scale* of *Equal Parts* ſet upon the *Æquinoctial Circle* from *æ* to *a*, towards the *Weſt*, becauſe the *Longitude* was *Weſtward*: And through that point *a*, draw a Right Line *n a s*, parallel to N *æ* S, the Firſt *Meridian*, ſo ſhall that Line *n a s*, be the *Meridian* under which the Harbour of *Jamaica* lieth.——Alſo the *Longitude* of the *Cape of Good Hope*, being 27. 50 deg. Reduce them into Miles (by Multiplying them by 60) and they make 1650 *Miles*, which ſet from Æ to *b*, and through *b*, draw a Right Line *n b s* parallel to N *æ* S, ſo ſhall this Line *n b s*, be the *Meridian* under which the *Cape of Good Hope* lyeth.

Thirdly, The *Latitude* of *Jamaica Harbour* being 18. 25 degrees, reduce them into *Miles*, and they make 1095, which ſet upon the Line *n a s* from *a* to H, towards *n* (becauſe the *Latitude* was *North*) ſo ſhall H be the Point repreſenting the *Harbour* of *Jamaica*. And, If you draw a Line through this point H, parallel to the *Æquinoctial*, as the Line H *b*, that Line ſhall repreſent the *Parallel* of the *Latitude* of the *Harbour* of *Jamaica*, *viz.* of 18. 25 deg.——Alſo, The *Latitude* of the *Cape of Good Hope* being 35. 85 deg. Reduce them into *Miles*, and they make 2151, which ſet upon the Line *n b s*, from *b* downwards towards *s*, (becauſe the *Latitude* is *South*) to C, ſo ſhall the point C, be the Point repreſenting the *Cape of Good Hope*: Through which, if you draw a Right Line C A, parallel to the *Æquinoctial*, it ſhall repreſent the *Parallel* of the *Latitude* of the *Cape of Good Hope*, *viz.* of 35. 85 deg. of *South* Latitude.

Laſtly, Draw the Lines H C, C A and A H; ſo ſhall you have conſtituted a *Right-lined Triangle*, Right-*angled* at A; from whence ſhall be deduced and reſolved all ſuch *Problems* and *Navigation*, as concern *Longitude*, *Latitude*, *Rumb* (or Courſe) and *Diſtance*.

Fig. LXII.

PROB. I.

TWO *Places*, H in 18. 25 de. of *North Latitude*, and 78. 35 d. of *Weſt Longitude*: And C, in 35. 85 deg. of *South Latitude*; and in 27. 50 de. of *Eaſt Longitude* being *Given*: To *Find*,

I. The *Rumb* leading from one *Place* to the other, the *Angle* H C A.

II. Their *Diſtance*: upon the *Rumb*; H C.

The *Difference* of the *Longitudes*, and *Latitudes* of the two *Places* being *Found*, and Reduced into *Miles*, as is ſhewed in the two foregoing *Problems*: Then, in the Triangle H A C Right-angled at Æ, you have given, (1.) The Perpendicular H A, the *Difference* of *Latitude* 3246 Miles: And, (2.) The Baſe A C, the *Difference* of *Longitude* 6351 Miles: By which you may find the *Rumb* H C A (by CASE I) and the *Diſtance* H C (By CASE VII) of R. A. P. T.

1. For the *Rumb* H C A.

As A C, the *Dif. of Longitude* 6351 M.	3.802842
Is to the *Dif. of Latitude* H A 3246 M.	13.511348
So is the *Radius*, Tang. 45 deg	10.
To the Tang. of 27. 07 deg.	9.708506

Which is the Angle H C A, whoſe *Complement* H C n——A H C, is the *Rumb*, 62. 93 deg. that is, *North-Eaſterly* 62. 93 deg. from C to H, and *South-Weſterly* 62. 93 deg. from H to C.

2. For the *Diſtance* H C.

As the Sine of A H C the *Rumb* 62. 93 deg.	9.949610
Is to A C, the *Dif. of Longitude*, 6351 M.	13.802842
So is the *Rad.* Sine 90 deg.	10.
To H C their *Diſtance*, 7132. 4 M.	3.853232

PROB. II.

THE *Rumb* C H A, 62. 93 deg. And the *Difference of Longitude* A C 6351 M. *Given*, to *Find*,

I. The *Difference of Latitude* H A, And

Fig. LXII.

II. The *Diſtance* H C.
By CASE II. and CASE V. Of R. A. P. T.

1. For the *Difference of Latitude* H A.

As the Sine of A H C, 62. 93 deg.	9.949610
Is to the Sine of H C A 27. 07 de.	9.658086
So is AC, the *Diff. of Longitude* 6351.	3.802842
	13.460928
To the *Diff. of Latitude* H A, 3246 M.	3.511318

2. For the *Diſtance* H C.

As the Sine of the *Rumb* A H C 62. 93 de.	9.949610
Is to A C, the *Diff.* of *Long.* A C, 6351.	13.802842
So is Rad. S. 90 d.	10.
To the *Diſtance* H C 7132. 4 M.	3.853232

PROB. III.

THE *Diſtance* H C 7132. 4 M. And the *Difference of Longitude* A C, 6351 M. *Given*; to *Find*,

I. The *Rumb* A H C, And

II. The *Difference of Latitude* A H.

By CASE III and CASE VI, of R. A. P. T.

1. For the *Rumb* A H C.

As the *Diſtance* H C, 7132. 4.	3.853174
Is to the *Dif. of Longitude* A C, 6351 M.	13.802842
So is *Rad.* S. 90 de	10.
To the Sine of the *Rumb* 62. 93 de.	9.949668

2. For the *Difference* of *Latitude* H A.

Having found the Rumb, by the Laſt, Say.

As the Sine of the *Rumb* A H C, 62. 93 de.	9.949610
Is to the Dif. of Longitude A C, 6351 M.	3.802842
So is the Sine of H C A, 27. 07 deg.	9.658086
	13.460928
To the Dif. of Latitude H A. 3246 M.	

Fig. LXII.

PROB. IV.

THE *Diſtance* C H, 7132. 4 M. And the *Difference of Latitude* H A 3246 M. *Given*: To find,

I. The *Rumb* H C A,

II. The *Difference of Longitude* A C.

By CASE III. and CASE VI. of R. A. P. T.

1. For the *Rumb*.

As the Diſtance H C 7132. 4 M.	3.853232
Is to the Dif. Latit. H A 3246	13.511348
So is the Radius Sine 90 de.	10,
To the Sine of 27. 07 de. H C A,	9.658116

Whoſe Complement is A H C the Rumb 62. 93 de.

2. For the *Difference of Longitude* A C.

As the Radius Sine 90 de.	10.
Is to the *Diſtance* H C 7132. 4 M.	3.853232
So is the Sine of H C A 27. 07 de.	9.658086
To the *Diff. of Longitude* A C 6351 M.	13.511318

PROB. V.

A *Ship* at H, the Harbour of *Jamaica*, in 18. 25 de. of *North Latitude*; and *Weſt-Longitude* 78. 35. de. being bound for the *Lyzard*, which lyes in 50. 34 de. of *North Latitude*; and 5. 96 de. of *Weſt Longitude*: And ſhe ſails upon theſe ſeveral *Courſes*, *viz.* (1.) *North-Eaſt* 16. 25 deg. 42. 16 *Miles* from H to K.——(2.) *North-Eaſt* 60. 00 deg. 25. 36 *Miles* from K to L——(3.) Directly *North*, 32. 00 Miles from L to M——(4.) *North-Eaſt*, 60. 12 deg. 28. 16 *Miles*, from M to N.——(5.) *North-Eaſt*, 46. 50 deg. 40. 25 *Miles*, from N to O.——(6.) Directly *Eaſt* 28. 00 *Miles*, from O to P——And now I would know,

I. The *Difference* of *Latitude*, P R, and *Difference* of *Longitude*, H R, at the end of each of theſe *Courſes*.

II. How the Ship being at P, *bears* from the *Harbour* at H, from whence ſhe came.

III. In what *Longitude* and *Latitude* ſhe then is.

Fig. LXII.

IV. The neareſt *Diſtance* from P to H.

I. *The* Geometrical Conſtruction *and (gradual)* Trigonometrical Calculation, *of the whole* Travers.

DRAW a Right Line N S for the *Meridian*, and another E W at Right-angles thereto, for the *Parallel* of the *Latitude* of *Jamaica*, croſſing each other at H, the point of the *Harbour*, where the *Voyage* begins. Then,

I. The Firſt *Courſe* being *North-Eaſt* 16. 25 de. make the Angle *a* H K equal thereto: And the *diſtance* Sailed being 42. 16 Miles, ſet them from H to K; and through K, draw *a* K parallel to W E: So ſhall you have conſtituted a *Triangle* H *a* K, right-angled at *a*, in which there is given, (1.) the *Rumb* (or *Courſe*) *a* H K 16. 25 deg. (2.) the *Diſtance* Sailed upon that *Rumb*, 42. 16 Miles: by which you may find, (1) the *Difference* of *Longitude a* K, 11. 80 M. And (2.) the *Difference of Latitude* H *a* 40. 47 M. By CASE IV. of R. A. P. T.

1. For the Difference of *Longitude* a K.

As the Radius Sine 90 deg.	10.
Is to the *Diſtance* ſailed H K, 42. 16 M.	1.624900
So is the Sine of the *Rumb* a H K. 16. 25 d.	9.446893
To *a* K 11. 80 M. the Difference of *Long.*	+1.071793

2. For the Difference of *Latitude* H *a*.

As the Radius, Sine 90 deg.	10.
Is to the Co-ſine of the *Rumb* a K H, 73. 75 de.	9.982294
So is the *Diſtance* ſailed H K, 42. 16 M.	1.624900
To H a 40. 47 M, the Dif. of *Latitude.*	+1.607194

And in ſuch difference of *Longitude* and *Latitude* will the Ship be, when arrived to K.

II, The Second *Courſe* being *North-Eaſt* 60. 00 de. through the point K, draw a Right Line *b e* parallel to N S; And on the point K protract an Angle of 60. 00 deg. as *b* K L, and ſet the diſtance ſailed 25. 36 M. from K to L; and through L, draw L *b* parallel to W E: Then in the *Triangle b* K L, there is given the Angle b K L the *Rumb* 60. 00 d. and the Side K L 25. 36

36 M. the diſtance upon the *Rumb*, whereby may be found *b* L the Differ. of *Long.* and K *b* the Difference of *Latitude*, as in the former *Courſe*. *Fig.* LXII.

1. For the Difference of *Longitude* b L.

As the Radius	10.
To the *Diſtance* Sailed K L 25. 36 M.	1.404149
So is the Sine of the *Rumb* b K L 60. 00 d.	9.937530
To the Difference of *Longitude* b L 21. 96 M.	✠1.341679

2. For the Difference of *Latitude* K b.

As the Radius	10.
To the *Diſtance* Sailed K L, 25. 36 M.	1.404149
So is the Co-ſine of the *Rumb* b L K 30 00 d.	9.698970
To the Difference of *Latitude* K b 12. 68 M.	✠1.103119

III. The Third *Courſe*, being *directly North*, and the *Diſtance* Sailed 32. 00 M. Through the Point L, draw a Right *Line* L c parallel to N S. And becauſe the *Courſe* was directly *North*, ſet 32. 00. M. (the *Diſtance* ſailed) from L to M; and to that Point is the Ship now arrived; which being in the ſame *Longitude* as when ſhe was at L, the difference of *Longitude* is nothing. But the difference of *Latitude* is the ſame with the *Diſtance* Sailed; *viz.* 32. 00 Miles, or *Minutes*. So that for this *Courſe* there is no need of any *Trigonometrical Calculation*.

IV. The Fourth *Courſe* being *North-Eaſt* 60. 12 deg. continue the Line L M to *c*. And on the Point M, protract the Angle of the *Rumb*, *c* M N 60. 12 m. and ſet the *diſtance* ſailed 28. 46 M. from M to N: And through N, draw N *c*, parallel to W E. Then, in the Triangle M *c* N, there is given the *Angle c* M N 60. 12 de. the *Courſe* or *Rumb*: And the Side M N 28. 46 M. the *Diſtance* Sailed; by which you may find the Side *c* N, the Difference of *Longitude*; and M *c* the Difference of *Latitude*: In the ſame manner as in the two firſt *Courſes*.

1. For the Difference of *Longitude* c M.

As the Radius	10.
Is to the *Diſtance* Sailed, M N 26. 46 M.	1.422589
So is the Sine of the *Rumb* c N M, 60 12 d.	9.938054
To the Difference of *Longitude* c N 22. 94 M.	✠1.360643

2. For

Fig. LXII.

2. For the Difference of *Latitude* M *c*.

As the Radius	10.
Is to the *Diſtance* Sailed M N, 26. 46 M.	1.422589
So is the Co-ſine of the *Rumb* c N M 29. 88 M.	9.697391
To the Difference of *Latitude* M c, 13. 18 M.	✠1.119980

V. The Fifth *Courſe* being *North-Eaſt* 46. 50 d. through the Point N, draw a Right Line *d* N *f*, parallel to the *Meridian* N S: And upon the Point N, protract the *Angle* of the *Rumb*, *d* N O, 46. 50 deg. And ſet the *Diſtance* ſailed 40. 25 M. from N to O; and through O, draw *d* O parallel to W E. Then, in the Triangle N *d* O, you have given the *Angle d* N O the *Rumb*, 46. 50 deg. And the Side N O, the *Diſtance* Sailed 40. 25 M. by which may be found, the difference of *Longitude d* O, and the difference of *Latitude* N *d*, as before.

1. For the Difference of *Longitude*, *d* O.

As the Radius Sine 90 de.	10.
To the *Diſtance* ſailed, N O 40. 25 M.	1.604766
So is the Sine of the *Rumb* d N O, 46. 50 d.	9.860562
To the Difference of *Longitude* d O, 29. 19 M.	✠1.465328

2. For the Difference of *Latitude* N *d*.

As the Radius	10.
To the *Diſtance* Sailed, N O, 40. 25 M.	1.604766
So is the Co-Sine of the *Rumb* d O N, 43. 50 d.	9.837812
To the Difference of *Latitude* N d 27. 70 M.	✠1.442578

VI. The Sixth *Courſe* being directly *Eaſt*: And the *Diſtance* Sailed 28. 00 Miles, continue the Line or *Parallel d* O, and ſet the *Diſtance* ſailed upon it, *viz.* 28. 00 M. from O to P, and to that Point is the Ship now arrived: which being in the ſame *Parallel* of *Latitude* as when ſhe was at O, the Difference of *Latitude* is nothing: But the Difference of *Longitude* is equal to the *Diſtance* Sailed; namely 28. 00 M. So that for this *Courſe*, there needs no *Trigonometrical Calculation*.

Fig. LXIII. Having thus finiſhed all the *Courſes*, you find the *Difference* of *Longitude* and *Difference* of *Latitude* to be ſuch as are ſet down in the *Scheme* of the *Travers*. And thus is the *Firſt part* of the *Problem* reſolved. Now for the Second.

VII. From

Fig. LXIII.

VII. From the Point P (in your *Travers*, *Fig.* LXIII) draw a Right Line to H, the *Harbour* from whence you began, and from the ſame Point P, let fall the *Perpendicular* P R, upon the Line W E: So ſhall you have conſtituted a *Right-angled Plain Triangle* H R P, Right-angled at R: By the Solution whereof, the other parts of the *Problem* may be *Trigonometrically* reſolved.

VIII. Collect the ſeveral *Differences* of *Longitude* and *Latitude*, into one Sum as is done in the following Table.

		Difference of Longitude is		Difference of Latitude is
		Miles		Miles
In the *Courſe* from H to K	a K	11. 80	H a	40. 47
K to L	b L	21. 96	K b	12. 68
L to M	M	———	L M	32. 00
M to N	c N	22. 94	M c	13. 18
N to O	d O	29. 19	N d	27. 70
O to P	O P	28. 00	O	———
	H R.	113. 89	P E.	126. 03

By the *Table* you find the *Sum* of all the *Differences* of *Longitude* to be 113. 89 *Miles*, or *Minutes*: Which reduced into *Degrees* and *Minutes* (by dividing them by 60) they make 1. 91 deg. Which ſubſtracted from the *Longitude* of *Jamaica*, 78. 35 deg. there remains 76. 34 deg. for the *Longitude* in which the *Ship* is, being arrived to P.

Alſo, you find by the *Table*, the *Sum* of all the *Differences* of *Latitude* to be 126. 33 *Miles*: which reduced into *Degrees*, makes 2. 10. deg. Which being added to 18. 25 deg. the *Latitude* of *Jamaica*, it makes 20. 35 deg. for the *Latitude* in which the *Ship* is at P. And thus is the *Second* part of the *Problem* reſolved.

Now for the *Third*.

IX. The next thing to be found is the *Rumb*, *Courſe*, or *Bearing* of the *Ship*, ſhe being at P, to H, the *Harbour* from whence ſhe came.——In the Triangle P R H, there is given, (1) The

Fig. LXIII. Side H R, the *Difference* of *Longitude* 113. 89 min. (2) The *Difference* of *Latitude* P R, 126. 03 min. To find the Angle H P R, the *Rumb*: by CASE I. of R. A. P. T.

As the Side P R, Dif. *Latit.* 126. 03 M.	2.100474
Is to the Side H R, Dif. *Lon.* 113. 89 M.	12.056485
So is Radius, Tang. 45. 00 de.	10.
To the Tangent of the *Rumb* H P R 42. 11 d.	9.956011

So that the *Courſe* from P to H is *South-Weſt*, 42. 11 deg. and from H to P, *North-Weſt* as much.

X. The laſt thing to be found is, the *ſhorteſt Diſtance* (or *Diſtance* upon the *Rumb*) between H and P: By CASE V. of R. A. P. T.

As the Sine of the *Rumb*, H P R, 42. 11 d.	9.826435
Is to the Difference of *Longitude* H R, 113. 89 M.	12.056485
So is the Radius, Sine 90 d.	10.
To the *Diſtance* upon the *Rumb*, P H, 169. 83 M.	2.230050

PROB. VI.

IF from the *Latitude* of 18. 25 deg. at H, I ſail upon ſome *Rumb* between the *North* and the *Eaſt*, as H P 169. 83 *Miles*; and then find that I have departed from the *Meridian* from whence I came 113. 89 min. I would then know,

I. Upon what *Rumb* I ſteered: And
II. How much I have *altered* my *Latitude*.

THIS *Problem* may be reſolved in the *Triangle* H R P: In which there is *Given*, (1) The Side H P, the *Diſtance Sailed* upon the unknown *Rumb*, 169. 83 M. (2) The Side H R, the *Departure* from the *Meridian* of H, from whence you ſailed, 113. 89 M. by which you may find the Angle P H R, the *Rumb* ſailed upon: By CASE III, of R. A. P. T.

As the *Diſtance* ſailed H P, 169. 83 M.	2.229014
Is to the Radius, Sine 90 de.	10.
So is the *Departure* H R, 113. 89 M.	12.053191
To the Sine of P H R, 42. 11 deg.	9.824177

Whoſe Complement 47. 89 deg. is the Angle P H R, that is *North Eaſt* 47. 89 de. the *Rumb*.

Then

Fig. LXIII.

Then for the Difference of *Latitude* P R.

As the Co-ſine of the *Rumb* H P R, 42, 11 d.	9.826435
Is to the *Departure* H R, 113. 89 M.	2.056485
So is the Sine of *Rumb* P H R, 47. 89 d.	9.870321
	11.926806
To the Side P R 126. 03 M. the *Diff.* of *Latit.*	2.100371

Which reduced into deg. makes 2. 10 deg. which is the difference of *Latitudes*, which being added to 18. 25 deg. The Latitude of H, from whence you came, makes 20. 35 de. For the *Latitude* in which you are at P.

The Eight foregoing *PROBLEMS*, were all reſolved by *Right-angled Plain Triangles*: Theſe which follow, fall under the CASES of *Oblique-angled Plain Triangles.*

PROB. VII.

Fig. LXIV.

A *Ship* at C, in 22 deg. of *North Latitude*, Sails *North-Eaſt* 22. 50 deg. 64. 50 *Miles* to D: And from D, ſhe Sails between the *South* and the *Eaſt* 124. 00 *Miles*; and then finds that ſhe is in the ſame *Latitude* from whence ſhe firſt came: I would now know.

I. What was the *Rumb*, (or *Courſe*) from D to F.
II. The Difference of *Longitude* betwen C and F.
III. What *Latitude* the Ship was in, when ſhe was at D.

The Geometrical Conſtruction of the *Figure.*

Firſt, Draw a Right Line N S for the *Meridian*; and at Right Angles thereto another W E, for the *Parallel* of *Latitude* 22. 00 deg. *North.*

Secondly, Upon E, Protract the Angle of the *Rumb North-Eaſt* 22. 50 deg. *a* C D and ſet thereon 64. 50 M. the *diſtance ſailed* from E to D; and through the point D, draw an obſcure Line D *a*, parallel to the Line W E.

Thirdly, Take the *Diſtance Sailed* from D to F 124. 00 Miles in your Compaſſes, and ſetting one foot in D, with the other deſcribe the Arch *b c*, croſſing the *Parallel* of *Latitude* W E, in the point F.

Fig. LXIV. Fourthly, Joyn C D and F D, ſo ſhall you have conſtituted an *Oblique-angled Triangle* D C F, In which there is *Given* (1) The *Side* C D, 64.50 M. (2.) The *Side* D F 124.00 M. and (3.) The *Angle* D C F, 67.50 de. the Complement of the *Rumb* ſailed upon from C to D; to find (1.) The Angles at D and F, the *Rumbs*: (2.) The Difference of *Longitude* between C and F: And (3.) The Difference of *Latitude*, C *a*.

By Trigonometrical Calculation.

1. For the *Rumb* (or *Courſe*) D F C, By CASE I. of O. A. P. T.

As the Diſt. ſailed from D to F 124.00 M.	2.093421
Is to the Co-ſine of the *Rumb*, from C to D, 22.50 d.	9.965615
So is the *Diſtance* ſailed, from C to D 64.50 M.	1.809559
	11.775174
To the Sine of the Angle D F C 28.72 d.	9.681753

Whoſe Complement 61.28 de. is the *Rumb* from F to D, that is *North-Weſt* 61.28 de: And *South-Eaſt* 61.28 de. from D to F.

2. For the *Difference* of *Longitude* C F.

The Angles at C and F being known, the Angle at D is alſo known, and will be found to be 96.22 de. Then,

As the Sine of D F C, 28.72 deg.	9.681720
Is to the *Diſtance* ſailed from C to D 64.50 M.	1.809559
So is the Sine of the Angle C D F, 83.78 de.	9.997411
	11.806970
To the *Diff.* of *Longitude* C F, 133.43 M.	1.125250

3. For the *Diff.* of *Latitude*, D G, equal to C *a*.
By CASE IV. of R. A. P. T.

As the Radius Sine 90 de.	10.
To the *Diſt.* ſailed from C to D, 64.50 M.	1.809559
So is the Co-ſine of the *Rumb* from C to D 67.50 d.	9.965615
To the Dif. of *Latitude* D G = *a* C, 59.59 L.	1.775174

PROB.

PROB. VIII.

TWO *Ships* ſet Sail, from the *Port* P, One of them Sails *North-Weſt* 70. 00 deg. 106. 00 *Miles* to O; The other Sails *South-Weſt* 74. 00 deg. 236. 60 Miles, to M. I demand, *Fig.* LXV.

I. How the two *Ships* at O and M, do *Bear* from each other: And

II. Their *Diſtance* M O.

The Geometrical Conſtruction.

Having drawn your *Meridian* N S, and *Parallel* W E, croſſing each other at Right Angles in the point P, which let be the *Port* from whence the two Ships ſailed. Then,

Firſt, Upon P; deſcribe an Arch of a Circle, as *n o m*: And becauſe the *Courſe* from P to O, was *North-Weſt* 70. 00 deg. ſet 20 deg. (the Complement thereof) from *o* to *m*; and through *m*, draw a Line, as P *m* O, ſetting thereon 106. 00 *Miles*, from P to O.

Secondly, Becauſe the *Courſe* from P to M, was *South-weſt* 74. 00 deg. ſet 16. 00 deg. (the Complement thereof) from *o* to *n*, And through *n*, draw P *n*, ſetting thereon; the Diſtance ſailed, *viz.* 236. 60 Miles from P to M.

Thirdly, Joyn O and M. So ſhall you have conſtituted an *Oblique-Angled Triangle* P O M: In which there is *Given*, (1.) The Angle O P M; 36 00 deg. the *Sum* of the Complements of the *Bearings* of the two Ships. (2.) The two Sides P O 106. 00 Miles and P M 236. 60 Miles. The ſeveral *Diſtances* which the two Ships ſailed: And by help of theſe, you may Find. (1) The *Bearing* of the two *Ships* one from the other: And (2.) Their *Diſtance*.

By Trigonometrical Calculation.

I. For the *Bearings*, O M P and M O P;

By CASE II. of O. A. P. T.

The *Sum* of the *Sides*, P O and O M, is,	342.06. M.
Their *Difference*.	130.06. M.
The *half Sum* of the Un-known *Angles* M and O, is	72.00. De.

Being thus prepared, ſay,

Fig. LXV.

As the *Sum* of the *Sides* 342. 06	2.534102
Is to the *Difference* of the *Sides* 130. 06	2.113144
So is the Tangent of the ½ Sum of the *Angles* 72 d.	10.448224
	12.561368
To the Tangent of 46. 80 degrees.	10.027266

Which 46. 80 deg. *Added* to 72. 00 deg. (the *half Sum* of the Angles O and M) gives 118. 80 d. for the Angle M O P; And 46. 80 d. ſubſtracted from 72 00 d. leaves 25. 20 deg. for the leſſer Angle O M P.

Now for the *Bearing*, ſubſtract the *Bearing* of the Ship from P to O, *viz.* 72. 00 deg. from 118. 80 deg. (the quantity of the M O P) there will remain 48. 80 deg. for the Angle M O Q, which is *South-Weſt* 48. 80 d. and ſo doth the Ship at O *Bear* from the Ship at M. And the Ship M, from O, *North-Eaſt* 48. 00 deg.

2. For the diſtance of the Ships M and O.

As the Sine of M O P, 118. 80 d. (61. 20)	9.942656
Is to the *Diſtance* ſailed from P to M. 236. 60 d.	2.374014
So is the Sine of O P M, 36. 00 deg.	9.769218
	12.143232
To the *Diſtance* O M 158. 70 M.	2.200576

PROB. IX.

Fig. LXVI. THERE are Two Iſlands, as K and M, which lie directly *North* and *South* of each other, and are diſtant 37. 75 Miles; And there is a third Iſland at L, which is diſtant from that at M, 59. 64 Miles; and from that at K, 80. 92 Miles: Now I would know, How the *Iſland* at L, bears from the other two Iſlands M and K.

The Geometrical Conſtruction.

Having drawn N S for the *Meridian*, and W E for a *Parallel* of *Latitude*, croſſing each other in the point K, which let be the place of one of the *Iſlands*; Then

The *Iſland* M, lying directly *North* therefrom, at the Diſtance of 37. 75 M. Set 37. 75 from K to M.—— And becauſe the *Iſland* at L is diſtant from the *Iſland* at K, 80. 92 M, with that diſtance, ſet one foot of the Compaſſes in K, and with the other deſcribe the Arch *a b*.——Alſo, the diſtance of the *Iſland* at L, being

Fig. LXVI.

being diſtant from M, 59. 64 Mil. with that diſtance ſet one foot in M, and with the other deſcribe the arch *c d*, croſſing the former arch *a b*, in L: ——Laſtly, Joyn L M, and L K, ſo ſhall you have conſtituted an *Oblique-Angled Triangle* K L M; in which all the three *Sides* are *Given* And the *Angles* at K and M are Required:

The *Trigonometrical* Calculation.

As the *Greateſt Side* K L; 80. 92 M.	1.908056
Is to the *Sum* of K M and L M, 97. 39 M.	1.988425
So is the *Difference* of thoſe *Sides*, 21. 89 M.	1.340246
	3.328671
To the Part of the *Greater Side* L O, 26. 34.	1.420615

This L O, 26. 34, ſubſtracted from K L 80. 92 leaves 54. 58 for the other part of the longeſt ſide K O; and the half thereof, 27. 29 is the part K P or P O. And now, the *Oblique-Angled Triangle* K L M, is Reduced into two *Right-angled Triangles* M P K, and M P L; in both which the *Hypotenuſes* and *Baſes* are given, by which the other *Angles* may be found, By CASE III. of R. A. P. T. According to which, you ſhall find the *Angle* M K P. to be 43. 71 de. and the *Angle* M L K 25. 94 deg.—So that L Bears from K *North-Eaſt* 43. 71 deg.——And M from *North-Eaſt* 69. 65 deg.

PROB. X.

Fig. LXVII.

There are two *Head-Lands* at B and C, which are diſtant 356.00 *Miles*; And there is a *Port* at A, from whence two *Ships* ſail to thoſe two *Head-Lands*, the Ship from A to C, ſails *South-Eaſt* 34. 90 deg. and that from A to B, *South-Weſt* 72. 10 de. And both the Ships together have ſailed 424. 12 Miles. I demand the *Diſtance* that each *Ship ſailed*; Or, how far are either of the *Head-Lands* from the *Port* at A?

The Geometrical Conſtruction.

ONE Ship ſailing *South-Eaſt* 34. 90 deg. and the other *South-Weſt* 72. 10 deg. the diſtance of thoſe *Rumbs* is 107. 00 de. Wherefore, *Fig.* LXVII. Firſt, Draw two Lines at pleaſure, as A B and A C, making the *Angle* at A, equal to 107. 00 de.

Secondly, Take 356. 00 Miles, in your Compaſſes, and enter them between the two Lines at pleaſure, as at B and C: conſtituting the *Triangle* A B C.

Thirdly, Continue the Side B A to D, making A D equal to A C, and joyn C D, conſtituting another Oblique-angled Triangle D C A, in which there is given. (1) The Angle at C A B, 73.00 deg. (the Complement of B A C to 180 deg.) And (2) The Angles A C D and A D C, each of them equal to half the Angle B A C, *viz.* to 53.50 deg. Then in the Oblique-Angled Triangle D C D, ſay,

As the given Side B C, 356.00 M.	2.551450
Is to the Sine of B D C, 53.50 de.	9.905178
So is the Side B D, 424.12 M.	2.627488
To the Sine of 73.27 deg.	12.532666
	9.981216

Which is the quantity of the Angle D C B: From which ſubſtract the Angle A C D, 53.50 de. there will remain 19.77 d. for the Angle A C B. Then in the Triangle B A C,

As the Sine B A C, 107.00 d. (73.00)	9.980619
Is to the Side B C, 356.00.	2.551450
So is the Sine of A C B, 19.77 de.	9.529231
	12.080681
To B A, 125.90 M.	2.100062

Which ſubſtracted from B D, 424.12, there remains 298 M, for the Side A C. the Diſtance that the other Ship ſailed. So that the Port A, is diſtant from the Head-Land C 298.22 M. And from that at B, 125.90.

PROB. XI.

Fig. LXVIII. Two Ships ſet ſail from two Head-lands at B and C, diſtant from each other 356. 00 Miles; and meeting together at A, they obſerve that the two *Head lands* bear ſo from their *Ship*, that they make an *Angle* of 107. 00 deg. Alſo, the way that both the Ships have made from B and C to A, doth amount unto 424. 12 Miles: I demand how much of this each Ship ſailed.

Fig. LXVIII.

The Geometrical Conſtruction.

Firſt, DRAW a Right-line at pleaſure, as B A D, and upon any point thereof, as A, protract an Angle of 107. 00 deg.

Secondly, The Diſtance of the two *Head-lands* being 356. 00 Miles enter them between the two ſides, at B and C, and joyn B C, making the Triangle B A C.

Thirdly, Make A D equal to A C, and joyn C D, ſo have you conſtituted another Triangle A C D.

Now in the Triangle A B C, is Given. (1) The Side B C, 356 00. (2) The Angle B A C 107. 00 deg: And in the Triangle A C D, the Angle B A C being 107. 00 de. the Angle D A C muſt be 73. 00 de. And the Sides A C and A D, being equal (by conſtruction) the Angles A D C and A C D, muſt be alſo equal, *viz.* (each of them) 53. 50 deg. And being thus far prepared, you may proceed to find the Angle A C B.

By Trigonometrical Calculation.

As the Side B C, 356. 00	2.551449
To the Sine of the Angle A D C, 53. 50 d.	9.905178
So is the whole Side B A D, 424. 12 M.	2.627000
	12.532178
To a fourth Sine, 73 06 d.	9.980729

This 73. 06 d. ſhould be the quantity of the Angle D C B, but (by the conſtruction) the Angle you ſee is Obtuſe; and therefore the Complement of 73. 06 deg. to 180, *viz.* 106. 94 deg. is the quantity of the whole Angle D C B. From which if you ſubſtract the Angle A C D, 53. 50 deg. there will remain 53. 44 deg. for the Angle A C B.

Then

As the Sine of B A C, 107. (73. 00d.)	9.980619
Is to B C 356. 00.	2.551449
So is Sine A C B, 53. 44 deg.	9.904842
	12.456291
To the Side B A, 299. 00———	2.475672

And ſo much did the Ship from B, ſail to A, and that ſubſtracted from 424. 12 there remains 175. 12, and ſo much did the Ship from C to A Sail.

II. *Of* MERCATOR's Sailing, *by the True* Sea-Chart.

ALL the foregoing *Problems* are performed by that kind of *Navigation*, commonly called *Plain Sailing*, or *Sailing* by the *Plain Sea-Chart*; in which *Chart*, the *Degrees* of *Longitude* and *Latitude*, in all Places, are ſuppoſed to be equal; which is *Erroneous*, though moſt practiſed: But there are two other ways of *Sailing*, both more exact than the former, the one called *Mercator's*, the other, *Sailing* by the *Middle Latitude*.

That of *Mercator's* requires that the *Degrees* of *Latitude* in that *Chart* be *inlarged* as they go farther from the *Æquinoctial* towards either of the *Poles*, which is done by *Reducing* them into *Meridional Parts*: but the *Degrees* of *Longitudes* into *Miles* and *Centeſms*, as in *Plain Sailing*: And for the *Reducing* of *Degrees* and *Minutes* of *Latitude* into *Meridional Parts*, I have here inſerted a *Table* for the ready performance thereof, to every *Degree* and *Quarter*, or 25 *Centeſms* of a *Degree*.

75
011
082
153
226
300
374
449
526
603
682
762
843
925
3009
3094
3181
3269
3359
3451
3545
3641
3739
3839
3940
4047
4155
4266
4380
4498
4619
4744

II.

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25. *C*

A Table of Meridional Parts.

	Centeſines.					Centeſines.			
Degrees	0	25	50	75	Degrees	0	25	50	75
0	0	15	30	45	31	1950	1976	1993	2011
1	60	75	90	105	32	2028	2046	2064	2082
2	120	135	150	165	33	2100	2117	2135	2153
3	180	195	210	225	34	2171	2190	2208	2226
4	240	255	270	285	35	2244	2263	2281	2300
5	300	315	330	346	36	2318	2337	2355	2374
6	361	376	391	406	37	2393	2411	2430	2449
7	421	436	451	466	38	2468	2487	2507	2526
8	482	497	512	527	39	2549	2564	2584	2603
9	542	557	573	588	40	2623	2642	2662	2682
10	603	618	634	649	41	2702	2722	2742	2762
11	664	679	695	710	42	2782	2802	2822	2843
12	725	741	756	771	43	2863	2884	2904	2925
13	787	802	818	833	44	2946	2967	2988	3009
14	848	864	879	895	45	3030	3051	3073	3094
15	910	926	942	957	46	3116	3137	3159	3181
16	973	988	1004	1020	47	3203	3225	3247	3269
17	1035	1051	1067	1082	48	3292	3314	3337	3359
18	1098	1114	1130	1146	49	3382	3405	3428	3451
19	1161	1177	1193	1209	50	3475	3498	3522	3545
20	1225	1241	1257	1273	51	3567	3593	3617	3641
21	1289	1305	1321	1338	52	3665	3690	3714	3739
22	1354	1370	1386	1402	53	3764	3789	3814	3839
23	1419	1435	1451	1468	54	3865	3890	3916	3940
24	1484	1509	1517	1533	55	3968	3994	4021	4047
25	1550	1567	1583	1600	56	4074	4101	4128	4155
26	1616	1633	1650	1667	57	4183	4211	4238	4266
27	1684	1700	1717	1734	58	4295	4323	4352	4380
28	1751	1768	1785	1802	59	4409	4439	4463	4498
29	1819	1837	1854	1871	60	4528	4558	4588	4619
30	1888	1906	1923	1941	61	4650	4681	4712	4744

A Table of Meridional Parts.

Degrees	Centeſmes.			
	0	25	50	75
62	4715	4807	4840	4872
63	4905	4939	4972	5006
64	5040	5074	5109	5144
65	5179	5215	5251	5287
66	5324	5361	5399	5436
67	5475	5513	5552	5592
68	5631	5672	5712	5754
69	5795	5837	5880	5923
70	5967	6011	6056	6101
71	6147	6193	6240	6288
72	6336	6381	6432	6484
73	6535	6587	6640	6693
74	6747	6802	6857	6914
75	6972	7030	7089	7150
76	7211	7274	7338	7403
77	7469	7536	7605	7675
78	7746	7819	7894	7970
79	8048	8127	8209	8292
80	8377	8465	8555	8647
81	8742	8839	8939	9042
82	9148	9258	9371	9488
83	9609	9735	9865	10000
84	10141	10288	10441	10601
85	10770	10946	11133	11330
86	11539	11761	11399	12255
87	12521	12821	14015	13513
88	13920	14381	14914	15545
89	16318	17316	18729	21170
90				

The Uſe of the Table of *Meridional Parts.*

What *Meridional Parts* do anſwer to 26 deg. of *Latitude.*

Latitude	26. 00 Deg—	1616	Merid. Parts.
	24. 25 Deg—	1509	M. P.
	47. 50 Deg—	3247	M. P.
	63. 00 Deg—	4905	M. P. &c.

2. What *Degrees* and *Ceteſms* do anſwer to.

679	Merid. Parts.	Anſwer	11. 25 Deg.
3522	M. P.		50. 50 Deg.
9488	M. P.		83. 75 Deg.

For the Reſolving of the following *Problems*, which (in kind) will be the ſame with thoſe in *Plain Sailing* as to the *Trigonometrical Work.* But in the performance of them, I will ſet the places propoſed down upon a *Sea-Chart*, made according to the *Projection* of *Mercator*, where the Degrees of *Latitude* are *enlarged* according as they tend nearer and nearer to the *Pole* : And on the out-ſide of ſuch a *Chart*, I have deſcribed a *Plain Sea-Chart* alſo, where the *Meridians* and *Parallels* of *Latitude* are every where of an equal diſtance; and ſo wrought the *Queſtions* according to both *Charts*, by which the *difference* will more plainly appear: And the *Chart* which I have here made to work the following *Examples upon*, begins (at the bottom of it) at about 49.50 deg. of *Latitude*, and extends upwards to 55. 50 deg. of *Latitude* ; and at the *Top* and *Bottom* to *Six* Degrees *Difference* of *Longitude* : But the *Parallels* of *Latitude* in the *Plain Chart*, (which are diſtinguiſhed by *Pricked Lines*) extend to above 59 deg. of *Latitude* within the ſame Bounds. *Fig.* LXIX.

PROB. I.

The *Latitudes* of two *Places*, and their *difference of Longitude* being known ; To find, (1) The *Rumb* leading from one to the other : And (2) Their *Diſtance* upon that *Rumb* : according to *Mercators Chart.*

By Trigonometrical Calculation

LET one Place be at A in the *Latitude* of 50 deg. and the other at C, in the *Latitude of* 55 deg. but differing in *Longitude* Eaſtward from B, 6. 50 deg. which two places thus ſet down in the

Fig. LXIX. the *Chart*, draw the Line A C, which is the *Rumb* (or *Courſe*) from A to C: And thus have you upon your *Chart* conſtituted a *Right-angled Plain Triangle* A B C, in which you have given, (1) B C, the *Difference of Longitude* 6.50 deg. which you muſt *Reduce* into *Miles* (or *Minutes*) of *Longitude* by multiplying them by 60, (as in *Plain Sailing*) and they make 330. 00 Miles.—— (2) A B, the *Difference of Latitude* 5. 00 deg. which muſt be Reduced into *Meridional Parts* thus:

The *Meridional Parts* for 50. 00 d. are	3475
For 55. 00	3968
Their *Difference*——	493

Which are the *Meridional Parts* anſwering to the 5. 00 deg. *Difference* of *Latitude* of the two *Places* A and C: And by theſe you may find;

(1.) The *Rumb* B A C.

(2) The *Meridional Diſtance* upon the *Rumb* A C.

By Trigonometrical Calculation.

(1.) For the *Rumb* B A C

As the Merid. Dif. of *Latitude* A B, 493 M. P.	2.692847
Is to the Dif. of *Longitude* B C 330 Miles	12.518514
So is the Radius, Tangent 45 deg.	10.
To the Tangent of B A C, 33. 80 degrees.	9.825667

Which is the *Rumb*, leading from A to C: whoſe Complement 56. 20 deg. is the Angle B C A, or the *Complement* of the *Rumb*:

(2.) For the *Meridional Diſtance* on the *Rumb* A C.

As the Co-ſine of the *Rumb*: B C A, 56. 20 d.	9.919592
Is to the *Merid. Differ. of Latitude* A B 493 M. P.	12.692847
So is Radius Sine 90 deg.	10.
To the *Merid. Diſtance* A C, 593.27 M. P.	2.773255

Now to find the *Meridional Degrees* anſwerable to theſe *Meridional Parts*, you muſt

Firſt, Subſtract the M P O of A B, from the M P of A C, their *Difference* is 100. 27 M. P. the half whereof is 50.13 M. P.

Secondly, *Add* this *half Difference* 50. 13, to the *Meridional Parts* of the *Greater Latitude* 55. 00 deg. *viz.* 3968, and it makes 4018. 13, which are the *Merid. Parts* anſwering to 55.50 deg. of *Latitude*.—— Alſo, *ſubſtract* this *half Difference* 50. 13, from the *Merid. Parts* of the *Leſſer Latitude* 50.00 deg. *viz.* 3475, and the

Fig. LXIX.

Remainder will be 3424. 87, which are the *Merid. Parts* anſwering to 49. 38 deg. of *Latitude.*

Thirdly, The *Difference* of theſe two *Latitudes* laſt ſound, *viz.* 55. 50 de. and 49. 38 deg. is 6. 12 deg. And that is the *true Diſtance* upon the *Rumb* between A and C, in *Degrees.*

And according to this Method, may all the other *Problems* (before wrought by the *Plain Sea-Chart*) be performed by *Tri. Calculation*; by the ſame *Canons*: Only remember, to *Reduce* the *Difference of Latitudes* and *Diſtance* on the *Rumb*, into *Meridional Parts*, but the *Difference* of *Longitude* into Miles, as in the other.

Now that you may ſee the *Difference* between theſe two ways; See the *Figure* of the *Chart*, wherein the two *Places* are laid down by the *Plain Chart*; as in the Triangle AOS, which Triangle being reſolved (as is before ſhewed) you will find

(1) The *Rumb* OAS, to be 47. 73 deg.
Differing from the truth 13. 93 deg.
(2) The *Diſtance upon the Rumb* AS, to be 7. 43 de.
Differing from the other 1. 31.

And let thus much ſuffice for *Sailing* according to *Mercator.*

There is a third way of *Sailing*, which differs not much from this way of *Mercator's*, but may be performed without *Reduction* or uſe of *Meridional Parts*; which is called *Sailing* by the *Middle Latitude*: of which a little.

III. *Of* Sailing *by the* Middle Latitude.

I Shall exemplefie this way of Sailing by *Four* of the moſt uſual *Problems* in *Navigation*: And they ſhall be the ſame as in the foregoing: whereby the Difference will the better appear.

PROB. I.

The *Longitude* and *Latitude* of two Places, A and C, Given to find,

(1) The *Rumb* BAC.
(2) The *Diſtance* upon the *Rumb*: AC.

LET one Place be in 50. 00 de. the other in 55. 00 de. of *North Latitude*: And 6. 50 deg. *difference in Longitude*: Then the *Middle Latitude* between 50. 00 deg. and 55.00 deg. is 52. 50 de. And its Complement 37. 50 deg. Then

(1) For the *Rumb*: The Proportion is,

As the *Difference of Lat.* 300. 00 Miles.	2.477121
Is to the *Dif.* of *Longit.* 330. 00 Miles.	2.518524
So is the Co-ſine of *Middle Latitude*, 37. 50 de.	9.784447
	12.302971
To the *Tangent* of 33. 81 deg.	9.825850
Which is the *Rumb* B A C.	

(2) For the *Diſtance* upon the *Rumb*.
The Proportion is,

As the Co-ſine of the *Rumb* 56.19 de.	9.919542
To Radius Sine 90 de.	10.
So is the *Difference* of *Latitude*, 300 M.	12.477121
To the *Diſtance* 361.06 M.	2.557579

PROB. II.

THE *Latitudes of two Places*, A and C, and the *Rumb*, B A C, (or *Courſe*) between them *Given*: to *Find*,

(1) Their *Diſtance* A C.

(2) Their *Difference of Longitude*, B C.

For the *Diſtance* it may be found as in the former *Problem*. But

(2) For their *Difference of Longitude*, This is
The Proportion.

As the Co-ſine of the *Middle Latitude*, 37.50 de.	9.784447
Is to the *Difference* of *Latitude* 300 M.	2.477121
So is the Tang. of the *Rumb*, 33.81 de.	9.825876
	12.302997
To the *Difference of Longitude* B C, 330.01 M.	2.518550

PROB.

PROB. III.

THE *Diſtance* A C, 361.06 M. And both the *Latitudes* Given, To find the *Rumb*.

The Proportion.

As the Diſtance A P, 361.06	2.557579
To the *Difference of Latitude* 300 M.	12.477121
So is *Radius* 90 deg.	10.
To the Sine of 56.19 deg.	9.919542

Whoſe Complement 33.81 deg. is the *Rumb* B A C. from the *Meridian*.

PROB. IV.

HAving one *Latitude*, the *Rumb*, and *Diſtance* upon the *Rumb*, to find the *Difference* of *Latitude*.

The Proportion.

As the Radius Sine 90 deg.	10.
To Co-ſine of the *Rumb*, 56.19 deg.	9.919542
So is the *Diſtance* upon the *Rumb*, 361.06	2.557579
To the *Difference of Latitude*, 300 M.	12.477121

Which Reduced into Degrees, is 5.00 deg.

PROB. V.

HAving the *Rumb* and both *Latitudes* given, to find the *Difference of Longitude*.

The Proportion.

As the Co-ſine of the *Mid. Latitude*, 37.50 de.	9.784447
To the *Difference* of *Latitude*, 300.00 M.	2.477121
So is the Tangent of the *Rumb*, 33.81 deg.	9.825876
	12.302997
To the Tangent of 300.03 M.	2.518550

Which is 5 Deg.

The Differences in theſe Three Ways of Sailing: In this Example.

	Plain Sail.	Mercator.	Mid. Latit.
Differ. of Long.	330.00 M.	330.00 M.	330.00 M.
Differ. of Latit.	300.00 M.	300.00 M.	300.00 M.
Rumb.	47.73 De.	33.80 Deg.	33.81 Deg.
Compl. Rumb.	42.27 De.	56.20 Deg.	56.19 Deg.
Diſtance.	445.97 M. Or 6.50 Deg.	593.27 M.P. Or 6.12 Deg.	361.06 M. Or 6.01 Deg.

JANUA

JANUA MATHEMATICA
VEL,
Trigonometria Practica.

SECTION VII.

OF
Theorical ASTRONOMY.

IN the Fourth *Section* of this Thrid PART, you have the *Doctrine* of *Triangles* applyed to *Practice* in that part of *Aſtronomy* which concerns the *Diurnal Motion* of the *Sun* and *Fixed Stars*: Now, in this Section, I ſhall diſcourſe ſomething of the other Stars or *Planets*, which have a *Secondary Motion* of their own; ſhewing alſo, how ſubſervient *Trigonometry* is for the *Calculating* of their true *Places* at any time, as alſo in computing of their *Magnitudes*, *Diſtances*, &c. And in order thereunto I ſhall firſt give you a brief deſcription of the Planetary *Syſtem*.

CHAP. I.

Of the Planetary Syſtem.

THIS *Syſtem* may be ſufficiently repreſented as in the *Figure* thereof: Wherein,

Firſt, The *Sun* being in the *Centre*, hath only a Rotation from *Weſt* to *Eaſt*, upon its own *Axis*, in the ſpace of 26 days, or much thereabouts: His *Centre* being an immoveable *Point*; to which the *Revolutions* of the other *Planets* are referred.

Secondly, The *Planets* are moved about the *Sun*, from *Weſt* to *Eaſt*, in ſeveral *Orbs*, which return into themſelves, every *Planet* in *Time* proportionable to the *Magnitude* of his *Orb* and *Diſtance* from the *Sun*; the Motions of the *Primary Planets*, ♄, ♃, ♂, ⊖, ♀, and ☿, being *uniform*, perpetually *conſtant* and *regular*.

The Figure of the Syſtem.

Fig. LXX. The Circle ☿ BCD ☿, denotes the *Way* and *Revolution* of the Planet *Mercury*, about the *Sun*, in 88 Days.——The Circle ♀ E F G ♀, the *Revolution* of *Venus* in 225 Days——The Circle ⊖ H I K ⊖, the *Revolution* of the *Earth*, with the *Moon* in one Year—The Circle ♂ L M N ♂, the *Revolution* of *Mars*——The Circle ♃ O P Q ♃, the *Revolution* of *Jupiter*, with his four *Companions* (or *Satellities*) in twelve Years——The Circle ♄ R S T ♄, the *Revolution* of *Saturn*, with his *Ring* and *Moon*, in Thirty Years.——The *Moon* alſo moves round the *Earth*, every Month——*Jupiters* four *Companions*, move about him according to their *Diſtances* from him.——The Firſt (and neareſt to him) in *One Day* and 18 *Hours*——The Second, in *Three Days* and 13 *Hours*——The Third, in *Seven Days* and 4 *Hours*.——The Fourth, in 26 *Days* and 5 *Hours*——*Saturns Moon* moveth about him in *Sixteen Days*: And all of them from *Weſt* to *Eaſt*, according to their *Planets Revolution* about the *Sun*.

Theſe Planets, whoſe *Revolutions* reſpect the *Sun* only, as *Saturn Jupiter*, *Mars*, the *Earth*, *Venus* and *Mercury*, are called *Primary Planets*: The other that move about *Saturn*, *Jupiter* and the *Earth*, *Secondary Planets*.

The *Secondary Planets*, are all of them much *Leſs* in *Magnitude* than their *Primary*; and all the *Planets* together, much *Leſs* than the ~~*Sun*, from whom they receive their *Light*, *Motion*, &c.~~

CHAP.

Fig. LXX.

CHAP. II.

Of the Theory of the Sun and the other Primary Planets.

THAT the Motion of the *Primary Planets* about the Sun were *Elliptical*, was firſt diſcovered by the learned *Kepler*, which he deduced from the acurate *Obſervations* of the Noble *Dane Tychobra*. And therefore, in ſuch an *Elliptical Figure*, may be deſcribed all ſuch *Points*, *Lines*, *Arches*, &c. as are requiſite to be known, and to inform the *Fancy* in the *Trigonometrical Calculation* of the *Planets Places*. But before I proceed ſo far, I muſt ſhew,

How to deſcribe an *Ellipſis*.

Concerning the *Definition* of this *Figure*, only that it is one of the *Sections* of a *Cone*, and the many ways that it may be Artificially *deſcribed*, I ſhall ſay nothing, referring the Reader to *Midorges*, and others that have largely written of the *Sections* of a *Cone*. But to *Deſcribe* ſuch a *Figure Mechanically*. Thus:

About any Two *Right-Lines* Given, for the Two *Diameters* of the *Ellipſis*, to deſcribe ſuch an *Ellipſis*.

Fig. LXXI. Let the Line P Y be the *Longer*, and R M the *Shorter Diameter* given: Let them croſs one another at *Right-angles* in B. This done, Take half of the *Longeſt Diameter* B P or B Y, in your Compaſſes, and ſetting one foot in R or M, the other will reach upon the *Longeſt Diameter* to the *Points* Z and S, which will be the *Centres* upon which the *Ellipſis* P R Y M muſt be deſcribed. Wherefore,

In the two Points Z and S, Fix two *Pins*, *Nails*, or the like; and about them put a *String* ſo long, that being doubled it may reach from the *Pin* at S, to the end of the Diameter at P; or from the *Pin* at Z to the end Y; ſo then the whole length of the String, will be twice as long as the Diſtance S P or Z Y; which *String* at that length, joyn at both ends: Then, putting this *String* over the two *Pins*, at Z and S, with a *Third Pin* (which may be a Black-Lead Pencil, or ſuch like) move the *String* about upon the two Fixed *Pins* Z and S; and it will by its *Motion* deſcribe an *Ellipſis*, ſuch as is the *Figure* P C R E Y F M D P.

Fig. LXXI.

CHAP. III.

Of the Motion *of the* Planets *in the* Ellipſis.

In this *Elliptical Figure*, the Line P Y is called the *Tranſverſe*, (or *Longer*,) And R M, the *Conjugate* (or *Shorter*) *Diameter* of the *Ellipſis*.— S the *Lower*, and Z the *Upper Focus* of the *Ellipſis*; upon which two Points the *Elliptical Figure* was deſcribed— And the *Elliptical Figure* it ſelf P C R E Y F M D P be the *Orbit* of the *Earth*, or any other of the *Primary Planets*:— S the Place of the *Sun*; (the common *Node*, and *Centre* of the *Planitary Orbs*, to which the *true Motions* of the *Planets* are referred,) in the *Lower Focus* of the *Ellipſis*.— Now, a Planet moving in the *Ellipſis*, when it ſhall be in P, it is in *Aphelion*, or at its *Greateſt Diſtance* from the Sun— When in Y *Perihelion*, or at its *Neareſt Diſtance* to the *Sun*— About the upper *Foci* of the *Ellipſis* Z, the *mean Motion* of the *Planet* is regulated: B is the *common Centre* of the *Figure*: B Z or B S, the *Excentricity*: And S Z the double *Excentricity*.

The *Figure* thus deſcribed, the next thing to be known is, after what manner the true Place of a Planet may be determined therein.

Firſt, The mean *Anomaly* of a *Planet*, is its *equal Diſtance* from P, the *Aphelion Point* of that *Planet*: And this *Anomaly* is to be accounted from P, to P again. And here it is to be obſerved, that a *Planet* moving in the *Elliptical Arch* P R Y M, upon the *Focus* Z, is as long time tracing the leſſer *Arch* C P D, as he is the greater C Y M; the Reaſon is, for that (the *mean Motion* being made upon Z) the *Planet* deſcribeth *equal Angles* in *equal* Intervals of *Time*: And it is alſo evident, That if the *Motion* be *equal* upon one *Focus* Z, of the *Ellipſis*, it muſt be *unequal* upon the other S, in which the Sun is ſeated.

For inſtance: Suppoſe the Planet to be in C, which is 90 deg. from the *Aphelion Point* P, then will the *Angle* of its *mean Anomaly* be P Z C; but the *Co-equated Anomaly* will be the *Angle* P B C: And the *Angle* at the *Sun* P S C, which is the *Angle* to be found.

C H A P. IV.

to find the Angle that a Planet (in any part of the lipſis) makes with the Sun ; and alſo, the Planets true ace in the Ellipſis.

Fig. LXXII.

t S be the Sun, X the Centre of the Planets *Mean Motion* ;
the Points H, A, T, ſeveral Places of the *Planet* in its *Orb*,
the *Aphelion* point P, then will the ſeveral *Angles* P X H,
A, P X T, repreſent the *Anomaly* of the *Planet* : And the
ſ P S N, and P S M ; the *Angles* at the *Sun*. Now theſe *An*-
ay be found as followeth.

ppoſe the *Mean Anomaly* of the *Earth* being at H, to be 30 deg.
repreſented by the Angle P X H, whoſe Complement B X H,
o deg. By help whereof, with the *Common Radius*, and *Ex*-
city, the true *Place* of the *Planet* may be found by *Trigono*-
al Calculation. For,

ſt, In the Triangle H X B, there is Given, (1) The Angle
B, 150.00 d. the Complement of the *Planets Mean Anomaly*
o deg. (2) The Side B H, the *Common Radius* 100000. And
he Side B X, 1685, equal to half X S, the *Excentricity*. By
t you may find the Angle B H X, By CASE I. of O. A.

Then,

ſ the *Common Radius*, B H, 100000 Parts	5.000000
X B (half the *Excentricity*) 1685 P.	3.226599
the Sine of H X B, the Complement of the maly H Z P, 30 d. to 180 deg.	9.698970
	12.925569
the Sine of B H X 0.483 deg.	7.925569

t therefore the Angle X B H is 29.518 deg. the double
hereof, 59.036 deg. is the *Anomaly* of *Variation*.

Then ſay, Secondly,

ſ the *Radius*, Sine 90 d.	10
the Sine of the *Greateſt Variation* which is) 0.24 deg.	6.594172
he *Variation* laſt found, 59.034 d.	9.933210
the Sine of 0. 019 deg.	16.527382

hich is the *Variation* required.

Now,

Fig. LXXII. Now, foraſmuch as the *Planets* are moved in *Ellipſes*, and not in perfect *Circles*, the next thing, therefore, to be enquired is, the *Planets* Place in the *Ellipſis*: And to that purpoſe, about Z, let there be deſcribed a little Circle K O N, upon which, from K to N, I number the *Anomaly* of the *Epicicle* 59.034 deg. before found, and then will the Angle P B Z, be equal to the *corrected Anomaly*; and ſo is N the place of the *Planet* in the *Ellipſis*; and the Angle Z B N, the *Æquation* of the *Epicicle*; or (which is the ſame) the *Difference* between the *Place* of a *Planet* in the *Circle*, and in the *Ellipſis*: And may be found *Trigonometrically*, thus. For,

Thirdly, In the Triangle B Z N, you have given, (1) The Side B Z, the *common Radius* 100000 P. (2.) The Side Z N, 00008 P, the *Semidiameter* of the *Epicicle*. (3) The Angle B Z N 120.98 Deg. by which you may find, the Angles B N Z and Z B N, (by CASE II. of O. A. R. T.) Thus,

The Angle N Z B being 120.98 deg. the Complement to 180 deg. is 59.02 deg. the half whereof is 29.51 deg. The Side B Z is 100000 P. the Side Z N 00008 P, their *Sum* 100008 P, their *Difference* 99992. Then,

(3) As the *Sum* of the Sides B Z and Z N, 100008	5.000035
Is to their *Difference* 99992	4.999965
So is the Tang. of half Z B N and Z N B 29.505 d.	9.752818
	14.752783
To the Tang. of the *half Difference* 29.502 d.	9.752748

This *half Difference*, added to the *half Sum*, gives 59.007 deg. for the *Angle* Z N B: And *ſubſtracted* from it, leaves 0.003 deg. for the *Angle* N B Z.

Then again ſay,

As the Sine of B N Z, 59.007	9.993121
Is to B Z 100000 P.	5.000000
So is the Sine of B Z N, 59.02 d.	9.933140
	15.933143
To the Side B N 100004 P.	5.000022

From the Angle B P Z 29.502
Subſtract N B Z 005
There remains P B N 29.497 Whoſe Complement to 180 deg. is 150.503 deg. and is the Angle S B N.

Fourthly, In the Triangle B S N, there is given, (1) B N 100004. (2) B S 1685. (3) The Angle contained by them, S B N 150.50 deg. *Fig.* LXXII.

Then,

As the Sum of B N and B S, 101689.	5.007278
Is to their Difference 98319	4.992637
So is the Tangent of half B S N and B N S 14.77d.	9.421029
	14.413666
To the Tangent of 14.30 d. the half difference.	9.406388

Which added and ſubſtracted from the half Sun, gives
29.07 deg. for B S N the *Angle at the Sun.*
And 0.47 deg. for the *Opt. Æquation* B N S.

Then from the Mean *Anomaly* P X H	30.00 deg.
Subſtract the Angle at the *Sun* B S N,	29.07
There remains the *Proſtapheriſis.*	0.93

Again,

As the Sine of B S N, 29.04 de.	9.686117
Is to B N, 100004.	5.000021
So is the Sine of B S N, 150.50 deg. (or 29.50.)	9.692338
	14.692359
To S N, 101564.	5.006232

Note that when a *Planet* is in any of the *Points* P, A, or Y, there is no *Variation*, but being out of any of theſe *Points*; it hath *Variation* more or leſs, and is at its *greateſt* when the *Elongation* is 45 deg. from any of thoſe *Points*.——And note alſo, That this *Variation augments* the *Places* of all the *Planets* (*Venus* and the *Moon* excepted) in the *Firſt* and *Third Quadrants* of the *Orbit*; But in the *Second* and *Fourth*; *it diminiſhes* the Planets place; according to the quantity of the *Angle* of *Variation*. And as this *Example* was wrought, the *Earth* or other *Planet* being ſuppoſed in the *Firſt Quarter* of the *Ellipſis*; the ſame is to be underſtood and performed, if the *Planet* were in the *Second*, *Third* or *Fourth Quadrant* thereof, as is plain by the *Figure*.

Fig. LXXII.

CHAP. V.

Of the Two-fold Inequality *of the other* Primary Planets.

THE *Sun* ſeems to be moved under the Plain of the *Eclip-tick*; But it is the *Earth* that performeth this *Motion* about the *Sun*, who is ſeated in the *Focis* of the *Earths Ellipſis*: So that, as the *Earth* is moved under the *Ecliptick Orb*, ſo much the *Sun* appears to move on the contrary part thereof: And from hence may be concluded, that the *Sun* (or *Earth*) can be ſubject to no other *Inequality*, than what is produced by their own *Simple Motion* in the *Ellipſis*: And this is true alſo, in all the *Primary Planets* as in reſpect ~~to the~~ *Sun*; who is ſeen, from them, to change his *Place*, according to the quantity of their Motions.

Now, For as much as the *Earth* is far diſtant from the *common Centre* of their *Orbs* (which is the *Sun*) therefore it is, that (by their different *Motions*, and various poſitions of their *Orbs*) they ſeem to us to be ſubject to *a ſecond Inequality*; which is not eſſential in their *Motions*, but accidental only; being cauſed through the *great diſtance* of the Point to which their *Places* are referred: So that the *Planets* ſeem to us to have a different ſpecies of *Motion* from the *Natural Motion* in their reſpective *Orbs*: they appearing at one time *Direct*, at other times *Retrograde*, and ſometimes not to move at all, but appear as *Stationary*: To be nearer to the *Earth* at one time than at another; and appearing to the Eye of a *Greater* and *Leſſer Magnitude*: The cauſe of all which various *Paſſions* will manifeſtly appear from the *Calculation* of their *Places*, which ſhall be the Work of the following *Chapter*.

CHAP. VI.

To Calculate the true Place of a Planet Trigonometrically.

REtaining the ſame Method deliver'd in the laſt Chapter, I ſhall give an Example in the finding out of the place of the Planet *Mars*. The Sun being in 5.85. deg. of *Scorpio*, and the mean Anomaly of *Mars* from his Aphelion point 55.47 deg.

The reſt, as in the following Table. And from theſe things given, the true place of the Planet *Mars* may be found. *Fig.* LXXII.

A Table of the Semidiameters *of the* Orbits *and* Epicicles; *the* Excentricities; *the Greateſt* Variations *and* Inclinations *of the* Planets, *in order to the* Trigonometrical Calculation *of their* Places.

	♄	♃	♂	◒	♀	☿
Semidiam. of the Circle P A Y.	952500	521300	152040	100000	72405	38192
Semidiam. of Epicicle Z N.	788	299	327	8	1	429
Excentricity B X=B S.	54800	24960	14115	1788	530	8100
The Greateſt Variation of the Ano	0.09 d	0.05 d	0.25 d	0.02 d	0.02 d	1.30 d
Angle of the greateſt Inclin. H B V.	2.51 d	1.36 d	1.85 d	0.00 d	3.38 d	6.90 d

The Trigonometrical Calculation.

In the Figure the Semicircle P H Y, is half the Orbit of *Mars*, P his *Aphelion* point, H his place in his Orbit, B the Centre, B X the Excentricity: H B the Semidiameter of the Orbit. *Fig.* LXXII.

I. Then, In the Triangle X H B, there is given, (1) The Side H B, the Semidiameter of the Orbit, 15204. (2) The Side X B, the Excentricity 1411. (3) The Angle H X B the Complement of the mean Anomaly P H: 55.47 to 180 deg. viz. 124.53 deg.

By which you may find the Angles B H X, and X B H.

(By CASE II. of O. A. P. T.

As the Sum of the Sides H B and X B 16615.	4.2205003
Is to their Difference 13793.	4.1396587
So is the Tang. of half X H B and X B H 27.73 d.	9.7207213
	13.8603800
To the Tangent of 23.50 d.	9.6398797

Which ſubſtracted from 27.73 (the half Sum) leaves 4.15 de. for the Angle at H: And added to 27.73 d. gives 51.31 deg. for the Angle X B H. And that doubled, makes 102.62 d. Which is the Anomaly of Variation.

Which found ; ſay,

Fig. LXXII.

As Radius	10.
To the Sine of the greateſt Variation of *Mars*, 0.25 d.	7.639816
So is the Sine of Variat. laſt found 102.62 (77.38)	9.989379
To the Sine of H B Z, 0.244 deg. the Variat.	x 7.629195

To the Angle X B H 51.32 deg.
Add the Angle H B Z 0.244 the Variation.
The Sum is X B Z 51.564 deg.
The double whereof is 103.128 And is the *Motion* of the *Epicicle* a Z N.

Deſcribe the *Epicicle*, and upon it ſet the *Motion* of the *Epicicle* 103.128 deg. from a to N, ſo will the Angle a O N be equal to the corrected *Anomaly* P B Z ; ſo that N is the Place of the Planet *Mars* in the *Ellipſis* : And the Angle Z B N the *Equation* of the *Epicicle* ; which is the difference between the place of a Planet in the *Circle*, and in his *Ellipſis* ; which may be thus found. For,

In the Triangle B Z N there is given, (1) The Side Z B, the Semidiameter of the Circle 100000, (2) The Side Z N, the Semidiameter of the Epicicle 33. (3) The Angle B Z N, the Complement of the Motion of the Epicicle N Z *a* (103.64) *viz.* 77.36 de.

And by theſe may be found, the Angle N B Z.

Thus :

As the Sum of the Sides Z B and Z N, 100008	5.00008
Is to their Difference 99992	4.99996
So is the Tang. of half Z N B and Z B N, 51.32 de.	10.09659
	15.09655
To the Tangent of 51.31 d.	10.09647

Which added to 51.32, gives 102.63 de. for the Angle Z N B ; and ſubſtracted therefrom, leaves 0.01 deg. for the Angle N B Z.

Then for the Side N B.

As the Sine of Z N B 102.63 de. (77.37)	9.989362
Is to the Sine of N Z B 77.36 deg.	9.989345
So is the Side Z B, 100000	5.000000
	14.989345
To the Side B N, 99996	4.999983

From

Fig. LXXII.

From the Angle P B Z	51.32 deg.
Subſtract N B Z	0.244
There remains the Angle X B N	51.076 whoſe Complement to 180 deg. is 128.924 for the Angle N B S.

And then,

In the Triangle B N S, there is Given, (1) The Side B N 99996. (2) The Side B S, 1411, And (3) The included Angle N B S 128.92 deg. From whence may be found the Angle at the Sun B S N; and the Side S N.

Thus,

As the Sum of the Sides N B and S B, 101407.	5.006082
Is to the Difference of thoſe Sides 98585.	4.993811
So Tang. of half the Angles B S N and B N S, 25.54 d.	9.679276
	14.673087
To the Tangent of 24.92 deg.	9.667005

Which added to 25.54 de. gives, 51.08 deg for the Angle B S N; and ſubſtracted, leaves 0.62 de. for the Angle B N S.

Then, For the Side N S,

As the Sine of S N B, 0.62 deg.	8.034260
Is to the Side B S, 1411.	3.149219
So is Sine N B S 128, 92 de. (51.08)	9.891115
	13.040334
To the Side S N 101405.	5.006074

Which is the Diſtance of ♂ from ☉.

Place of the Aphelion.	150.35 deg.
The Angle at the Sun.	51.08.
Heliocentrick Place of *Mars*.	201.43
The North Node Sub.	47.55
The Argument of Latitude.	153.88

CHAP.

CHAP. VII.

To find the Place of a Planet in the Ecliptick, &c.

Fig. LXXIII.

FOR that a Planets Place in Longitude in the Ecliptick, differeth ſomething from the Longitude in its own *Orb*, I ſhall therefore ſhew (by Trigonometry,) How to find the *Reduction*, and the *Ecliptick Place*, with the *Curtation* and *Parallax* of the *Orb*, and conſequently, the *Geocentrick Place* and *Latitude*.

In order whereunto, in the Scheme; Let S repreſent the *Sun*, H the Place of *Mars* in his *Orbit*; V his place in the *Ecliptick*; E the *Earth*: A the *North Node* of *Mars*; B the *South Node*; (the two Interſections of the *Planets Orb* with the *Ecliptick*; A D B, a *Semicircle* of the *Ecliptick*: D R, the limit of the Planets *Greateſt* North *Latitude*. Theſe things premiſed, I proceed to

The Trigonometrical Calculation.

1. In the Spherical Triangle B H V, Right-angled at V, there is given. (1) The Side B H 26.65 de. (the Complement of the *Argument of Latitude.*)——(2) The Angle H B V, the *Greateſt Inclination* of *Mars*, 1.85 de. By which you may find the Side B V: (By CASE I. of R. A. S. T. Thus

As Radius, 90 de.	10.
To Co-ſine H B V, 88. 15 de.	9.999773
So is the Tangent of B H, 26.63 de.	9.700577
To the Tangent of B V 26.64 de.	9.700350

The *Difference* between B H and B V, is 0.01 de. is to be ſubſtracted from 201.43 d. the place of *Mars*, becauſe the Arch B V is leſſer then the Arch B H.

Heliocen. Place of *Mars*, in his Ellipſis.		201.43 deg.
Reduction	Subſt.	.01
Heliocen: Place Corrected		201.42
Place of the Sun		215.85
Difference		14.43

Which is the Anomaly of Commutation.

2. In the ſame Triangle, there is given as before, whereby the *Inclination* of the *Orbit* from the *Plain* of the *Ecliptick* H V, may be found (by CASE II. of R. A. S. T. Thus

As Radius, 90.00 de.	10.
Is to the Sine of B H, 26.65 de.	9.651800
So is the Sine of B H V, 1.85 de.	8.508974
To the Sine of H V, 0.83 de.	8.160774

Fig. LXXIII.

Unto which, the Angle HSV is equal.

3. For the *Curtated Diſtance* S.V. In the Right-angled Plain Triangle H S V, there is given, (1) The Angle V S H, 0.83 d. (2) The Side S H 15204, the *diſtance* of *Mars* in his *Orbit*, from the *Sun*: By which you may find the Side S V, (By CASE IV. of R. A. S. T.) Thus

As Radius, Sine 90.00 de H V S,	10.
Is to the Side HS, 15204.	4.181958
So is the Sine of V H S, 89.17 d.	9.999954
To the Side S V, 15202.	14.181912

The *Ecliptic Place* of the Planet *Mars*, with his *Inclination* and *Curtation* thus attained: The next thing to be enquired after is, the *Parallax* of the *Earths Orb*; and his *Geocentrick Place*, in *Longitude* and *Latitude*.

4. In order whereunto, in Figure LXXIII. the Circle O X E Z, representing the *Earths Orbit*; Number the quantity of the *Anomaly* of *Commitation* 14.43 de. from X (the oppoſite Place of *Mars* from the *Sun*) to E, drawing the Line E V; which will conſtitute an Oblique-angled Plain Triangle S V E: in which there is Given, (1) The Side S V 15202. (2) The Side S E, 10000, (3) The included Angle V S E 165.57 de. (the Complement of the *Angle of Commitation* to 180 deg.) By which you may find (1.) the Angles S V E and V E S. And (2) the Side V E. *Fig.* LXXIII.

As the Sum of V S and S E 25202.	4.401435
Is to their Difference 5202.	3.716170
So is the Tang. of half the Angles at E and V, 8.22 d.	9.159743
	12.875913
To the Tangent of 1.71 deg. their difference.	8.474478

Which added to the half Sum of the Angles at V and E (8.22 d.) gives 9.03 de. for the Angle V E S: And ſubſtracted therefrom, leaves 6.51 de. for the Angle S V E: Which is the *Parallax of the Orb*.

Then for the Side V E,

Fig. LXXIII. As the Sine of V E S 9.93 de.	9.236650
Is to the Sine of V S E 163.56 (16.44)	4.181900
So is the Side V S, 15202	9.451803
	13.633703
To the ſide V E 24959	4.397053

Then,

To the *Heliocentric* Place of *Mars.*	201.43
Add the *Parallax* of the *Orb* S V E	6.51
The Sum is	207.94

Which is the *Geocentric Long.* of *Mars.*
That is 6 s. 27 deg: Or 27 deg. of ≏.

CHAP VIII.

Of the Proportions of the Semidiameters *of the* Sun, Earth, Moon, *and the other* Planets.

I. Of the *Sun*, *Earth*, &c. *Moon*;

BY the beſt Teleſcope-Obſervations.

To the mean diſtance of the *Earth* from the *Sun*	10.00000
The Semidameter of the *Sun* is of thoſe Parts	46300
The Semidiameter of the *Earth*	727
To the mean diſtance of the *Moon* from the *Earth*	1.00000
The Semidiameter of the *Earth* is	1650
The Semidiameter of the *Moon*	446

And from hence, at all times, the Diſtance of the *Luminaries* being firſt found, their apparent Semidiameters may be obtained: And for the Semidiameter of the *Earths Shadow*, in *Lunar Eclipſes*; I have here inſerted the Diagram of *Hyparcus*.

In which Diagram.

Fig. LXXIV. A denotes the *Centre* of the *Sun*.
A D his *Semidiameter*.
B the *Centre* of the *Earth*.

Fig. LXXV.

B E her *Semidiameter*.
A E D, or A B D, the apparent *Semidiameter* of the *Sun*.
A E H, or B D E, the *Horizontal Parallax*.
C G F equal to H E D, the *Semi-Angle* of the *Cone* of the *Earths Shadow*.
B C and B F, being equal to the *Diſtance* of the *Moon* from the *Earth*.
B T E, her *Horizontal Parallax*.
C B F, the apparent *Semidiameter* of the *Earths Shadow*.

From hence;

1. The Semidiameter of the *Sun*, the *Horizontal Parallax* being ſubſtracted, is equal to the *Semi-angle* of the *Cone* of the *Earths Shadow*.
   So A E D — A E H = H E D
2. The *Horizontal Parallax* of the *Moon*, the *Semi-angle* of the *Cone* of the *Earths Shadow* being ſubſtracted, is equal to the apparent *Semidiameter* of the *Shadow*.
   So B F E — C G E = G B F.
3. The Sum of the *Horizontal Parallaxes* of the *Sun* and *Moon*, is equal to the Sum of the apparent *Semidiameter* of the *Sun* and *Shadow* of the *Earth*.
   So, B D F + B F D = A B D + C B F.

Therefore, From the Sum of the *Horizontal Parallax* of the *Sun* and *Moon*, ſubſtract the apparent *Semidiameter* of the *Sun*; and there will remain the apparent *Semidiameter* of the *Earths Shadow*.

So, B F D + B D F — A B D = C B F.

II. For the Proportional Magnitudes of theſe Three Bodies.

The *Semidiameter* of the { *Sun* 46300 Log.	4.665581
{ *Earth* 727	2.861534
The Log. Difference	1.804047
Multiply by	3
The Logar. of 258309	5.412141

So that the *Body* of the *Sun* exceeds the *Body* of the *Earth* 258309 times.

Fig. LXXV.

The *Semidiameter* of the *Earth* 1650 Log.		3.217484
Moon 446 Log.		2.649335
	The Logar. Difference	0.568149
	Multiply by	3
To the Logar. of 50.63		1.704447

So that the *Earth* is greater than the *Moon* 50 times, and $\frac{63}{100}$ parts.

III. Of the *Semidiameters*, and *Proportions* of the other *Primary Planets* to the *Earth*.

From the accurate Obſervations of *Hugenius*, *Gaſſendus*, and *Horrox* are determined.

		D.	D.			
The apparent Semidiameters of theſe 5 Planets in their mean diſtance from the Earth,	♄	10	30	Which in ſuch Parts as the Sediameter of the Earth is 727, gives the Semidiameter of	♄	4855
	♃	24	0		♃	6054
	♂	4	0		♂	296
	♀	10	30		♀	509
	☿	5	0		☿	242.

From hence may be gathered, That the *Body* of *Saturn* is *Greater* than the *Earth* 298 times.

The *Body* of *Jupiter* 577 times.

But the *Earth* is *Greater* than the other Three: For it

Exceeds	♂	15	Times.
	♀	3	
	☿	27.	

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GEOMETRICAL ASTRONOMY,

SHEWING

A Plain and Eaſie Way to Project Theories of the Planets (in Plano:) And by them to find their true Places, both in Longitude and Latitude, at any time.

And their Diſtances from the Sun or Earth.

CHAP. I.

How to lay down (*in* Plano) *the* Theory *of the* Earth, *or other* Planet.

FOR the performance hereof you muſt have a *Scale* of *Chords*, and a *Scale* of 1000 *equal Parts* to the ſame *Radius*: (Or, rather, a *Sector*, with ſuch *Scales* upon it.) Then,

I. How to deſcribe and divide the *Ecliptick*.

With 60 deg. of your *Scale* of *Chords*, deſcribe a *Circle*; which divide into 12 equal Parts, for the 12 *Signs*, noting them with their reſpective *Characters*, ♈, ♉, ♊, *&c.* This *Circle* thus deſcribed and charactered, repreſents the *Ecliptick Circle* in the Heavens; whoſe *Centre is* ☉, the fixed place of the *Sun.*

In this *Ecliptick Circle*, every of the *Planets* hath a *Point*, which is called the *Aphelion* of that *Planet*: For when the *Planet* is in that Point, it is at its greateſt diſtance from the *Sun*. Now, the *Aphelial Points* of each *Planet*, are theſe which are exhibited in this little Table. Fig. LXXVI.

Fig.
LXXVI.

			Deg.
The Aphelial Point of	♄ Saturn	Is in	27.50 ♐
	♃ Jupiter		7.82 ♎
	♂ Mars		0.35 ♍
	♀ Venus		2.82 ♒
	☿ Mercury		14.95 ♐
	⊖ The Earth		7.00 ♑

II. To draw the *Aphelial Line*.

From the *Aphelial Point* of the *Planet* (as in our *Example* MARS,) 0.35 de. ♍ draw a Right-line to the Centre ☉, as the Line *Ap.* ♂ ☉; and that ſhall be the *Aphelial* Line of MARS.

For the *Aphelial Line* of the *Earth*: The EARTHS *Aphelial Point* being in 7.00 deg. of ♑ *Capricorn*, from that Point draw a Right-line to ☉ the Centre, as the Line ☉ *Ap.* ⊖, and that ſhall be the Earth's *Aphelial Line*, in the *Theories* of all the *Planets*.

III. To find the *Excentric Points* of any of the 3 Superior Planets ♄, ♃, ♂, they will be found by the numbers in this little Table. For,

If from the *Aphelial Point* of	♄	to the Centre ☉ be 1000,	946
	♃	the *Excentric* Point ſhall be	954
	♂	from the *Aphelial Point*	915

Wherefore, take the length of the *Aphelial Line Ap.* ♂. ☉ in your Compaſſes, and open the *Sector* in 10, and 10 of the equal Parts. Then, if from the *Sector* you take 915, (the *Excentric* of *Mars*) it will reach from the *Aphelial Point Ap.* ♂, upon the *Aphelial Line* of *Mars*, to X, ſo ſhall X be the *Excentric* of MARS; upon which Point (at the diſtance of *Ap.* ♂ X) if you deſcribe a Circle, as *Ap.* ♂, ☊, ♂, that Circle ſhall be the *Orbit* of the *Planet* MARS, in which it moveth.

For the *Aphelial* and *Excentric Points* of the EARTH in the Theories of the three Superiour Planets, they will be at the Numbers exhibited in this little Table. For,

The

Fig. LXXVI.

The diſtance of the *Aphelial Point* of the ⊖ from the *Centre*, ſhall be in the *Theory* of	♄	101	And the diſtance of the *Excentric* of the ⊖ from the *Aphelion* of the ⊖, ſhall be in the *Theory* of	♄	099
	♃	186		♃	183
	♂	611		♂	600

Wherefore, Out of your *Sector*, (it being ſtill opened to the former diſtance) take the diſtance of the *Aphelial Point* of the *Earth* (for MARS 611) and ſet it (upon the *Earths Aphelial Line*) from the Centre ⊙ to *Ap.* ⊖, ſo ſhall the point *Ap.* ⊖ be the *Aphelial Point* of the *Earth* —— And the diſtance of the *Excentric* of the *Earth*, from the *Aphelial Point* of the *Earth* being 600, take 600 out of the *Sector*, and ſet it from the *Earths Aphelial Point Ap.* ⊖ downwards towards the Centre ⊙, and that ſhall be the Centre whereon to deſcribe the *Orbit* of the *Earth* on the *Theory* of MARS. Which Point falling ſo near the Centre ⊙, you may deſcribe the *Earths Orbit* upon the Centre ⊙ it ſelf, the difference being only $\frac{011}{1000}$ Parts: And in the *Theories* of the other two *Superior Planets*, the difference wil be much leſs; for in the *Theory* of *Saturn* it will be $\frac{002}{1000}$ Parts: And in the *Theory* of *Jupiter*, but $\frac{003}{1000}$ Parts: So that in ſmall *Theories* they need not be much regarded.

4. For the *Aphelial* and *Excentric Points* of the *Earth*, and the two *Inferior Planets* ♀ and ☿, the *Tables* following will exhibit.

1. For the *Aphelial Points.*

If the diſtance of the Aphelial Point of the Earth to ⊙ be 1000, the Excentric Point of	⊖	Shall be from the Centre ⊙	1.000
	♀		.716
	☿		.461

2. For the *Excentric Points.*

And from the *Aphelial Point* of	⊖	to the Excentrick Point of	⊖	ſhall be	.982
	♀		♀		.711
	☿		☿		.381

5. For

Fig. LXXVI.

5. For the Nodes of the Planets.

The Aſcendant Nodes of the ſeveral Planets (for the Earth hath none:) This Table will tell you to what Points in the Ecliptic they are to be drawn. For

			d.	
The Node of	♄	Is in	22.45	♋
	♃		5.50	♋
	♂		17.55	♉
	♀		14.00	♊
	☿		14.15	♉

Wherefore, the Node of the Planet ♂ being in 17.55 de. of ♉, lay a Ruler from the Centre ☉ to 17.55 de. of *Taurus*, and there draw an obſcure Line: This Line is the common Section of plain Planets Excentric; with the Plain of the *Ecliptic*; and where this Line croſſeth the Orbit of *Mars*, there write ☊ *Dragons Head*; at which Point, when any Planet is in its Orbit, it goes into *North Latitude*: And at the oppoſite Point, through the Centre, is the place of ☋ the *Dragons Tail*, where it goes into *South Latitude*.

Of the Inclination of the *Planets*.

The Greateſt Inclination of the Planets are ſuch as are expreſſed in this Table.

For	♄	2.53	Degrees.
	♃	1.32	
	♂	1.84	
	♀	3.37	
	☿	6.90	

The *Aphelian Lines* and *Orbits* of the *Earth*, and any of the *Planets* being drawn within the *Ecliptick Circle*, the next thing to be done is, to ſhew how to place the *Earth* and any *Planet* in their reſpective *Orbits*, for any *Time* propoſed: And alſo, How to *Reduce* their Places ſo found in their *Orbits* at that time, to their reſpective Places in the *Ecliptick Circle*.

In order whereunto it will be neceſſary to ſhew,

I. How *Time* is to be *Accounted*.

II. To *Collect* the *Anomalies* of the *Earth* and *Planet* for any *Time* propoſed. *Fig.* LXXVI.

III. How *Degrees* of *Anomaly* in the *Orbits*, are to be *reduced* to *Degrees* in the *Ecliptick.*

CHAP. II.

How Time *is to be Accounted.*

TIME is to be thus Accounted: Any *Day* begins upon its own *Noon*: So that 12 at *Noon* of the firſt Day of *January*, is the *Common Term* of the *Old* and *New Years*; it being the *End* of the *Former*, and the *Beginning* of the *Latter*.

Before the *Longitude* or *Place* of a *Planet* in the *Ecliptic* can be found; the *equal Motion* of the *Anomaly* for the *Time* propoſed muſt be known; and that *Motion* for any propoſed *Time* may be collected out of theſe TABLES.

Epoches

Æpoches *of the* ANOMALIES *for the* YEARS.

Epoches for	Earth	♂	♃	♂	♀	☿
		100	100	100	100	100
Years.	De. Pts.	De. Pts.	De. Pts.	De. Pts.	De. Pts.	De. Pts.
1692	194. 34	346. 17	246. 08	126. 89	247. 04	167. 86
1700	194. 26	83. 88	128. 88	218. 08	248. 42	245. 58
08	194. 18	181. 59	11. 68	309. 26	249. 80	323. 30
16	194. 11	279. 30	254. 48	40. 45	251. 17	41. 01
24	194. 03	17. 01	137. 28	131. 63	252. 55	118. 73
32	193. 95	114. 72	20. 08	222. 81	253. 93	196. 45
40	193. 87	212. 43	262. 88	314. 00	255. 31	274. 17
48	193. 80	310. 14	145. 68	45. 18	256. 68	351. 89
56	193. 72	47. 86	28. 48	136. 40	258. 06	69. 61
64	193. 64	145. 57	271. 28	227. 58	259. 43	147. 33
72	193. 57	243. 28	154. 08	318. 76	260. 81	225. 05
80	193. 49	341. 08	36. 88	49. 94	262. 19	302. 77
88	193. 41	341. 00	279. 68	141. 12	263. 57	20. 49
96	193. 34	78. 71	162. 48	232. 30	264. 94	98. 21
1800	193. 30	225. 27	283. 88	277. 90	85. 63	317. 35
1	359. 74	12. 21	30. 33	191. 27	224. 27	53. 69
2	359. 49	24. 41	60. 66	22. 53	89. 54	197. 38
3	359. 23	36. 62	90. 99	213. 80	314. 32	161. 08
4	359. 96	48. 86	121. 40	45. 59	180. 69	218. 86
5	359. 71	61. 06	151. 73	236. 86	45. 46	272. 55
6	359. 45	73. 27	182. 06	68. 13	270. 23	326. 24
7	359. 19	85. 47	212. 39	259. 39	135. 00	19. 93

Motion

Motion of the Anomaly, in Months Of the Common YEAR.

	Earth		♄		♃		♂		♀		☿	
		100		100		100		100		100		100
	De.	Pts.	De.	Pts.	De.	Pts.	De.	Pts.	De.	Pts.	De.	Pts.
Janua.	30.	55	1.	04	2.	58	16.	24	49.	67	126.	86
Febru.	58.	15	1.	97	4.	90	30.	72	94.	52	241.	45
March	88.	17	3.	01	7.	48	47.	16	144.	19	8.	31
April	118.	27	4.	01	9.	97	62.	88	192.	25	131.	08
May	148.	03	5.	05	12.	55	79.	13	241.	92	257.	94
June	178.	58	6.	05	15.	04	94.	85	289.	98	20.	71
July	208.	95	7.	09	17.	62	111.	09	339.	55	147.	57
Auguſt	239.	50	8.	13	20.	19	127.	34	29.	31	274.	43
Septem	269.	07	9.	13	22.	68	143.	06	77.	38	37.	20
Octob.	299.	62	10.	17	25.	26	159.	30	127.	04	164.	06
Novem	329.	19	11.	17	27.	75	175.	02	175.	11	286.	83
Decem.	359.	74	12.	21	30.	33	191.	27	224.	77	53.	69

Months in the Leap-Year.

	Earth.		♄		♃		♂		♀		☿	
Janua.	30.	55	1.	04	2.	58	16.	24	49.	67	126.	86
Febru.	59.	14	2.	01	4.	99	31.	44	96.	13	245.	54
March	89.	69	3.	04	7.	56	47.	69	145.	79	12.	40
April	119.	26	4.	05	9.	95	63.	41	193.	86	135.	17
May	149.	81	5.	08	12.	63	79.	65	243.	52	202.	03
June	179.	38	6.	09	15.	12	95.	37	291.	58	24.	80
July	209.	93	7.	12	17.	70	111.	62	341.	25	151.	66
Auguſt	240.	49	8.	16	20.	27	127.	86	30.	92	278.	52
Septem.	270.	05	9.	16	22.	77	143.	58	78.	98	41.	20
Octob.	300.	61	10.	20	25.	34	159.	83	128.	64	168.	15
Novem	330.	18	11.	20	27.	84	175.	55	176.	71	290.	92
Decem.	360.	73	12.	24	30.	41	191.	79	226.	37	57.	78

DAYS.

	☉	♄	♃	♂	♀	☿
Days	D. P.	D. P.	D. P.	D. P.	D. P.	D. P.
1	0. 99	0. 03	0. 08	0. 52	1. 60	4. 09
2	1. 97	0. 07	0. 17	1. 05	3. 20	8. 18
3	2. 96	0. 10	0. 25	1. 57	4. 81	12. 28
4	3. 94	0. 13	0. 33	2. 10	6. 41	16. 37
5	4. 93	0. 17	0. 24	2. 62	8. 01	20. 46
6	5. 91	0. 20	0. 50	3. 14	9. 61	24. 55
7	6. 90	0. 23	0. 58	3. 67	11. 21	28. 65
8	7. 88	0. 27	0. 66	4. 19	12. 82	32. 74
9	8. 87	0. 30	0. 75	4. 72	14. 42	36. 83
10	9. 86	0. 33	0. 89	5. 24	16. 02	40. 92
11	10. 84	0. 37	0. 91	5. 76	17. 62	45. 00
12	11. 83	0. 40	1. 00	6. 29	19. 23	49. 11
13	12. 81	0. 43	1. 08	6. 81	20. 83	53. 20
14	13. 80	0. 47	1. 16	7. 34	22. 43	57. 29
15	14. 78	0. 50	1. 25	7. 86	24. 03	61. 38
16	15. 77	0. 53	1. 33	8. 38	25. 63	65. 48
17	16. 76	0. 57	1. 41	8. 91	27. 24	69. 57
18	17. 74	0. 60	1. 50	9. 43	28. 84	73. 66
19	18. 73	0. 63	1. 58	9. 96	30. 44	77. 75
20	19. 71	0. 67	1. 66	10. 48	32. 04	81. 85
21	20. 70	0. 70	1. 75	11. 00	33. 64	85. 94
22	21. 68	0. 73	1. 83	11. 53	35. 25	90. 03
23	22. 67	0. 77	1. 91	12. 05	36. 85	94. 17
24	33. 65	0. 80	1. 99	12. 58	38. 45	98. 22
25	24. 64	0. 83	2. 08	13. 10	40. 05	102. 31
26	25. 63	0. 87	2. 16	13. 62	41. 66	106. 40
27	26. 61	0. 90	2. 24	14. 15	43. 26	110. 49
28	27. 60	0. 93	2. 33	14. 67	44. 86	114. 58
29	28. 58	0. 97	2. 41	15. 20	46. 46	118. 68
30	29. 57	1. 00	2. 49	15. 72	48. 06	122. 77
31	30. 55	1. 04	2. 58	16. 24	49. 67	126. 86

The manner of *Collecting* the *equal Anomalies* from theſe TABLES, is thus:

Firſt, Exſcribe the *Epocha* which belongs to that Year which moſt nearly precedeth the Year wherein you ſeek the Place of any *Planet*.

Secondly, Under that *Epocha* (or number) write the Motions belonging to ſo many *Years*, *Months* and *Days*, as are compleatly expired ſince the *Year* of the *Epocha*; all which Numbers muſt be taken out of their proper *Tables*, and ſet orderly one under another; which the diſjunction of the Numbers will give Direction enough how to do.

Thirdly, All theſe Numbers muſt be added into *One*; and their *Sum* ſhall give the *Anomaly* for the *Time* propoſed —— And if the *Sum* riſe to be above a whole *Circle*, or 360 Degrees; you muſt then caſt away the number of 360 as oft as you may, and the remaining Number muſt be taken for the *Anomaly*.

Fourthly, Theſe things are to be done both in the *Earth* and the *Planet* ſeverally; as is done in the Example following.

CHAP. III.

How to Collect the Anomalies *of* Saturn *and the* Earth, *for the 16th Day of* May, *at Noon*, *in the Year of* Chriſt 1712, *being* Biſſextile, *or* Leap-Year.

	Anomaly. ♁	Anomaly. ♄
	Deg.	Deg.
Æpocha for the Year 1700	194. 26.	83. 88
Motion in Years Compleat { 4	359. 96	48. 86
{ 7	359. 19	85. 47
April Compleat B.	119. 26	4. 05
Days Compleat 15	14. 78	. 50
The Sum of the Anomalies	1947. 45	222. 76
The 2 Circles Subſt.	720.	
Equal Anomalies for the Time.	327. 45	222. 76
Which reduced to the Eclipt ick, are	♐ 15. 50	♌ 15. 00

Firſt, Out of the Table of *Years*, exſcribe the *Æpocha* for 1700: And under it 4 *Years* and 7 *Years*, (which together make 11 compleat *Years*:) Under them exſcribe *April Compleat* (putting B to it, for that it is a *Leap-Year*:) And under that exſcribe *Days Compleat* 15. Thus is the firſt part of your Work performed.

Secondly, Repairing to your *Tables*, you ſhall find the *Anomaly* of the *Earth* which ſtands againſt 1700, to be 194.26 de. and the *Anomaly* of ♄ to be 83.88 deg. both which ſet in their proper *Columns* under ⊖ and ♄, againſt 1700. Alſo, Take out of their proper *Tables*, the *Anomalies* belonging to 4 *Years* and 7 *Years*; and write them in their proper *Columns* againſt their reſpective *Times*: As 359.96 deg. for the ⊖, and 48.86 deg. for ♄; which ſet down againſt 4 *Years*: Do the like for 7 *Years*: And for the *Month* of *April*, which muſt be taken out of the *Table* of *Months* in the *Leap-Years*, becauſe 1712 will be *Leap-Year*: And for the 15 *Days*, take the *Anomalies* anſwering to them out of the Table of *Anomalies* for *Days*.

Thirdly, all the *Anomalies* being thus exſcribed out of the *Tables*, and ſet orderly one under another; you muſt add them together, and you ſhall find the Sum of the ⊖ *Anomaly* to be 1047.45 deg. And the *Anomaly* of ♄ to be 222.76 deg. But, ~~becauſe the Sum of the *Earths Anomalies* is above 360, you muſt~~ take that Number, as oft as you can (which is twice, *viz.* 720 deg.) and the Remainder 327. 45 deg. is the *Anomaly* of the ⊖ *Reduced*: But, the *Anomaly* of ♄ being leſs than 360 deg. needs no ſuch *Reduction*. And thus have you computed the *Anomalies* of the ⊖ and ♄, for the 16th Day of *May*, in the Year 1712 at Noon.

CHAP. IV.

How the Degrees *of* Anomaly *of the* Earth, *or any of the* Planets *(found as before) in their Orbits; are to be* Reduced *to* Degrees *in the* Ecliptick.

FOR the performance hereof, the following *Table* is ſubſervient, for it will readily (by inſpection only) anſwer your deſire.

A

A TABLE, ſhewing what Point of the *Ecliptick* anſwers to any Degrees of Anomaly, that the *Earth*, or any of the other *Planets* is found to have in its own *Orbit*.

Anomalies in their Orbits.

Anomaliæ Degrees.	☉		♄		♃		♂		♀		☿	
	S.	D.	S.	D.	S.	D.	S.	D.	S.	D.	S.	D.
0	♑	7. 00	♐	27. 50	≏	7. 82	♍	0: 35	♒	2: 82	♐	14: 97
10		16. 65	♑	6. 44		16. 92		8: 70		12: 69		21: 64
20		26. 32		15. 42		26. 04		17: 09		22: 59		28: 37
30	♒	6. 00		24. 44	♏	5. 20		25: 54	♓	2: 42	♑	5: 20
40		15. 70	♒	3. 50		14. 42	≏	0: 10		12: 19		12: 19
50		25. 45		12. 72		23. 75		12: 80		22: 23		19: 39
60	♓	5. 24		22. 05	♐	3. 20		21: 69	♈	2: 14		26: 84
70		15. 09	♓	1. 50		12. 74	♏	0: 80		12: 07	♒	4: 64
80		25. 00		11. 07		22. 44		10: 15		22: 03		12: 85
90	♈	4. 94		21. 0	♑	2. 30		19: 80	♉	2: 01		21: 55
100		15. 00	♈	1. 00		12. 34		29: 75		12: 07	♓	0: 85
110		25. 05		11. 22		22. 05	♐	10: 02		22: 07		10: 85
120	♉	5. 20		21. 65	♒	2. 89		20: 64	♊	2: 14		21: 65
130		15. 40	♉	2. 26		13. 42	♑	1: 60		12: 20	♈	3: 35
140		25. 65		13. 07		24. 10		12: 87		22: 30		16: 00
150	♊	6. 00		24. 01	♓	4. 90		24: 48	♋	2. 42		29: 62
160		16. 27	♊	5. 12		15. 82	♒	6: 27		12: 55	♉	14: 18
170		26. 64		16. 29		26. 80		18: 26		22: 68		29: 37
180	♋	7. 00		27. 50	♈	7. 32	♓	0: 35	♌	2: 82	♊	15: 00
190		17. 37	♋	8. 72		18. 84		12: 44		13: 00	♋	0: 55
200		27. 73		19. 88		29. 82		24: 42		23: 09		15: 76
210	♌	8. 05	♌	0. 99	♉	10. 74	♈	6: 24	♍	3: 23	♌	0: 29
220		18. 35		11. 94		21. 54		17: 82		13: 34		13: 90
230		28. 60		22. 74	♊	2. 22		29: 10		23: 46		26: 55
240	♍	8. 80	♍	3. 55		12. 75	♉	10: 07	≏	3: 50	♍	8: 25
250		19. 00		13. 79		23. 9		20: 69		13: 57		19: 05
260		29. 04		24. 00	≏	3. 30	♊	1: 09		23: 60		29: 05
270	≏	9. 07	♋	4. 02		13. 33		10: 90	♏	3: 62	≏	8: 35
280		19. 02		13. 84		23. 20		20: 55		13: 60		17: 05
290		28. 92		23. 49	♌	2. 90		29: 90		23: 57		25: 26
300	♏	8. 76	♏	2. 95		22. 45	♋	9: 01	♐	3: 50	♏	3: 07
310		18. 55		12. 29		21: 89		15: 90		13: 42		10: 52
320		28. 30		21. 49	♍	1: 23		26: 60		23: 32		17: 73
330	♐	8. 02	♐	0. 34		10: 44	♌	5: 17	♑	3: 23		24: 72
340		17. 69		9. 60		19: 66		13: 62		13: 09	♐	1: 52
350		27. 35		18. 57		28· 43		22: 0		23: 60		8: 26
350	♑	7. 00	♑	27. 50	≏	7: 82	♍	0: 35	♒	2: 82		14: 95

The

The Uſe of the *Table*.

The *Table* conſiſts of Seven *Columns* : In the firſt, towards the Left-hand, is placed the Degrees of *Anomaly*, from 1 to 360, by every 10th Degree. And in the other Six *Columns* noted at the Head with ⊖, ♄, ♃, ♂, ♀, ☿, you have the Degrees, and 100 Parts of the *Ecliptick* which anſwer thereunto. So that if you ſeek the Degrees of *Anomaly* of any *Planet* (found as by the former *Table*) in the firſt *Column*, you ſhall find againſt thoſe Degrees of *Anomaly*, the *Sign*, *Degree*, and 100 *Part* of the *Ecliptick* anſwerable thereunto.

So in the foregoing *Example* of the *Earth* ⊖ and *Saturn* ♄, where the *Anomaly* of the *Earth* was found to be 328, and of *Saturn* 223 (neareſt)——Now, if you look for theſe *Numbers* in the firſt *Column* of this *Table*, you ſhall find, that againſt 330, (which is the neareſt Number to 328) in the *Earths Column*, ♐ 8.02 de. And that Degree of the *Ecliptick* anſwers to 330 deg. of the *Earths Anomaly* in its *Orbit*——Alſo, againſt 220 in the firſt *Column* (which is the neareſt to 223, the *Anomaly* of *Saturn*) you ſhall find in the *Column* for *Saturn*, ♌ 11.93 deg. And that Degree of the *Ecliptick* anſwers to 320 deg. of *Saturns Anomaly* in his *Orbit* : And ſo of any other.

But in the Uſe of this *Table* you are to *note*, That there being but every *Tenth Degree* of *Anomaly* in the firſt *Column*, the *Degrees* in the other *Columns* for the *Planets* give thoſe Signs and Degrees of the *Ecliptick*, of every *Tenth Degree* of *Anomaly* : And therefore you muſt make Proportion (which how to perform, we ſuppoſe to be known) and therefore in our Example.

					D. M.
The Anomaly of	the Earth	being	328	the Ecliptick	15.52 ♐
	Saturn		223	Degrees anſwering, are	15.02 ♌

CHAP.

CHAP. V.

The Uſe of the Theories, and by them to find the Places *of the* Earth *and* Planets, *both in* Longitude *and* Latitude, *&c. for any time propoſed. As for MARS and the EARTH, on the 18th of* October 1705.

THE *Orbits* of the EARTH and MARS being deſcribed in the *Theory*, as is before ſhewed how to do;

Firſt, Collect the equal *Anomalies* for the *Earth* and *Mars*, for the Time propoſed, and their *Reductions* to the *Ecliptick*, as is here done.

The Equal Anomalies *of the* Earth *and* Mars, *for the 18th Day of* October 1705 *at Noon.*

	⊖ Anomaly.	♂ Anomaly.
Æpoche for Years 1700	194. 26	218. 08
Years Compleat 4	359. 96	45. 59
Month of *September* Compleat	269. 07	143. 06
Days Compleat 17	16. 76	8. 91
The Sum	840. 05	415. 64
Circle Subſtr.	720	360.
The Equal Anomalies	120. 05	55. 64
Their Reduction to the Ecliptick	5. 12 ♉	17. 68 ♎

Secondly, The *Equal Anomaly* of the *Earth*, reduced to the *Ecliptick*, being in 5. 12 de. of *Taurus*, lay a Ruler to the Centre ⊙, and 5.12 deg. ♉, (as at *c*) and where it croſſeth the *Orbit* of the *Earth*, make ⊖, for that is the Place of the *Earth* in his *Orbit* at that time.

Thirdly, The *Equal Anomaly* of MARS, reduced to the *Ecliptick Circle*, being in 17.68 deg. of *Libra*: Lay a Ruler to the

Centre ☉, and 17.68 deg ♎ (as at *d*,) and where it croſſes the *Orbit* of MARS, make ♂; for that is the place of MARS in his *Orbit* at that time.

Fourthly, Draw the Right-Lines, ☉◓, ☉.♂ and ♂◓, conſtituting the Right-lined Triangle ☉ ♂ ◓.

The *Theory* thus finiſhed, I come now to ſhew,

I. To find the *Sun's true Place* in the *Ecliptick*, for the time propoſed.

Lay a Ruler upon the Line ☉ ◓, and it will cut the *Ecliptick Circle* in b ☉ b, in 5. 51 deg. of *Scorpio*; and that is the *Sun's Longitude* (or *Place* in the *Ecliptick*) at the time propoſed.

II. To find the *Longitude* (or true *Place* in the *Ecliptick*) of MARS for the ſame time.

Through the Point ☉ in the Centre, draw a Right-Line parallel to the Line ♂ ◓, as the ſhort Line *a* ♂ *a*, which will croſs the *Ecliptick Circle* in 25.00 deg of *Libra* ♎, where make the Character of MARS; for that is his true *Place* of *Longitude* in the *Ecliptick*, for the time propoſed.

Mark now the Triangle ☉ ♂ ◓, made upon the *Theory*, by the Centre ☉, and the Point of ◓ and ☉ in their *Orbits*: For in that Triangle,

The Side	☉ ♂	is the diſtance of	♂	from the	Sun.
	☉ ◓		◓		Sun.
	◓ ♂		♂		Earth.

All which are to be *Meaſured* upon *Proper Scales* for the ſeveral *Planets*. And

The Angle	♂ ☉ ◓	is the Angle at the	Sun	or of	*Commitation.*
	♂ ◓ ☉		Earth		*Elongation.*
	☉ ♂ ◓		Planet		*Paral. of the Orb.*

Theſe *Angles* may be Meaſured (very near the Truth) by help of your *Scale* of *Chords*; but more accurately (when the *Sides* are known) by *Trigonometrical Calculation*: of which hereafter.

But for the meaſuring of the *Sides*, *Proper Scales* muſt be made for each *Planet*, and the *Earth*. And, *Fig.* LXXVI.

The *Proportions* between the *Length* of the *Aphelial Lines* of the ſeveral *Planets*, (taken from the *Center* of the *Theories*, to the *Aphelial Points*) and the *Scales* by which to meaſure their *Diſtances* from the *Sun* or *Earth*; are theſe exhibited in this *Table*, viz.

			Parts.
For	Saturn	as 100 to	85. 63
	Jupiter		92. 87
	Mars		56. 73
	Earth		69. 38

Which Numbers will make *Proper Scales* for meaſuring the *Diſtances* in each *Theory*. And the *Scale* for the *Earth*, will ſerve, alſo, for the *Theories* of *Venus* and *Mercury*.

And note farther, That all *Diſtances* meaſured upon theſe *Proper Scales*, are

In	Saturn	to be *multiplied* by	400
	Jupiter		200
	Mars		100
	Venus		50
	Mercury		50

And theſe *Products* ſhall *Reduce* the *Diſtances* meaſured upon the *Proper Scales*, into *Semidiameters* of the *Earth*.

III. To find the *Diſtance* of MARS from the *Sun* and from the *Earth*, for the Time propoſed.

Take in your *Compaſſes* the length of the *Aphelial Line Ap.* ♂, ☉, and opening the *Sector* in the Points 56.73 and 56.73. The *Sector*, as ſo opened, ſhall be the *Scale* whereby to meaſure the *Diſtances* in the *Theory* of MARS. Wherefore, if you take in your Compaſſes (out of the Theory) of *Mars*.

Fig. **LXXVI.**

					Parts.
The Line	⊙ ♂	And apply it to	⊙ ♂	to contain	55. 50
	⊙ ◓	the *Sect.* ſo open'd,	⊙ ◓		34. 75
	◓ ♂	you ſhall find	◓ ♂		88. 20

Which Parts multiplied by 100, will give

For	⊙ ♂	3550	*Semidiameter* of the *Earth.*
	⊙ ◓	3475	
	◓ ♂	8820	

IV. To find the *Angle* of *Commination*, or *Angle* at the *Sun*) ♂ ⊙ ◓.

Lay a Ruler to ⊙ the Centre, and the Point ◓ in its *Orbit*; and it will cut the *Ecliptick Circle* in the Point *c*: Alſo lay a Ruler upon ⊙ the Centre, and the Point *Mars* in its *Orbit*, and it will cut the *Ecliptick* in the Point *d*; then will the Degrees of the *Ecliptick*, intercepted between *c* and *d*, be the quantity of the *Angle* ♂ ⊙ ◓, the *Angle* at the *Sun* (or of *Commination*) and will be found to be 162.00 deg.

V. To find the *Angles* of *Elongation* and *Parallax* of the *Orb* (or the *Angles* at the *Earth* and *Planet*) ⊙ ◓ ♂ and ⊙ ◓ ♂.

Theſe (as I ſaid before) may be meaſured upon the *Theory* by the aſſiſtance of the *Scale* of *Chords*: But the *Angle* of *Commination* and the *diſtance* of the *Earth* and *Mars* being known, they may more exactly be found by *Trigonometrical Calculation*: Thus;

As the Sum of the Sides ♂ ⊙ and ◓ ⊙.	9025 ——	3.954447
Is to their Difference 2075.		3.317018
So is the Tang. of half the Angles at ♂ and ◓, 9. 00 d.		9.199712
		12.516730
To the Tangent of 2,09 deg.		8.562283

Which added to 9.00 deg. gives 11.09 deg. for the *Angle* at the *Earth* (or of *Elongation*) ⊙ ◓ ♂ —— And ſubſtracted from 9.00 deg. leaves 6.91 deg. for the *Angle* at *Mars* (or of the *Parallax* of the *Orb*.

Fig.
LXXVI.

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Fig.
LXXVI.

Fig. LXXVI.

VI. To find the Latitude of MARS at the ſame time.

Before the *Latitude* can be found, the *Argument of Latitude* muſt be had: To the attaining whereof, Lay a Ruler to the Centre ☉, and the Point ☊ in the *Planets Orbit*; ſo will the Ruler cut the *Ecliptick* in the Point *e*: And a Ruler laid from the Centre ☉ to ♂, will cut the Ecliptick in the Point *d*: So that the *Angle* *d* ☉ *e* is the *Argument of Latitude*; and in this our *Example* will be found (by meaſuring the Degrees of the *Ecliptick Circle* between *d* and *e*) to be 149.75 deg.

The *Argument of Latitude* thus attained, the true *Latitude* may be found by the following Canon.

As the Radius, Sine 90 de.	10.
Is to the the Sine of the *Arg. of Latit.* (149.75 d.)	9.7022357
So is the Tang. of *Mars Greateſt Incl.* 1.84 de.	8.5068445
To the Tangent of 0.92 deg.	8.2090802

Which 0.92 Degrees is the *Latitude* of MARS ſeen at the *Sun*; and is *North*, becauſe the *Inclination* of MARS was *Northward*.

But to find the *Latitude* as ſeen at the *Earth*,

The Proportion is,

As the *Diſtance* ♂ ☉, 8820	3.945468
Is to the *Diſtance* ☉ ♂, 5550	3.744293
So is the Tang. of the *Latit.* ſeen at the *Sun* 0.92 de.	8.205702
	11.949995
To the Tangent of 0.58 deg.	8.004527

Which is the *Latitude* of *Mars* ſeen from the *Earth*.

The

Figure LXXVII.

The Equal Motions of the Anomalies of the Earth, and the other Five Planets, ♄, ♃, ♂, ♀, ☿, For the 18th Day of *October*, at Noon, *Anno* 1705.

	⊖	♄	♃	♂	♀	☿
Æpocha for Years 1700	194:26	83:88	128:88	218:08	248:42	245:58
Motion in Years Compleat 4	359:96	48:86	121:40	45:59	180:69	218:86
September Compleat	269.07	9:13	22:68	143:06	77:38	37:20
Days Compleat 17	16:79	0:57	1:41	8:91	27,24	69:57
The Sums	840:08	142:44	274:37	415:64	533:73	571:21
Circles Subſt.	720:			360:	360:	360:
The Equal Anomalies	120:08	142:44	274:37	55:64	173:73	211:21
Their Reduction to the Eclipt.	5:12 ♉	15:19 ♉	17:70 ♋	17:83 ♎	26:41 ♋	16:10 ♌
The Planets Longitudes	6:00 ♓	17:00 ♌	29:00 ♋	25:00 ♎	27:00 ♍	17:50 ♎
Diſtances from the Sun	44:30	76:25	88:00	55:50	51:08	24:50
Earth		67:51	86:00	88:20	78:03	77:02
Latitude		2:56 S	0:25 N	0:57 N	1:42 N	2:06

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